Cosmic Large-scale Structure Formations

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1. Gaussian Random Field/ Power spectrum/Correlation function/ Phase
2. BAO
3. Galaxy Clustering
4. RSD
5. Lensing: WL/ Strong Lensing
6. Linear Growth
7. Nonlinear growth (spherical collapse)
8. Halo model: Press-Schesther formalism, merge tree

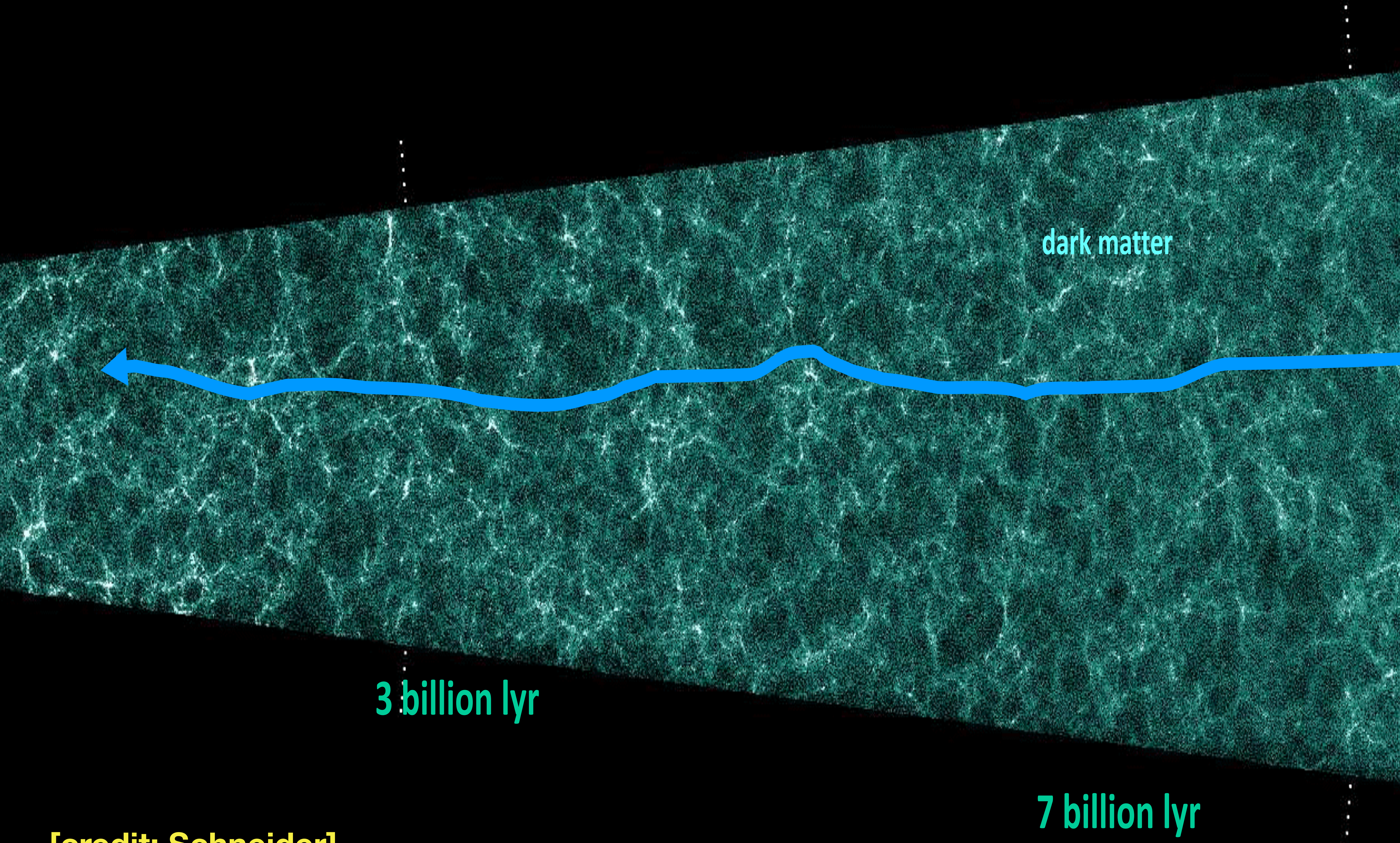
Strong Lensing

given by Dr. Tao YANG

General picture of WL

1. shear field
2. convergence field (magnification/number count)

mass structure vs cosmic time



[credit: Schneider]

Weak lensing power spectrum

The lensing convergence is related to the projected Laplacian of the projected gravitational potential (note factor of 2 difference from 3D Poisson equation),

$$\kappa = \frac{1}{2}\nabla^2\psi \qquad \delta(\chi'\theta, \chi') \equiv \nabla_\perp^2 \Phi(\chi'\theta, \chi')$$

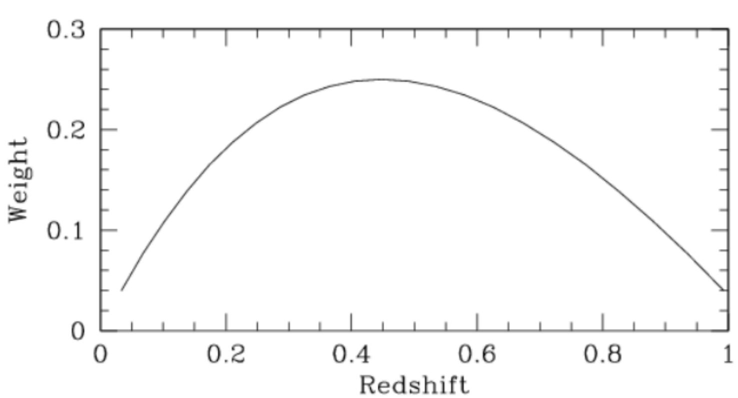
$$\kappa(\vec{\theta}) = \frac{3H_0^2\Omega_m}{2c^2}\int_0^{\chi_\infty} \frac{\chi\, d\chi}{a(\chi)} g(\chi)\delta(\chi\vec{\theta}, \chi)$$

The convergence is a weighted projection of the 3D cosmological mass density perturbations along the line-of-sight. We call the weighting function the **lensing efficiency**,

Source redshift distribution

$$g(\chi) \equiv \int_\chi^{\chi_\infty} d\chi' \frac{\chi' - \chi}{\chi'} n(\chi')$$

No source clustering



The lensing efficiency is very broad and most sensitive to mass mid-way between observer and source.

Credit: M. White

The convergence power spectrum is defined as,

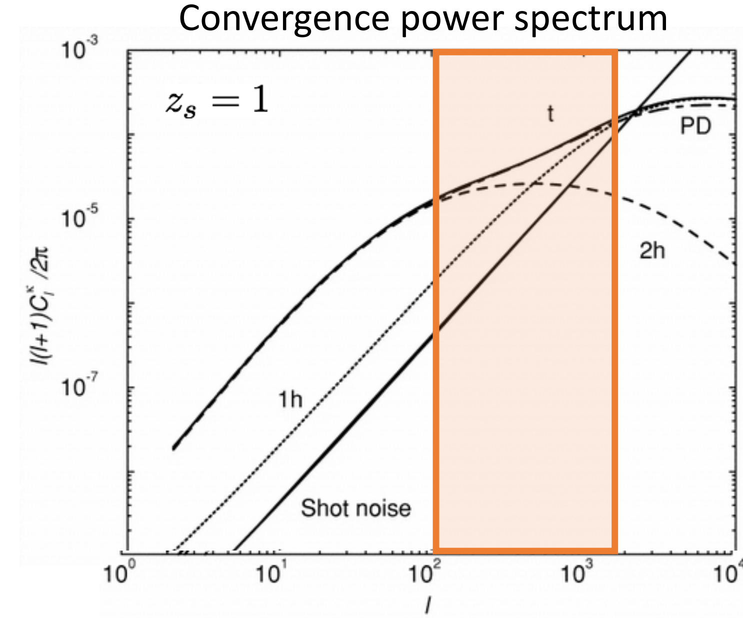
$$\langle \tilde{\kappa}(\ell)\tilde{\kappa}^*(\ell')\rangle = (2\pi)^2\delta_D(\ell - \ell')P_\kappa(\ell)$$

And is related to the 3D mass power spectrum as,

$$P_\kappa(\ell) = \frac{9}{4}\Omega_m^2\left(\frac{H_0}{c}\right)^4 \int_0^{\chi_\infty} d\chi\, \frac{g^2(\chi)}{a^2(\chi)} P_\delta\left(k = \frac{\ell}{\chi}, \chi\right)$$

