Cosmic Large-scale Structure Formations

Bin HU

bhu@bnu.edu.cn

Astro@BNU

Office: 京师大厦9907

18 weeks

outline

Background (1 w)

- universe geometry and matter components (1 hr)
- Standard candle (SNIa) (0.5 hr)
- Standard ruler (BAO) (0.5 hr)

Linear perturbation (9 w)

- relativistic treatment perturbation (2 hr)
- primordial power spectrum (2 hr)
- linear growth rate (2 hr)
- galaxy 2-pt correlation function (2 hr)
- Baryon Acoustic Oscillation (BAO) (2 hr)
- Redshift Space Distortion (RSD) (2 hr)
- Weak Lensing (2 hr)
- Einstein-Boltzmann codes (2 hr)

Non-linear perturbation (6 w)

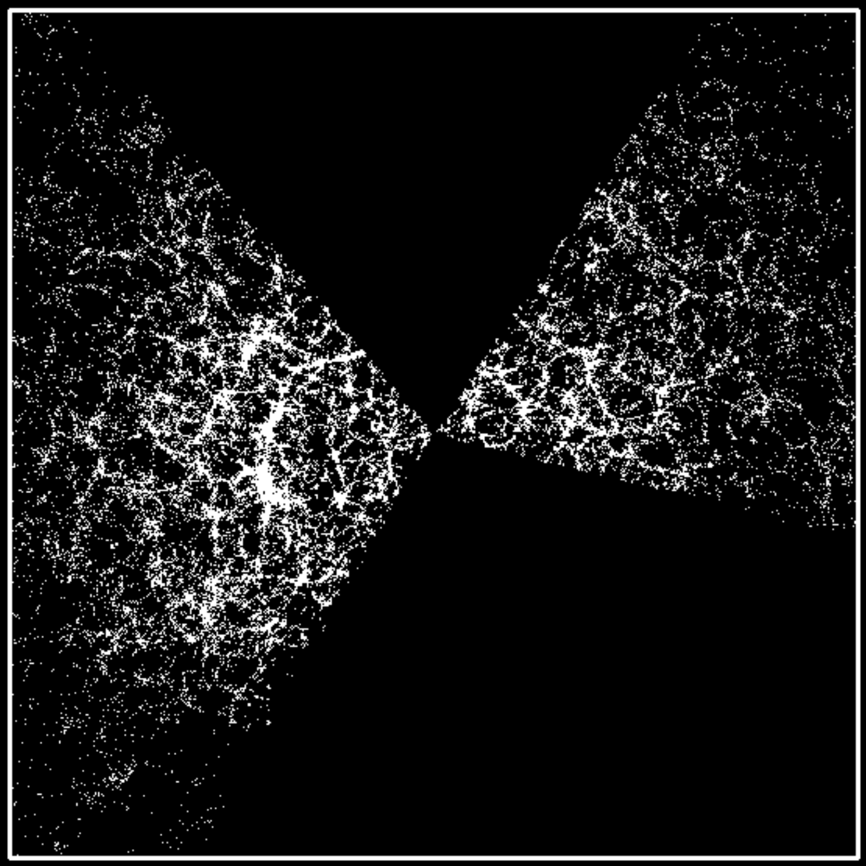
- Non-linear power spectrum (2 hr)
- halo model (2 hr)
- N-body simulation algorithms (2 hr)
- Press-Schechter (PS) halo mass function (2 hr)
- Extended-PS (EPS) halo mass function (2 hr)
- halo bias & halo density profile (2 hr)

Statistical analysis (2 w)