Limber approximation

Convergence power spectrum

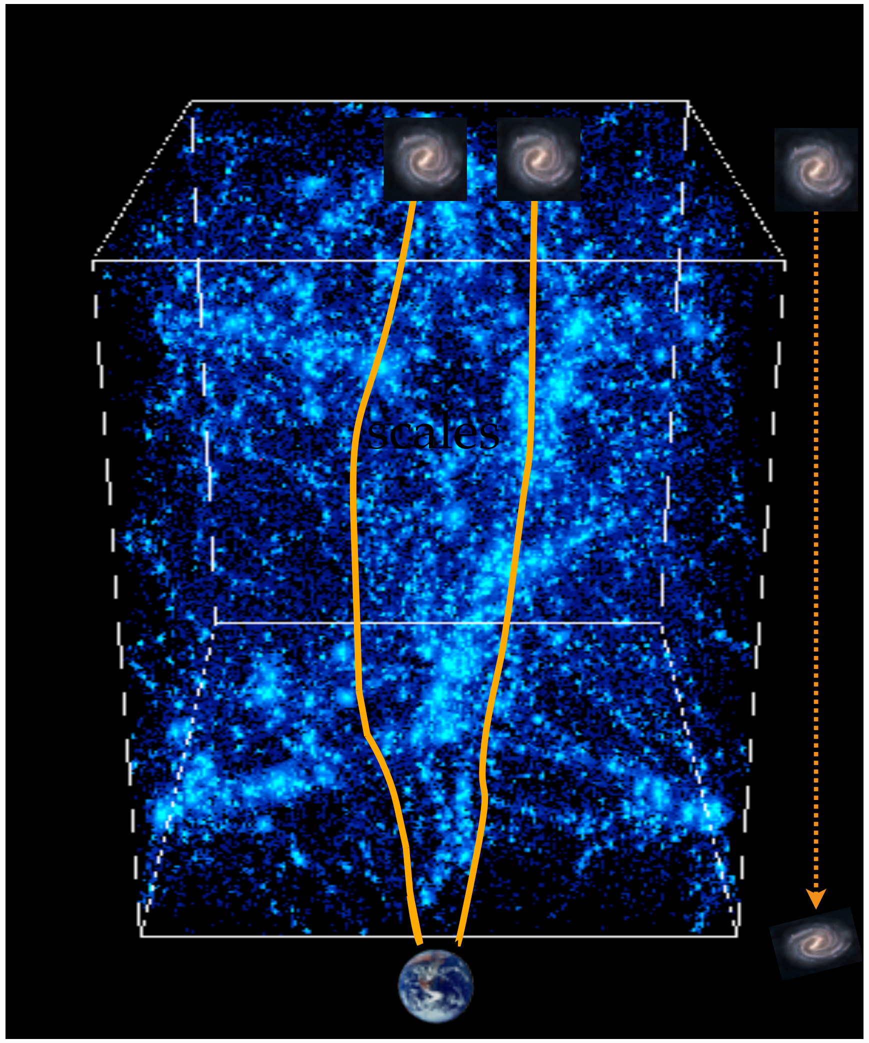


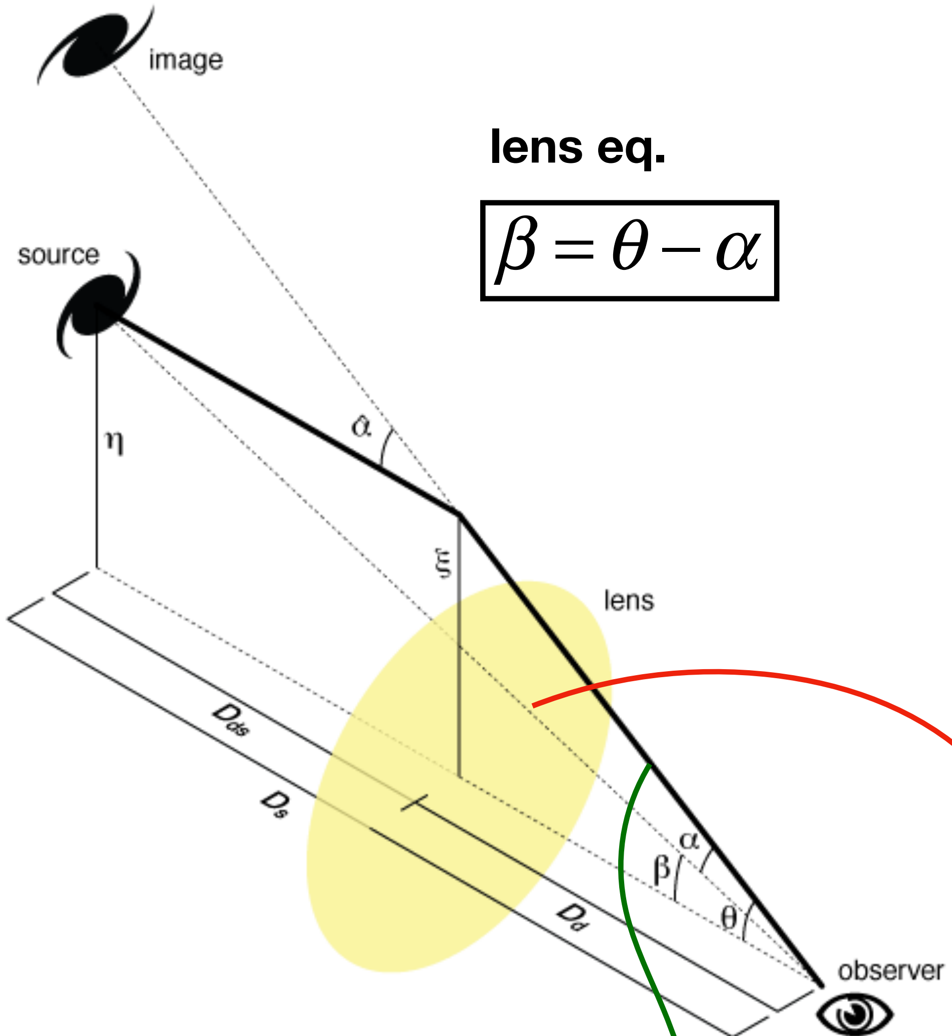
Cooray & Hu (2001)

Cosmic shear, or weak cosmological lensing

Light of distant galaxies is deflected while travelling through inhomogeneous Universe. Information about mass distributions is imprinted on observed galaxy images.

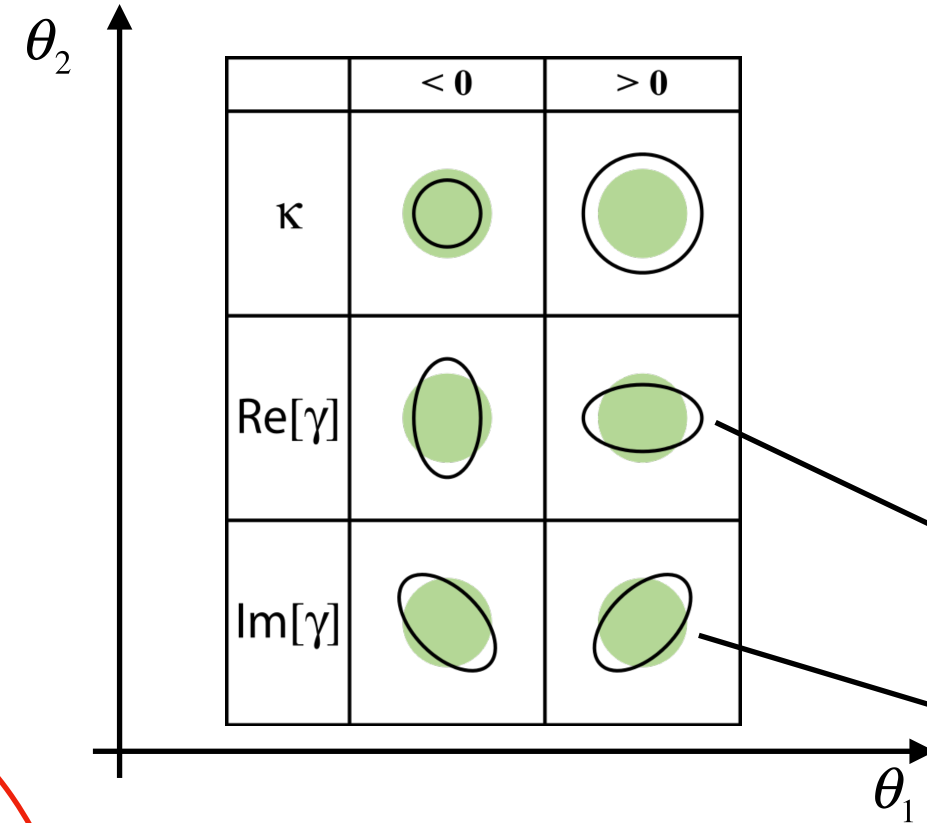
- Continuous deflection: sensitive to projected 2D mass distribution.
- Differential deflection: magnification, distortions of images.
- Small distortions, few percent change of images: need statistical measurement.
- Coherent distortions: measure correlations, scales few Mpc ... few 100 Mpc.





lens eq.

$$\boxed{\beta = \theta - \alpha}$$



$$\kappa = \psi_{,11} + \psi_{,22}$$

$$\gamma_1 = \psi_{,11} - \psi_{,22}$$

$$\gamma_2 = \psi_{,12}$$

e.g.

$$\psi(\theta_1, \theta_2) = \theta_1^{\,2} + \frac{1}{2}\theta_2^{\,2}$$

$$\psi \sim \theta_1 \bullet \theta_2 \Rightarrow \gamma_2$$

photon intensity matrix (2x2)

$$A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j}$$

$$\boldsymbol{\alpha} = \frac{2}{c^2} \int_0^{\chi} \mathrm{d}\chi' \frac{\chi - \chi'}{\chi} \left[\boldsymbol{\nabla}_{\perp} \Phi(\boldsymbol{x}(\chi'), \chi') - \boldsymbol{\nabla}_{\perp} \Phi^{(0)}(\chi') \right]$$

Jacobi (symmetric) matrix

$$A = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$

- convergence κ: isotropic magnification
- shear γ: anisotropic stretching

Born app

background geometry

$$\psi(\boldsymbol{\theta}, \chi) = \frac{2}{c^2} \int_0^{\chi} \mathrm{d}\chi' \frac{\chi - \chi'}{\chi \chi'} \phi(\chi' \boldsymbol{\theta}, \chi')$$

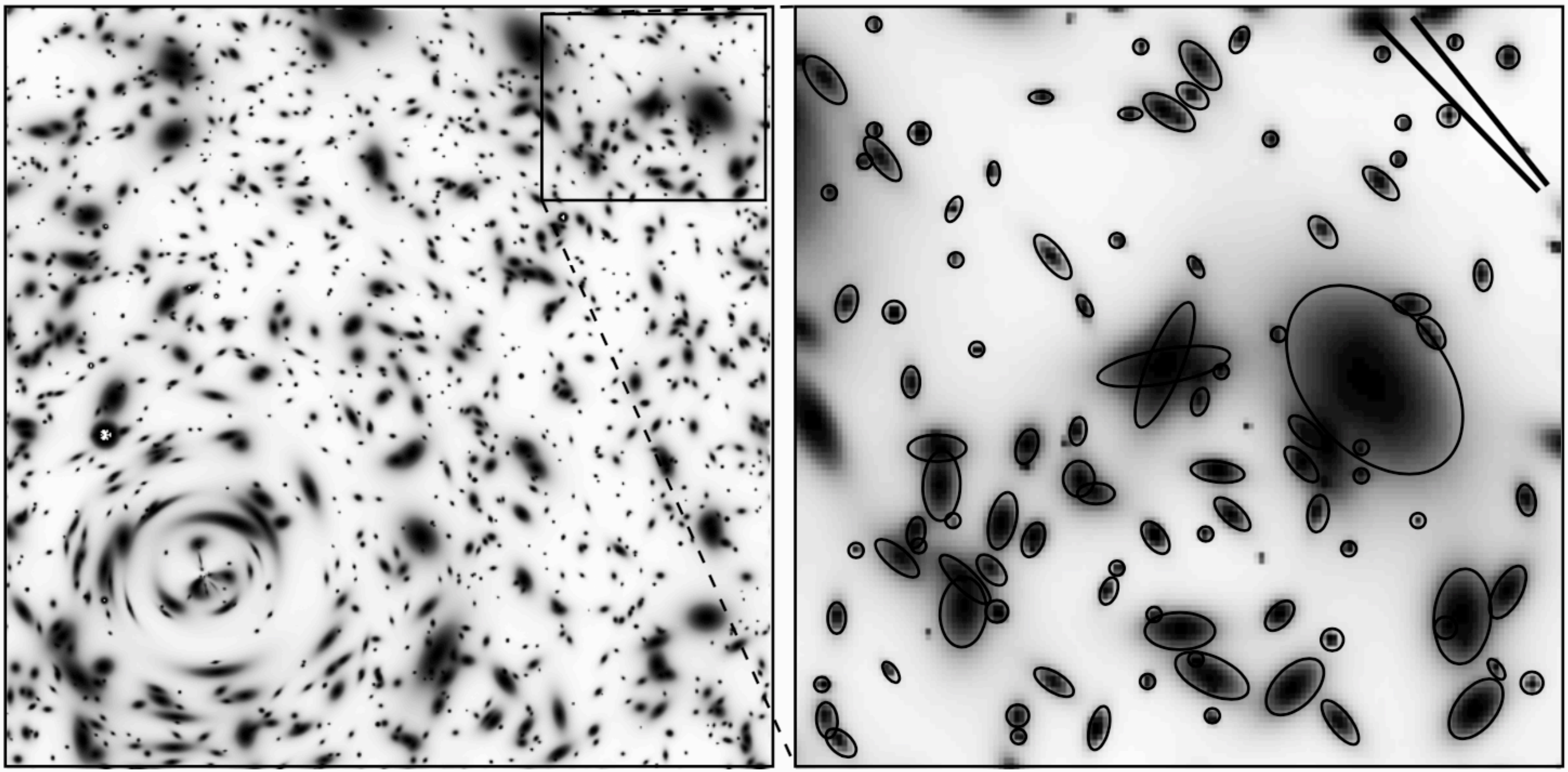
linear pert.

validation: a few Mpc~100Mpc

Galaxy Lensing

1. Shear measurement
2. number counts

Ellipticity and local shear



[from Y. Mellier]

Galaxy ellipticities are an estimator of the local shear.

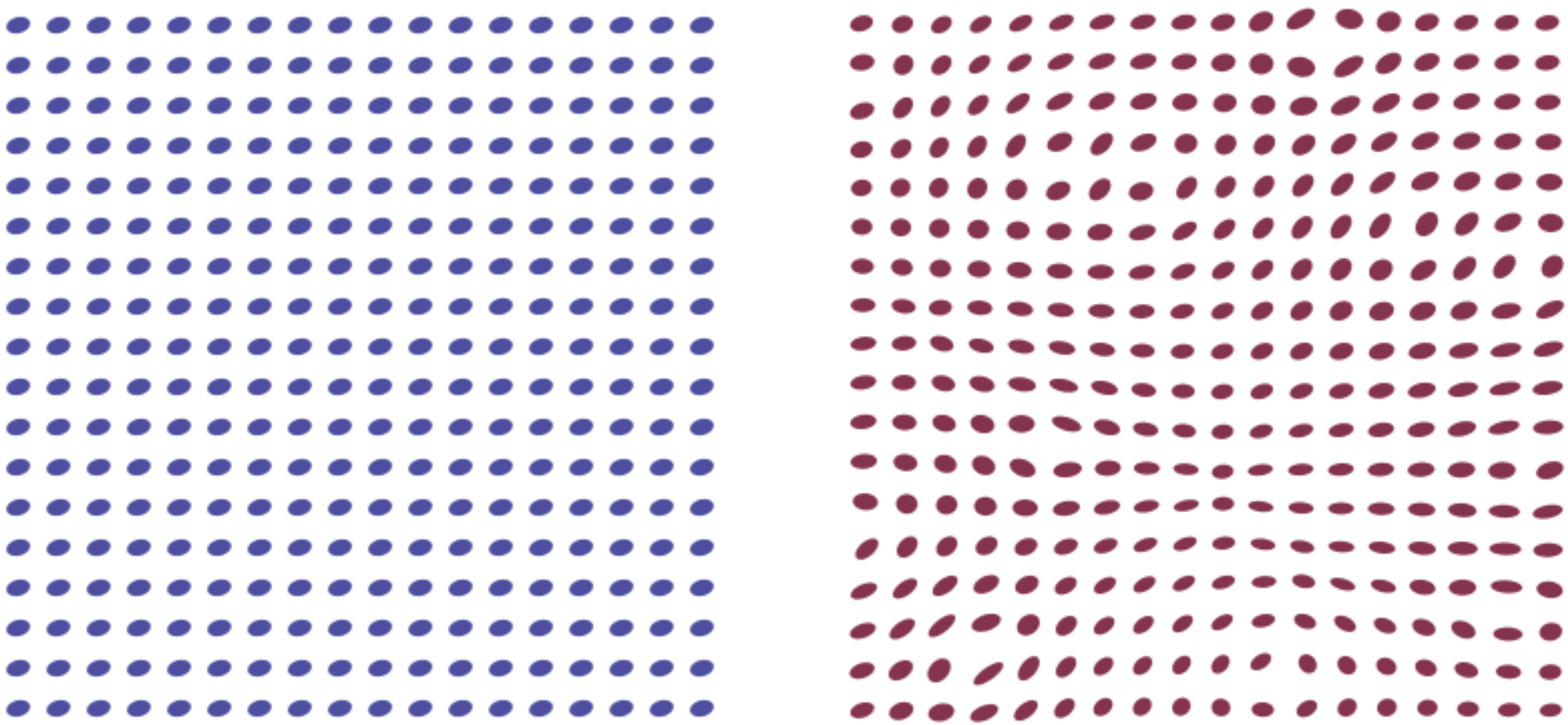
Convergence and shear III

Further consequence of lensing: magnification.
Liuville (surface brightness is conserved) + area changes ($d\beta^2 \neq d\theta^2$ in general) $\rightarrow$ flux changes.

$$\text{magnification} \quad \mu = \det A^{-1} = [(1-\kappa)^2 - \gamma^2]^{-1}.$$

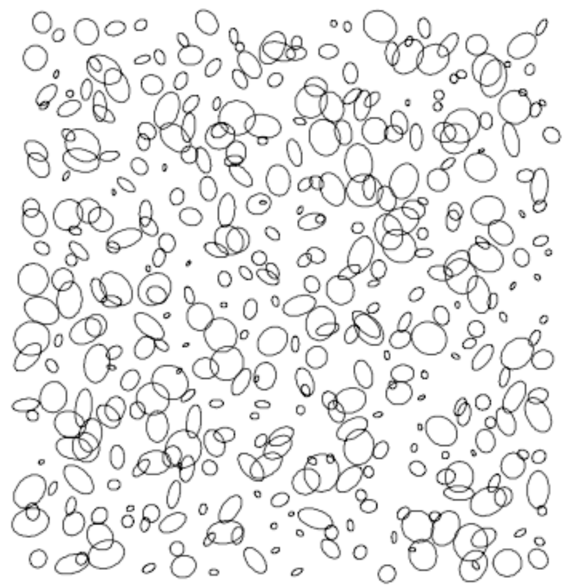
Summary: Convergence and shear linearly encompass information about projected mass distribution (lensing potential ψ). They quantify how lensed images are magnified, enlarged, and stretched. These are the main observables in (weak) lensing.