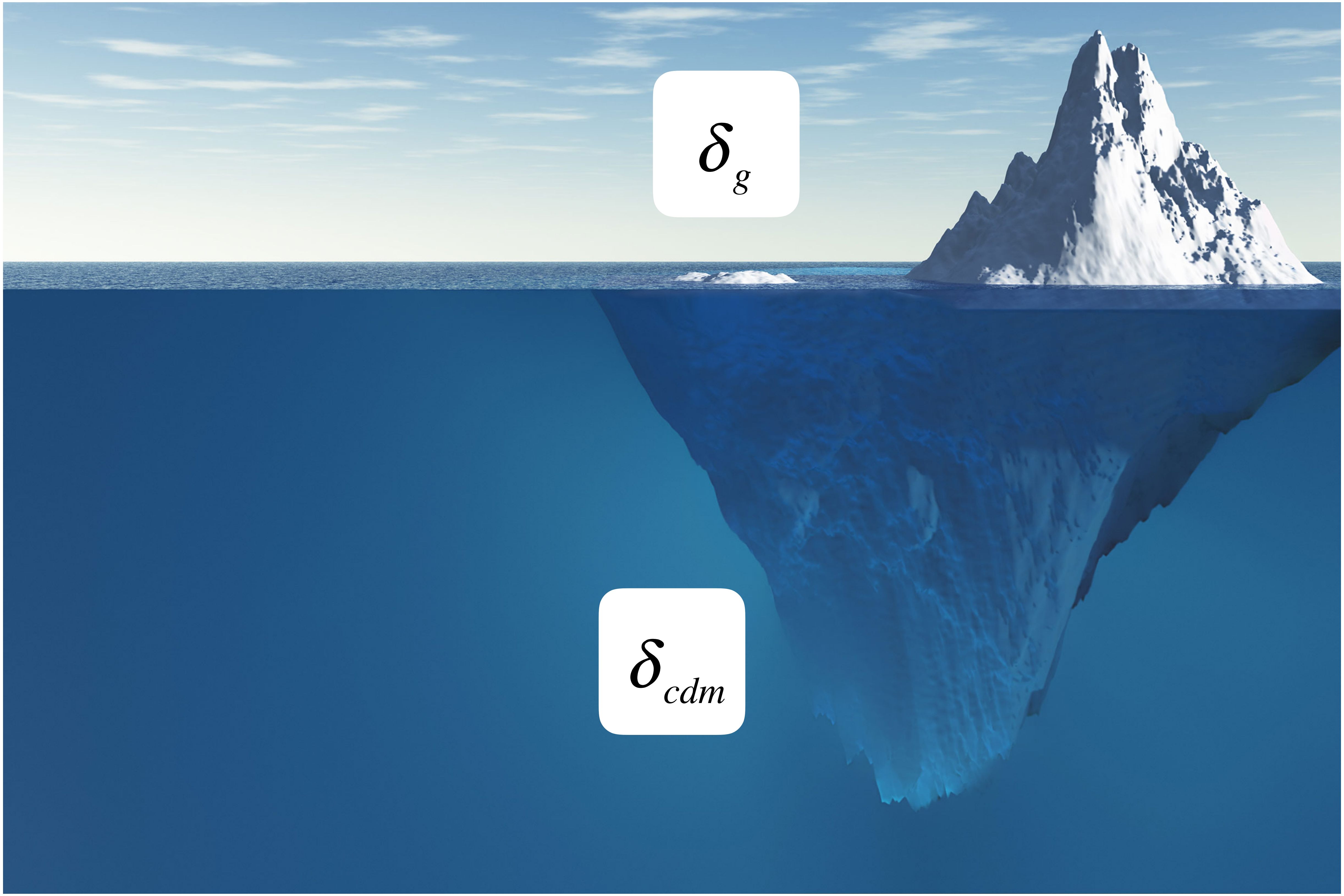
- Monte-Carlo Markov Chain sampler (2 hr)
- CosmoMC use (2 hr)