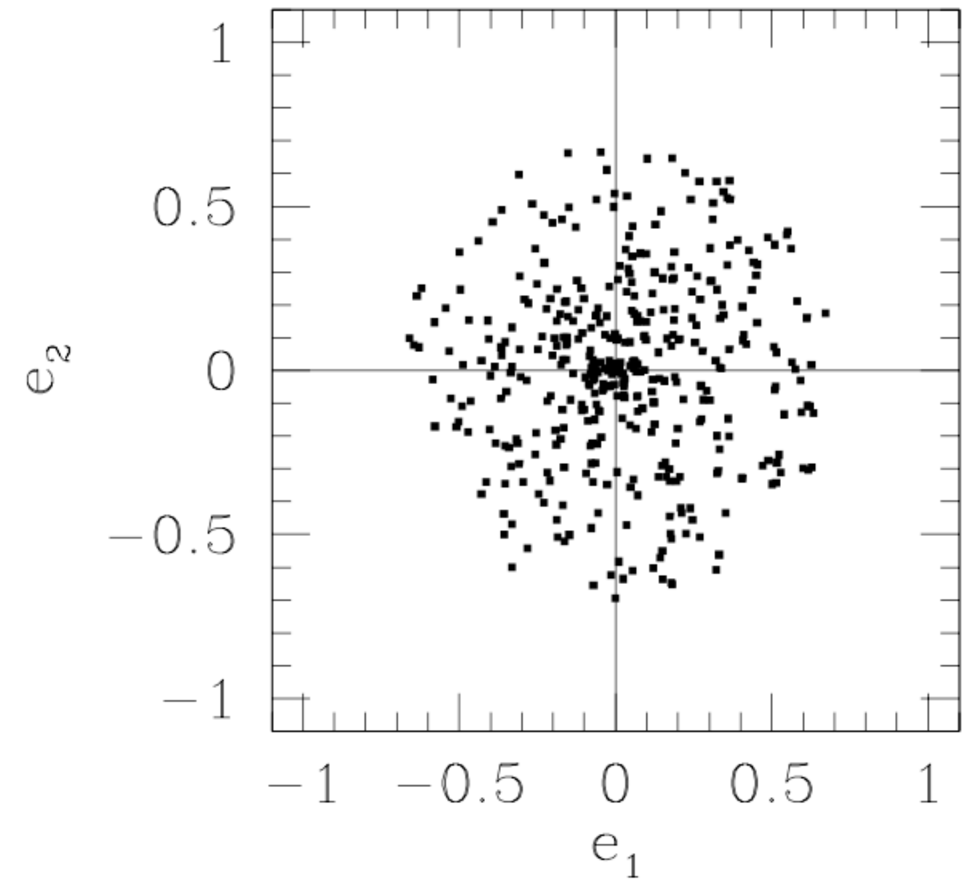
Coherently distorted!



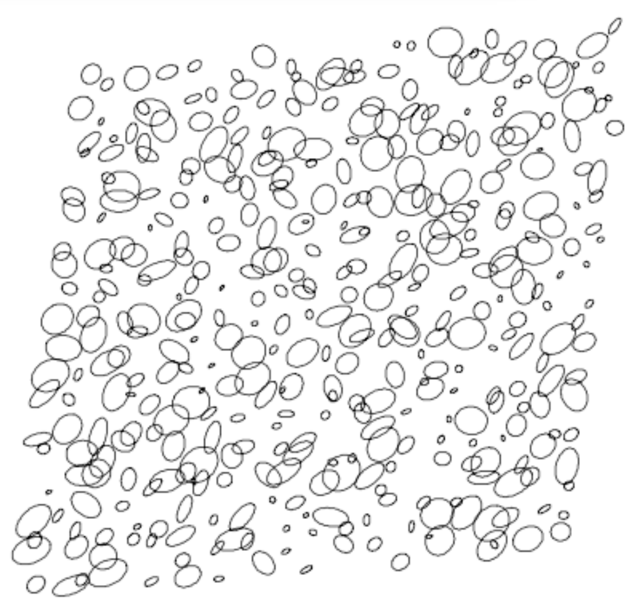
Unlensed sources

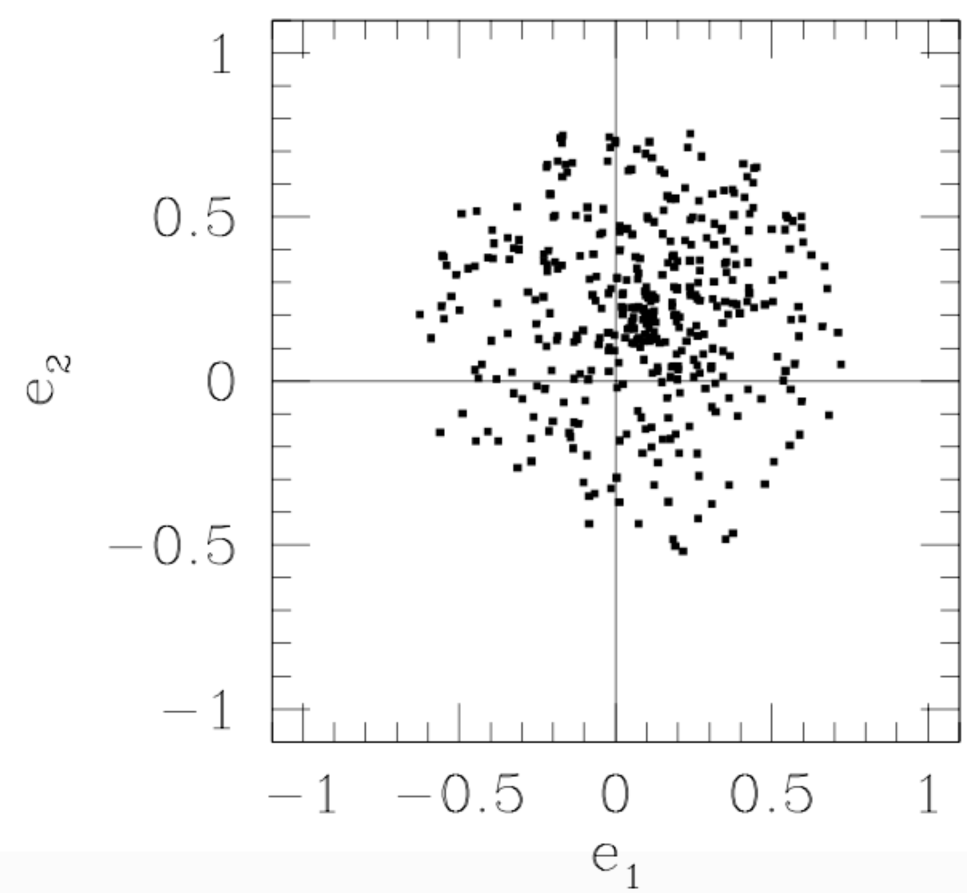
Weak lensing



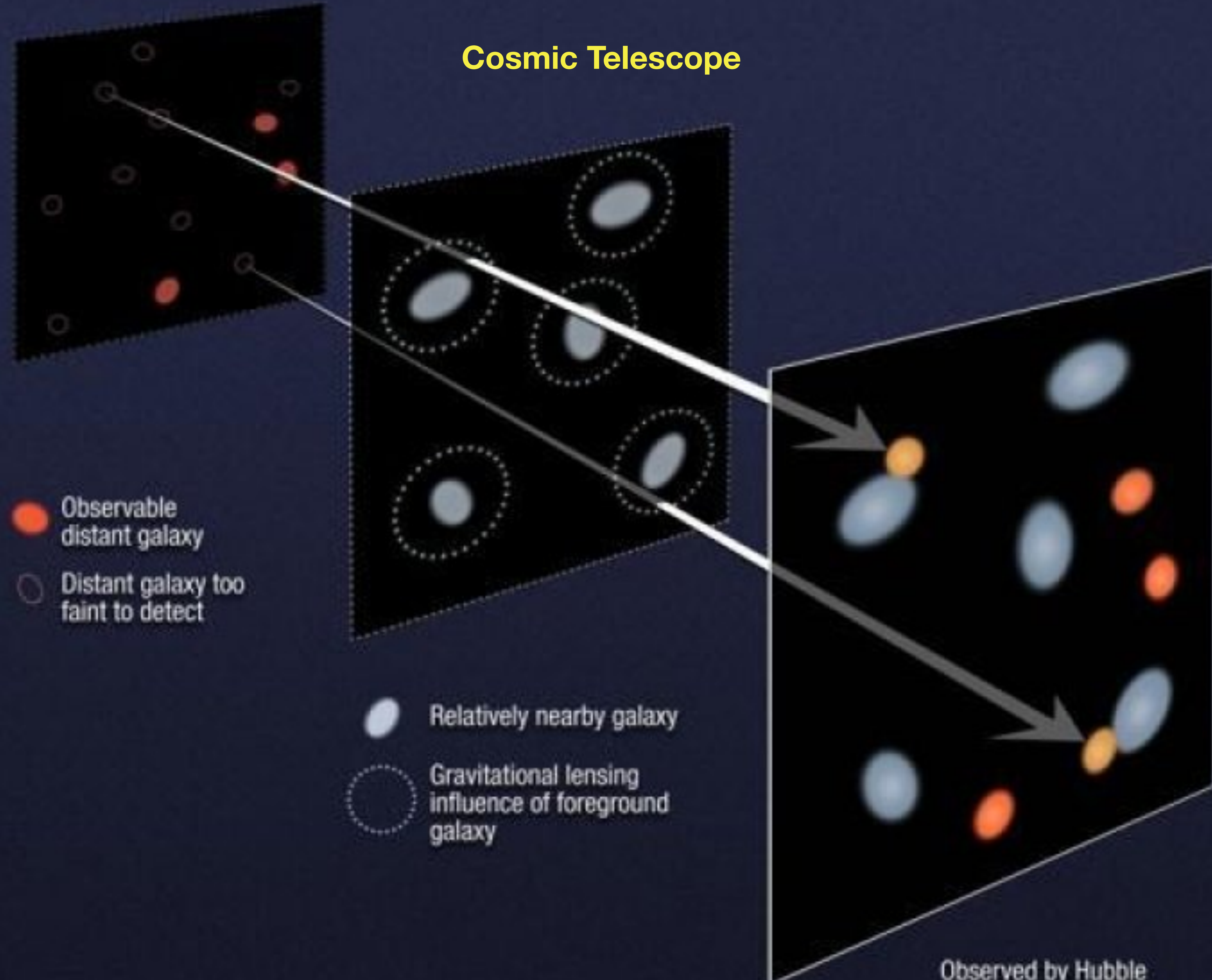


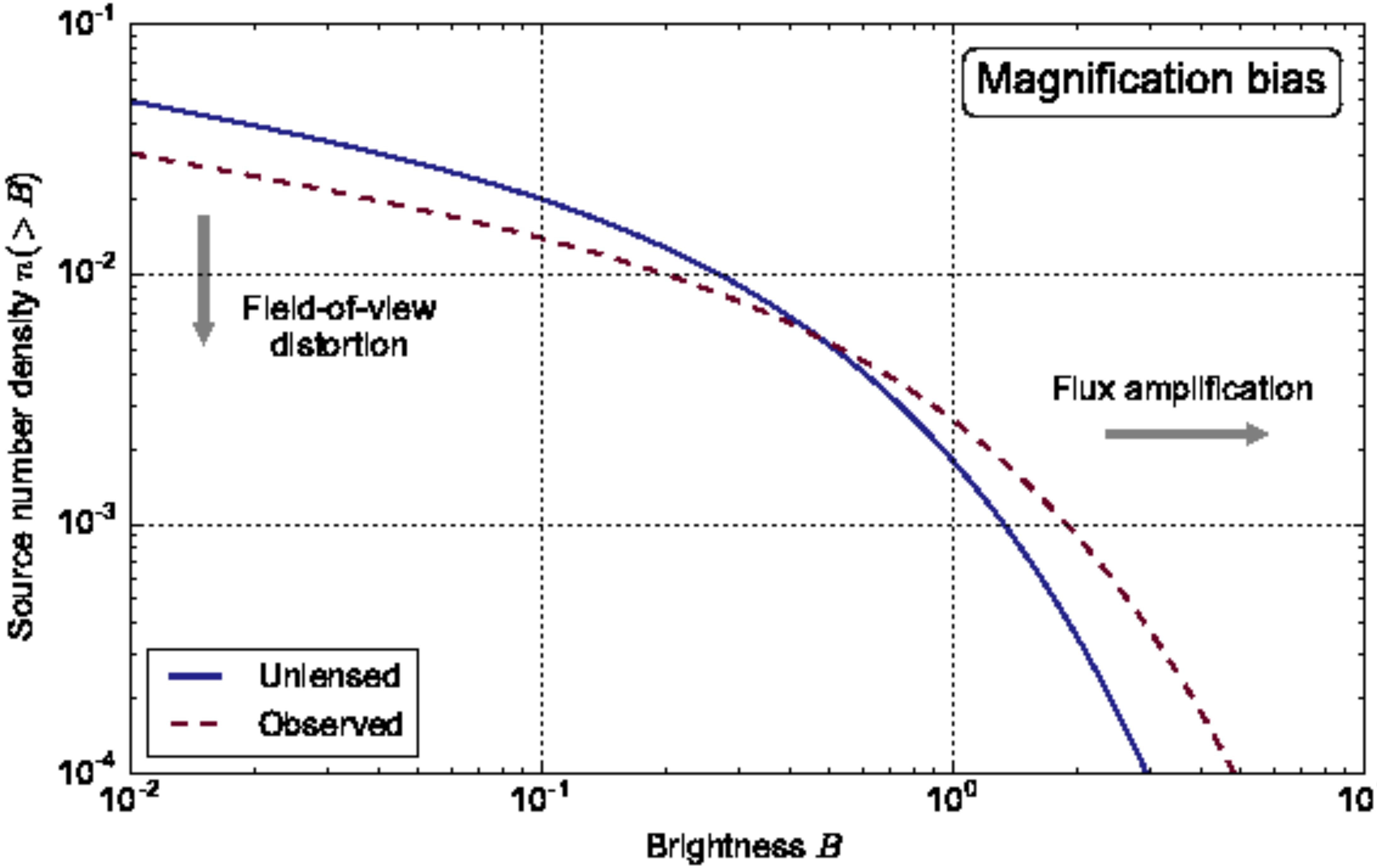
$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{1}{I_{11} + I_{22}} \begin{pmatrix} I_{11} - I_{22} \\ 2I_{12} \end{pmatrix}$$





Cosmic Telescope

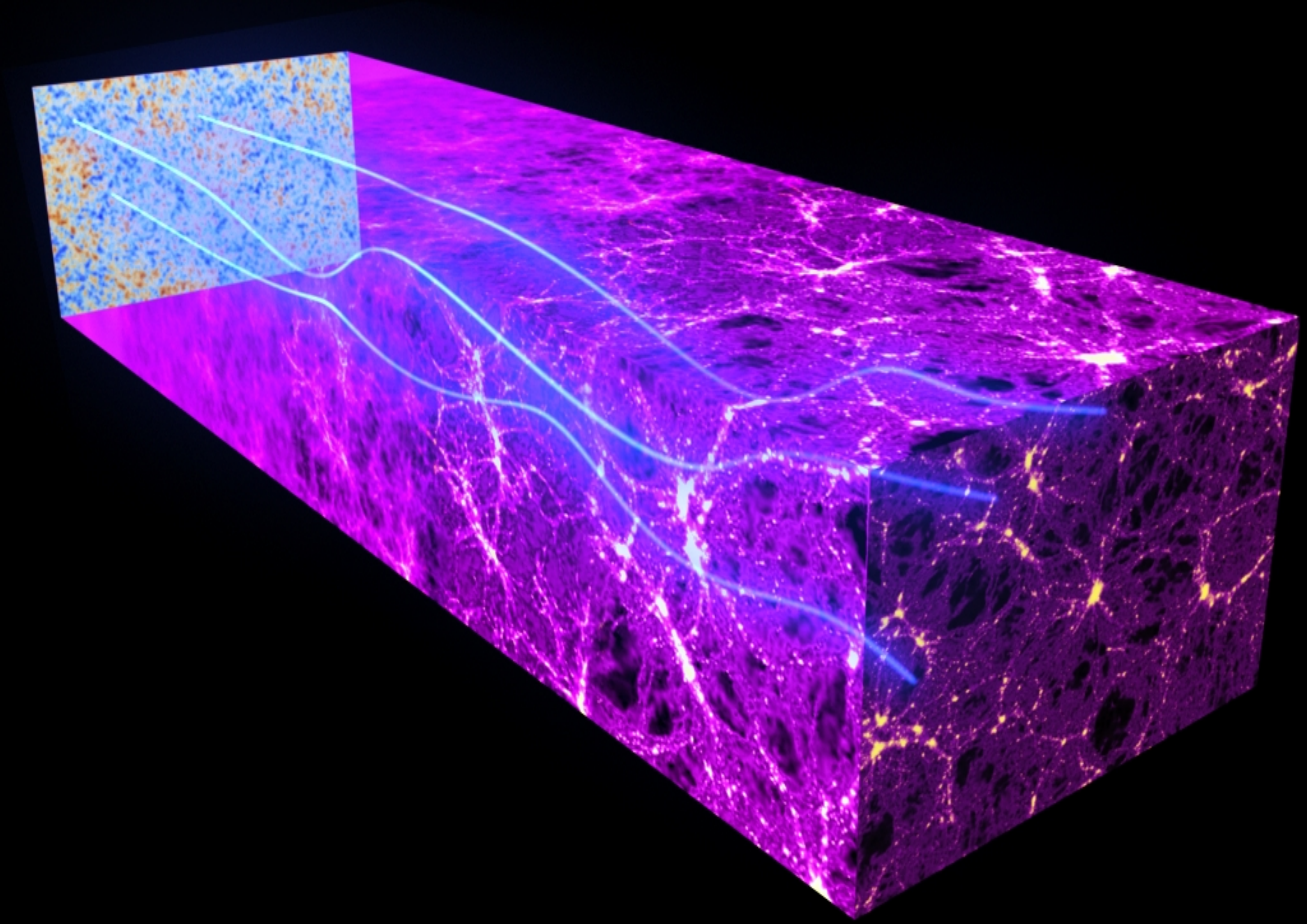




Magnification bias
Field-of-view distortion
Flux amplification
Unlensed
Observed
Source number density $n(>B)$
Brightness B
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-2}
10^{-1}
10^{0}
10^{1}