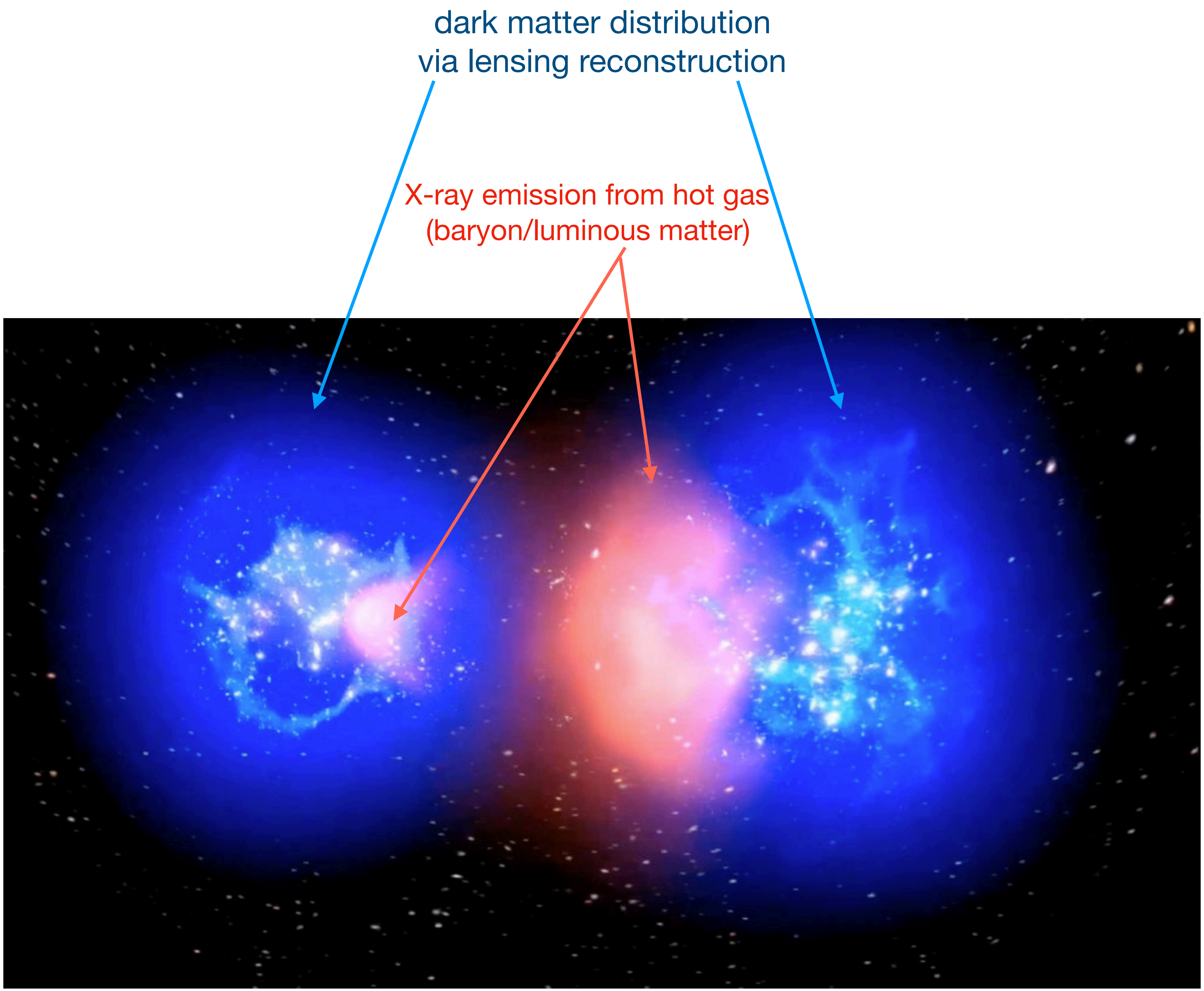
our telescope can only
receive light signal.

we can only measure
the luminous matter distribution



nearby galaxy distribution


δ_g
δ_{cdm}

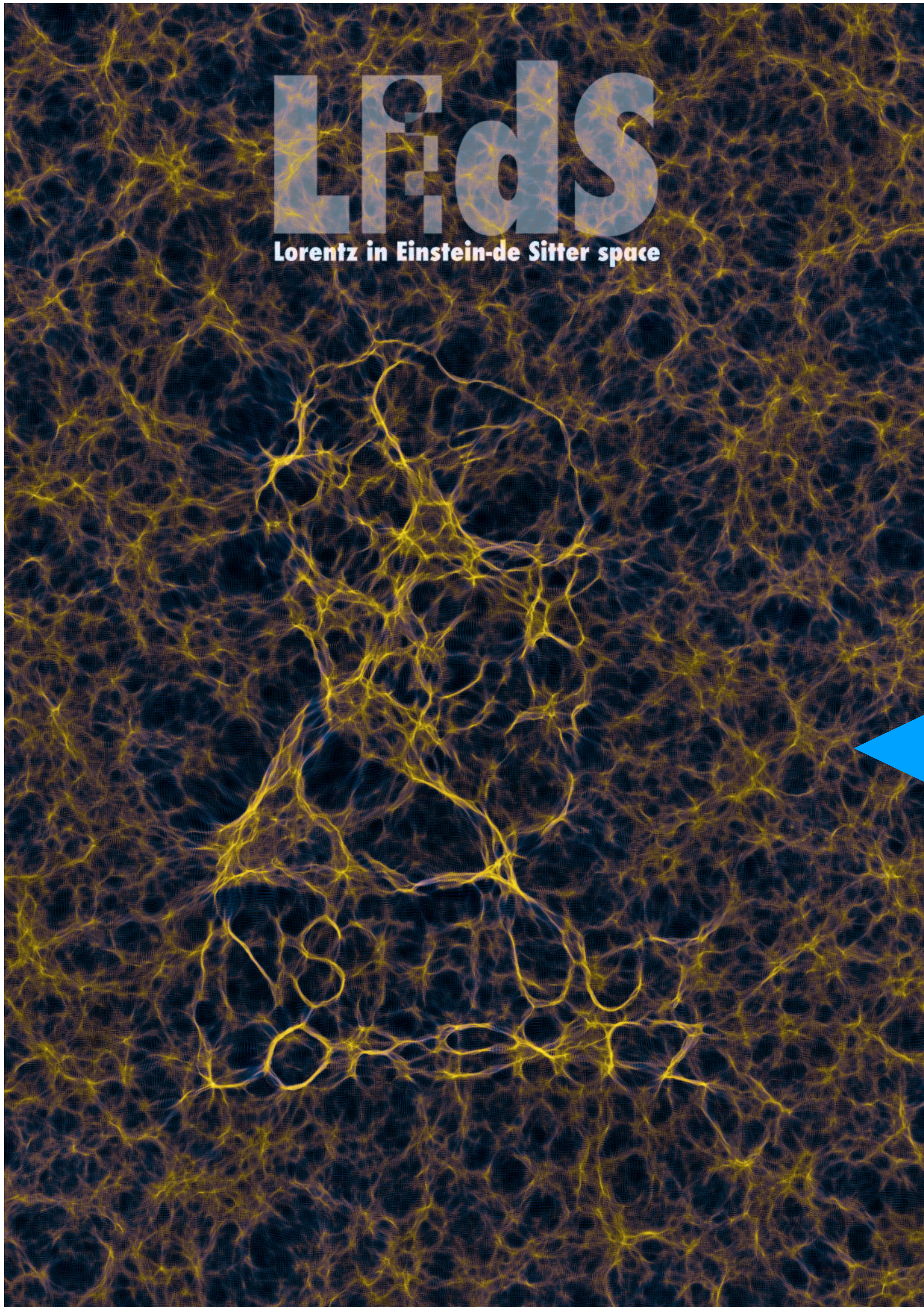

dark matter distribution
via lensing reconstruction
X-ray emission from hot gas
(baryon/luminous matter)