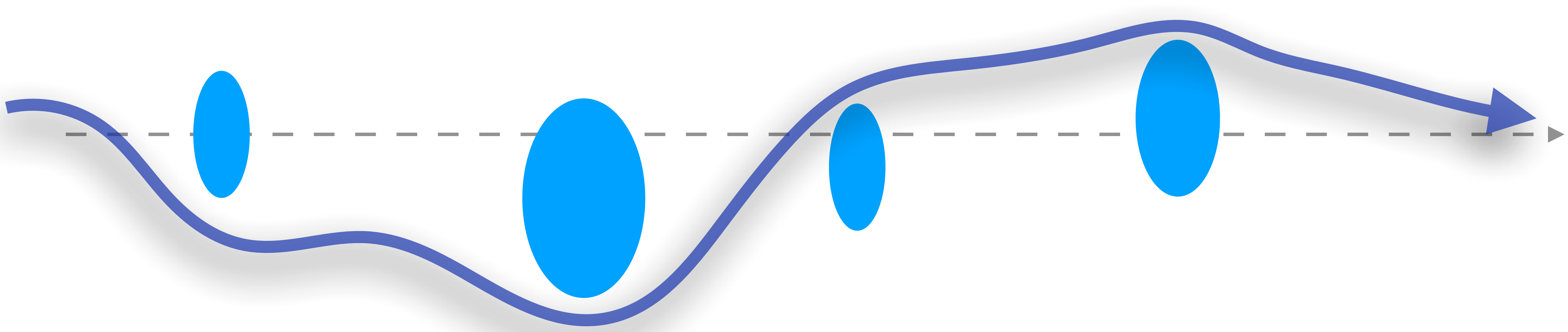
CMB Lensing

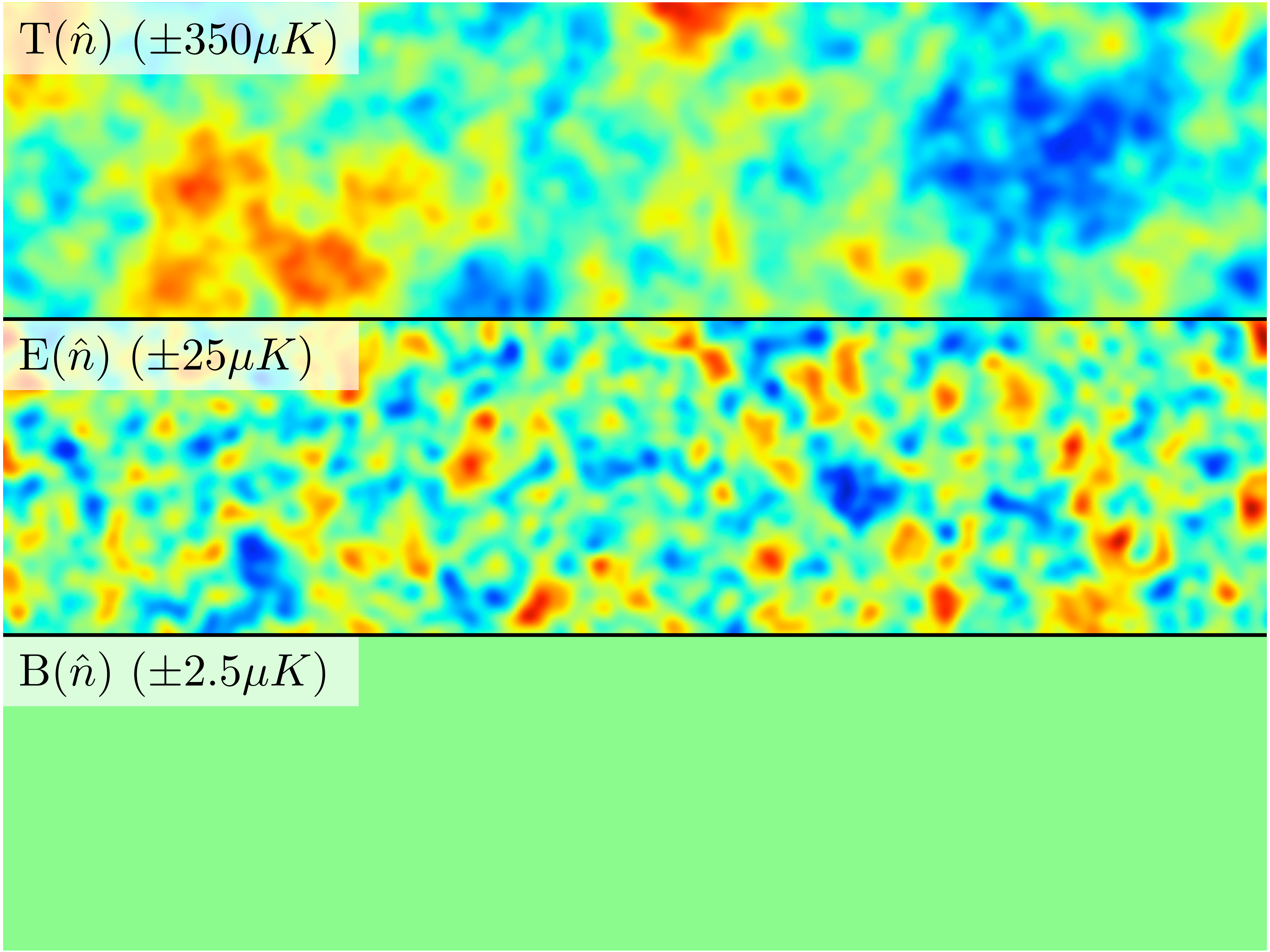
1. convergence measurement
2. For CMB, we don't measure the shear field (?)



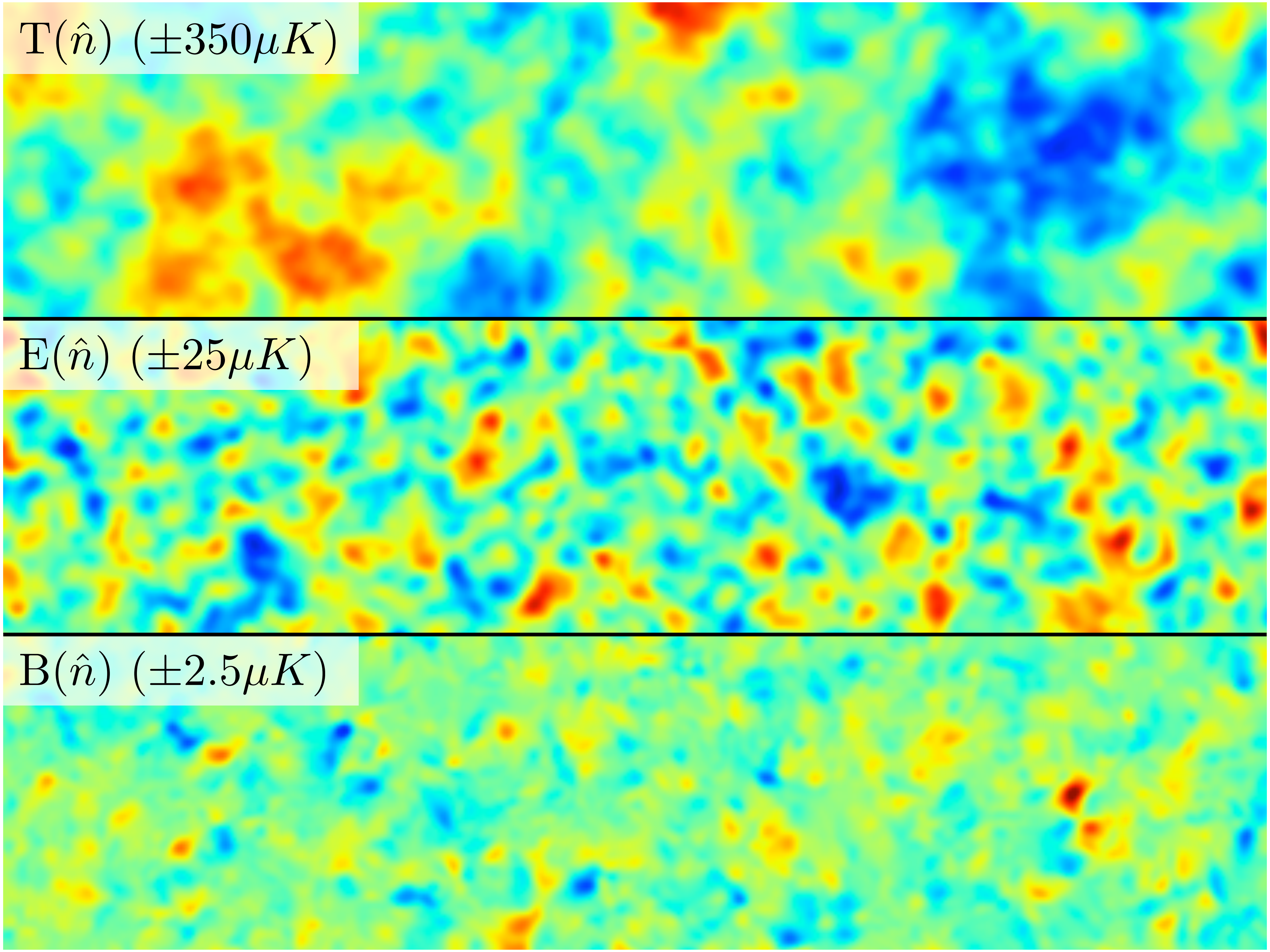
[Pb]