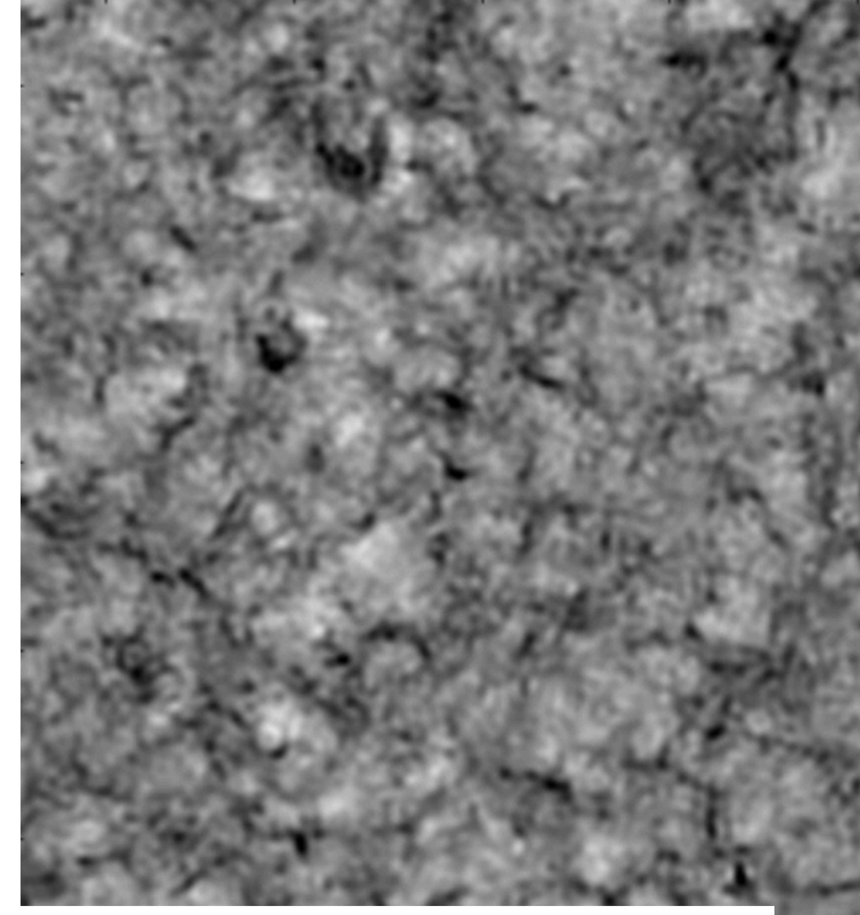
Major task of this lecture is to

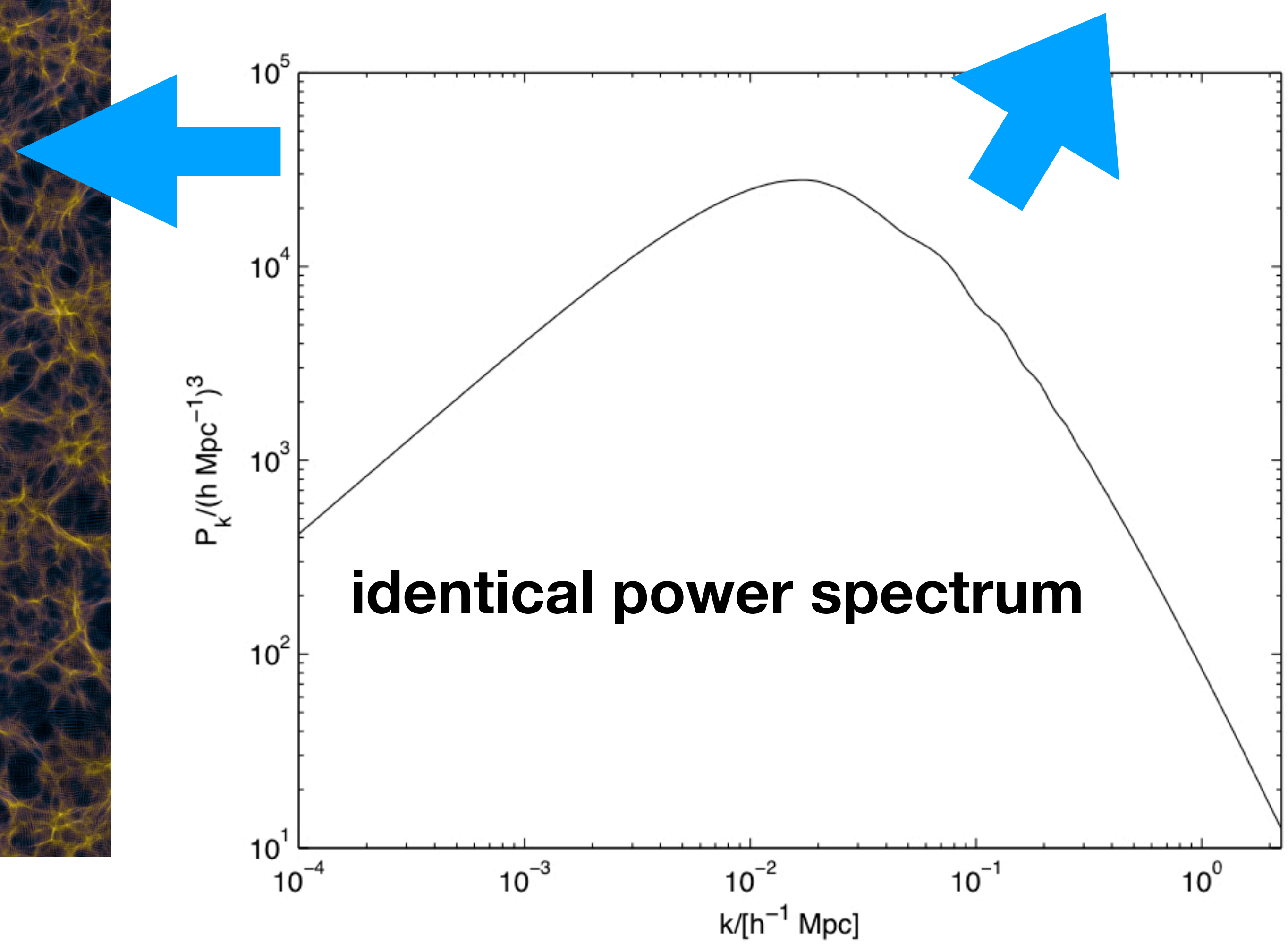
study how to use the galaxy

as the proxy of the total

matter distribution!


Lorentz in Einstein-de Sitter space




10^5
10^4
10^3
10^2
10^1
$P_k/(h\ \mathrm{Mpc}^{-1})^3$
identical power spectrum
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
$k/[h^{-1}\ \mathrm{Mpc}]$

Cosmic density field

For a given cosmology, the density field at a cosmic time t, is described by

$$\delta(\mathbf{x}, t) \quad \text{or} \quad \delta_{\mathbf{k}}(t).$$

How to specify a linear density field? to specify $\delta(\mathbf{x})$ for all $\mathbf{x}$ or to specify $\delta_{\mathbf{k}}$ for all $\mathbf{k}$? NO!

- We consider the cosmic density field to be the realization of a random process, which is descibed by a probability distribution function:

$$\mathcal{P}_x(\delta_1, \delta_2, \cdots, \delta_N)\, \mathrm{d}\delta_1\, \mathrm{d}\delta_2 \cdots \mathrm{d}\delta_N, \quad (N \to \infty)$$