where in matter domination the potentials due to these perturbations are constant in the linear regime. The depth of the potentials is $\sim 2 \times 10^{-5}$, so we might expect each potential encountered to give a deflection $\delta\beta \sim 10^{-4}$. The characteristic size of potential wells given by the scale of the peak of the matter power spectrum is $\sim$ 300Mpc (comoving), and the distance to last scattering is about 14000Mpc, so the number passed through is ~ 50. If the potentials are uncorrelated this would give an r.m.s. total deflection $\sim 50^{1/2} \times 10^{-4} \sim 7 \times 10^{-4}$, corresponding to about $\sim$ 2 arcminutes. We might therefore expect the lensing to become an order unity effect on the CMB at $l \gtrsim 3000$. In fact the unlensed CMB has very little power on



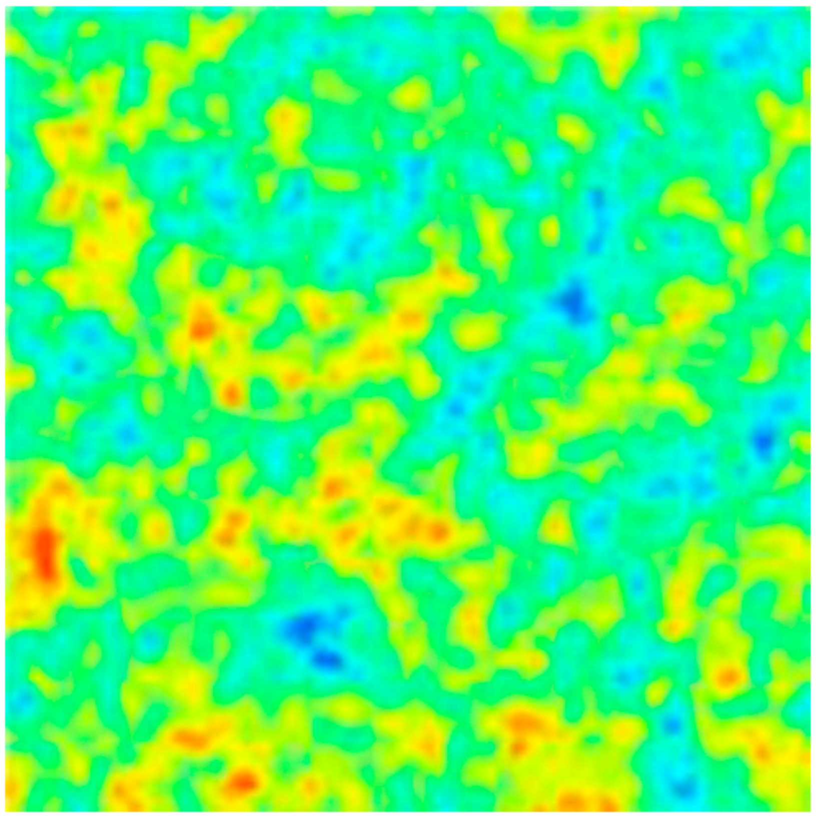


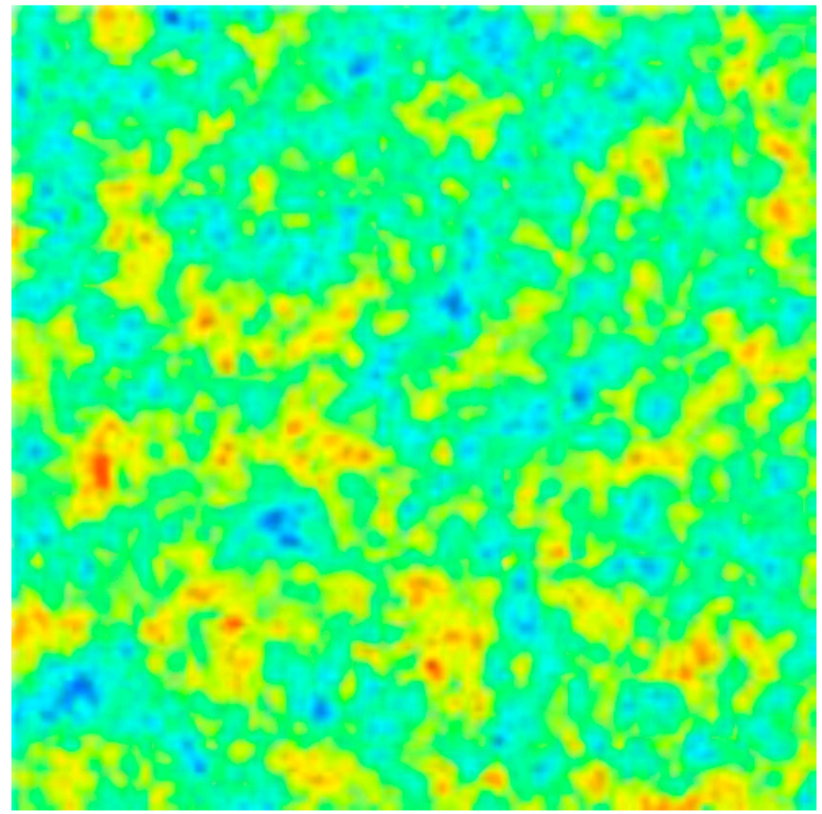
[credit: Lewis]

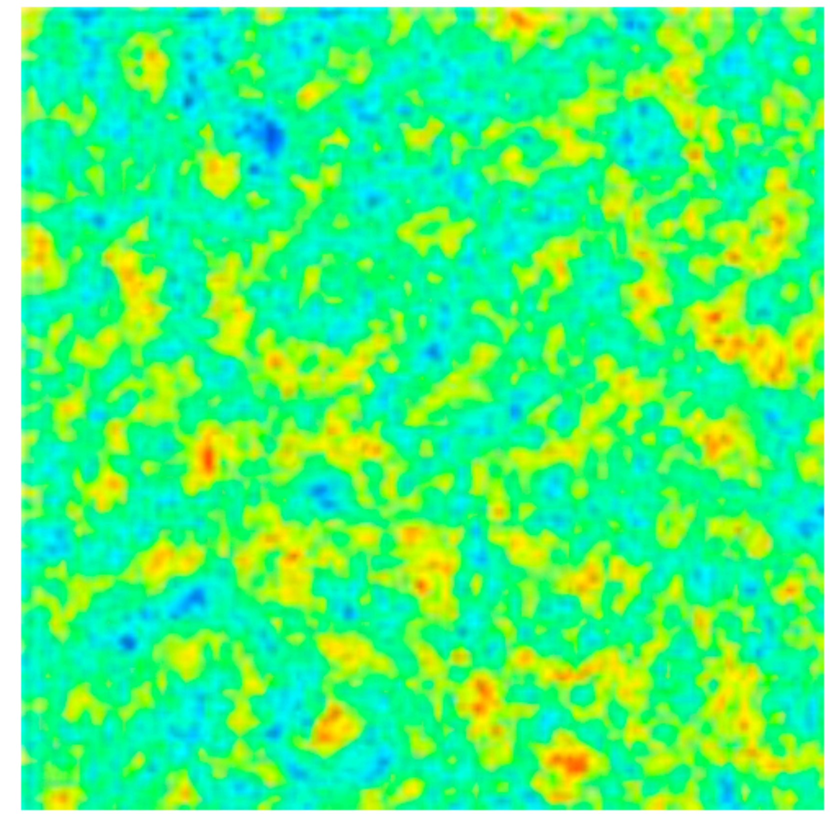


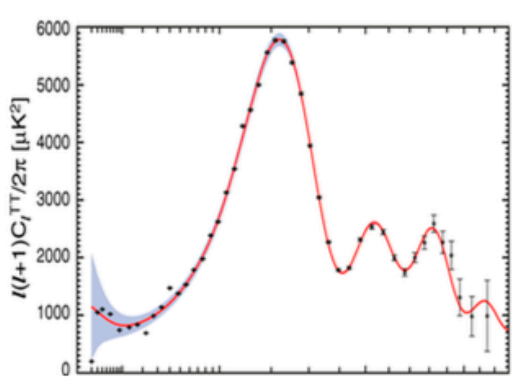
[credit: Lewis]