Thus, we emphsize the properties of $\mathcal{P}_x$, rather than the exact form of $\delta(\mathbf{x})$.

[from Houjun Mo]

- The form of $\mathcal{P}_x(\delta_1, \delta_2, \cdots, \delta_N)$: is determined if we know all of its moments:

$$\left\langle \delta_1^{\ell_1}\delta_2^{\ell_2}\cdots\delta_N^{\ell_N}\right\rangle \equiv \int \delta_1^{\ell_1}\delta_2^{\ell_2}\cdots\delta_N^{\ell_N}\mathcal{P}_x(\delta_1, \delta_2, \cdots, \delta_N)\,\mathrm{d}\delta_1\,\mathrm{d}\delta_2\cdots\,\mathrm{d}\delta_N,$$

where $(\ell_1, \ell_2, \cdots, \ell_N) = 0, 1, 2, \cdots$.

In real space:

$$\langle\delta(\mathbf{x})\rangle = 0, \quad \xi(x) = \langle\delta_i\delta_j\rangle, \quad \text{where} \quad x \equiv |\mathbf{x}_i - \mathbf{x}_j|.$$

In Fourier space:

$$\langle\delta_{\mathbf{k}}\rangle = 0, \quad P(k) \equiv V_{\mathrm{u}}\left\langle|\delta_{\mathbf{k}}|^2\right\rangle \equiv V_{\mathrm{u}}\langle\delta_{\mathbf{k}}\delta_{-\mathbf{k}}\rangle = \int \xi(\mathbf{x})\exp(i\mathbf{k}\cdot\mathbf{x})\,\mathrm{d}^3\mathbf{x},$$

In general, it is quite difficult to describe a random field.