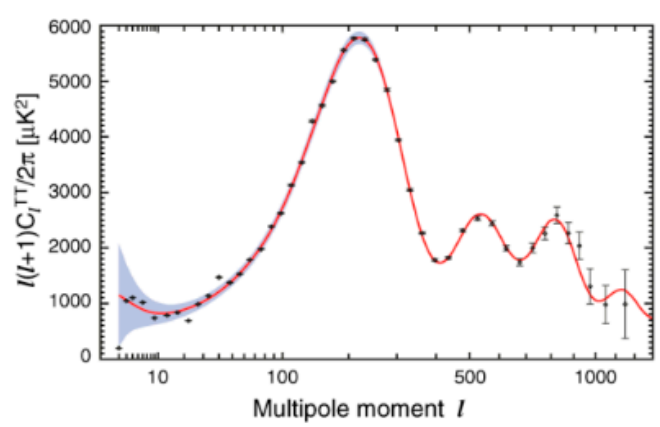
Magnification | Unlensed | Demagnification

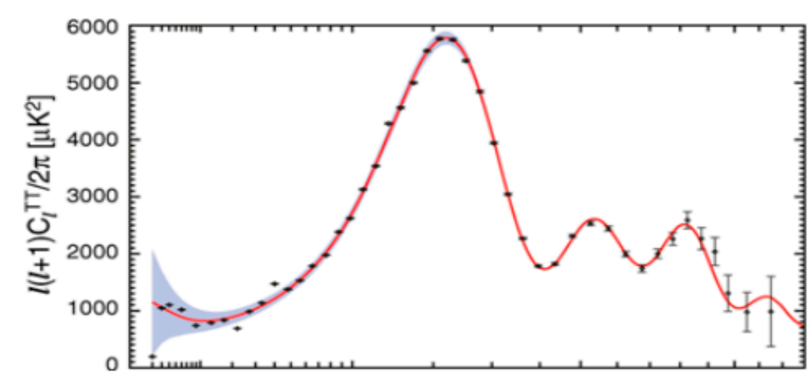








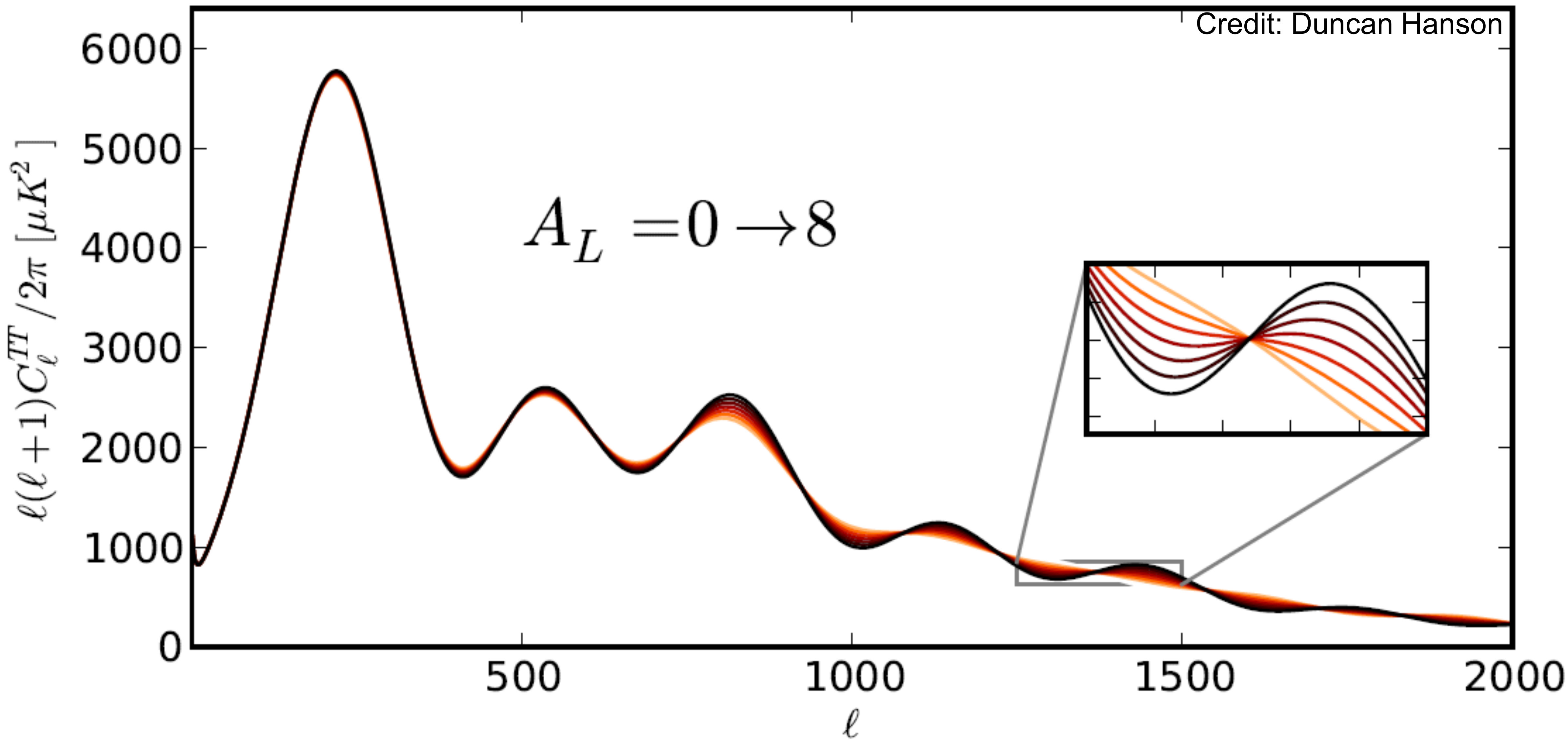




声学峰的位置发生平移！

[credit: Lewis]

Averaged over the sky, lensing smooths out the power spectrum



[credit: Lewis]

CMB Lensing: coupling the light bundles from different direction!

$$\begin{aligned}\tilde{\Theta}(\mathbf{x}) &= \Theta(\mathbf{x}') = \Theta(\mathbf{x} + \boldsymbol{\nabla}\psi) \\ &\approx \Theta(\mathbf{x}) + \nabla^a\psi(\mathbf{x})\nabla_a\Theta(\mathbf{x}) + \tfrac{1}{2}\nabla^a\psi(\mathbf{x})\nabla^b\psi(\mathbf{x})\nabla_a\nabla_b\Theta(\mathbf{x}) + \ldots\end{aligned}$$

$$\boldsymbol{\nabla}\psi(\mathbf{x}) = i\int\frac{\mathrm{d}^2\mathbf{l}}{2\pi}\,\mathbf{l}\psi(\mathbf{l})e^{i\mathbf{l}\cdot\mathbf{x}}, \qquad \boldsymbol{\nabla}\Theta(\mathbf{x}) = i\int\frac{\mathrm{d}^2\mathbf{l}}{2\pi}\,\mathbf{l}\Theta(\mathbf{l})e^{i\mathbf{l}\cdot\mathbf{x}}.$$

Taking the Fourier transform of $\tilde{\Theta}(\mathbf{x})$ and substituting we get the Fourier comp
second order in ψ

$$\begin{aligned}\tilde{\Theta}(\mathbf{l}) \approx \Theta(\mathbf{l}) &- \int\frac{\mathrm{d}^2\mathbf{l}'}{2\pi}\,\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\psi(\mathbf{l}-\mathbf{l}')\Theta(\mathbf{l}') \\ &- \frac{1}{2}\int\frac{\mathrm{d}^2\mathbf{l}_1}{2\pi}\int\frac{\mathrm{d}^2\mathbf{l}_2}{2\pi}\,\mathbf{l}_1\cdot[\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}]\,\mathbf{l}_1\cdot\mathbf{l}_2\Theta(\mathbf{l}_1)\psi(\mathbf{l}_2)\psi^*(\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}).\end{aligned}$$

Idea of reconstruction: using the mode-coupling!

$$< \tilde{\Theta}(\mathbf{l_1})\tilde{\Theta}(\mathbf{l_2}) > \neq 0 \qquad \text{for} \qquad \mathbf{l_1} \neq \mathbf{l_2}$$

do some calculation:

1. different primary CMB map lensed by A fixed lensing field

2. estimate the 1pt function of the lensing potential

3. calculate the 2pt function of the lensing potential
(understand the noise nature of the lensing potential)