[from Houjun Mo]

Gaussian Random Fields

- In real space:

$$\mathcal{P}(\delta_1, \delta_2, \cdots, \delta_n) = \frac{\exp(-Q)}{[(2\pi)^n \det(\mathcal{M})]^{1/2}}; \quad Q \equiv \frac{1}{2}\sum_{i,j} \delta_i \left(\mathcal{M}^{-1}\right)_{ij} \delta_j,$$

where $\mathcal{M}_{ij} \equiv \langle \delta_i \delta_j \rangle$. For a homogeneous and isotropic field, all the multivariate distribution functions are invariant under spatial translation and rotation, and so are completely determined by the two-point correlation function $\xi(x)$!

[from Houjun Mo]

- In Fourier space:

$$\delta_{\mathbf{k}} = A_{\mathbf{k}} + iB_{\mathbf{k}} = |\delta_{\mathbf{k}}| \exp(i\varphi_{\mathbf{k}}).$$

Since $\delta(\mathbf{x})$ is real, we have $A_{\mathbf{k}} = A_{-\mathbf{k}}$, $B_{\mathbf{k}} = -B_{-\mathbf{k}}$, and so we need only Fourier modes with $\mathbf{k}$ in the upper half space to specify $\delta(\mathbf{x})$. It is then easy to prove that, for $\mathbf{k}$ in the upper half space,

$$\langle A_{\mathbf{k}} A_{\mathbf{k}'} \rangle = \langle B_{\mathbf{k}} B_{\mathbf{k}'} \rangle = \frac{1}{2} V_{\mathrm{u}}^{-1} P(k) \delta^{(\mathrm{D})}_{\mathbf{k}\mathbf{k}'}; \quad \langle A_{\mathbf{k}} B_{\mathbf{k}'} \rangle = 0,$$

Thus As a result, the multivariate distribution functions of $A_{\mathbf{k}}$ and $B_{\mathbf{k}}$ are factorized according to $\mathbf{k}$, each factor being a Gaussian:

$$\mathcal{P}(\alpha_{\mathbf{k}})\, \mathrm{d}\alpha_{\mathbf{k}} = \frac{1}{[\pi V_{\mathrm{u}}^{-1} P(k)]^{1/2}} \exp\left[-\frac{\alpha_{\mathbf{k}}^2}{V_{\mathrm{u}}^{-1} P(k)}\right] \mathrm{d}\alpha_{\mathbf{k}},$$

[from Houjun Mo]

In terms of $|\delta_{\mathbf{k}}|$ and $\varphi_{\mathbf{k}}$, the distribution function for each mode, $\mathcal{P}(A_{\mathbf{k}})\mathcal{P}(B_{\mathbf{k}})\,\mathrm{d}A_{\mathbf{k}}\,\mathrm{d}B_{\mathbf{k}}$, can be written as

$$\mathcal{P}(|\delta_{\mathbf{k}}|,\varphi_{\mathbf{k}})\,\mathrm{d}|\delta_{\mathbf{k}}|\,\mathrm{d}\varphi_{\mathbf{k}} = \exp\left[-\frac{|\delta_{\mathbf{k}}|^2}{2V_{\mathrm{u}}^{-1}P(k)}\right]\frac{|\delta_{\mathbf{k}}|\,\mathrm{d}|\delta_{\mathbf{k}}|}{V_{\mathrm{u}}^{-1}P(k)}\frac{\mathrm{d}\varphi_{\mathbf{k}}}{2\pi}.$$

Thus, for a Gaussian field, different Fourier modes are mutually independent, so are the real and imaginary parts of individual modes. This, in turn, implies that the phases $\varphi_{\mathbf{k}}$ of different modes are mutually independent and have random distribution over the interval between 0 and 2π.

$P(k)$ is the only function we need!

φ_{k} **: is uniformly distributed between 0 and 2pi**

[from Houjun Mo]

Although power spectrum can NOT tell us ALL the statistics, still it is informative

real gauss random field

$$\hat{s}(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \hat{s}_{\vec{k}}$$

complex gauss random field

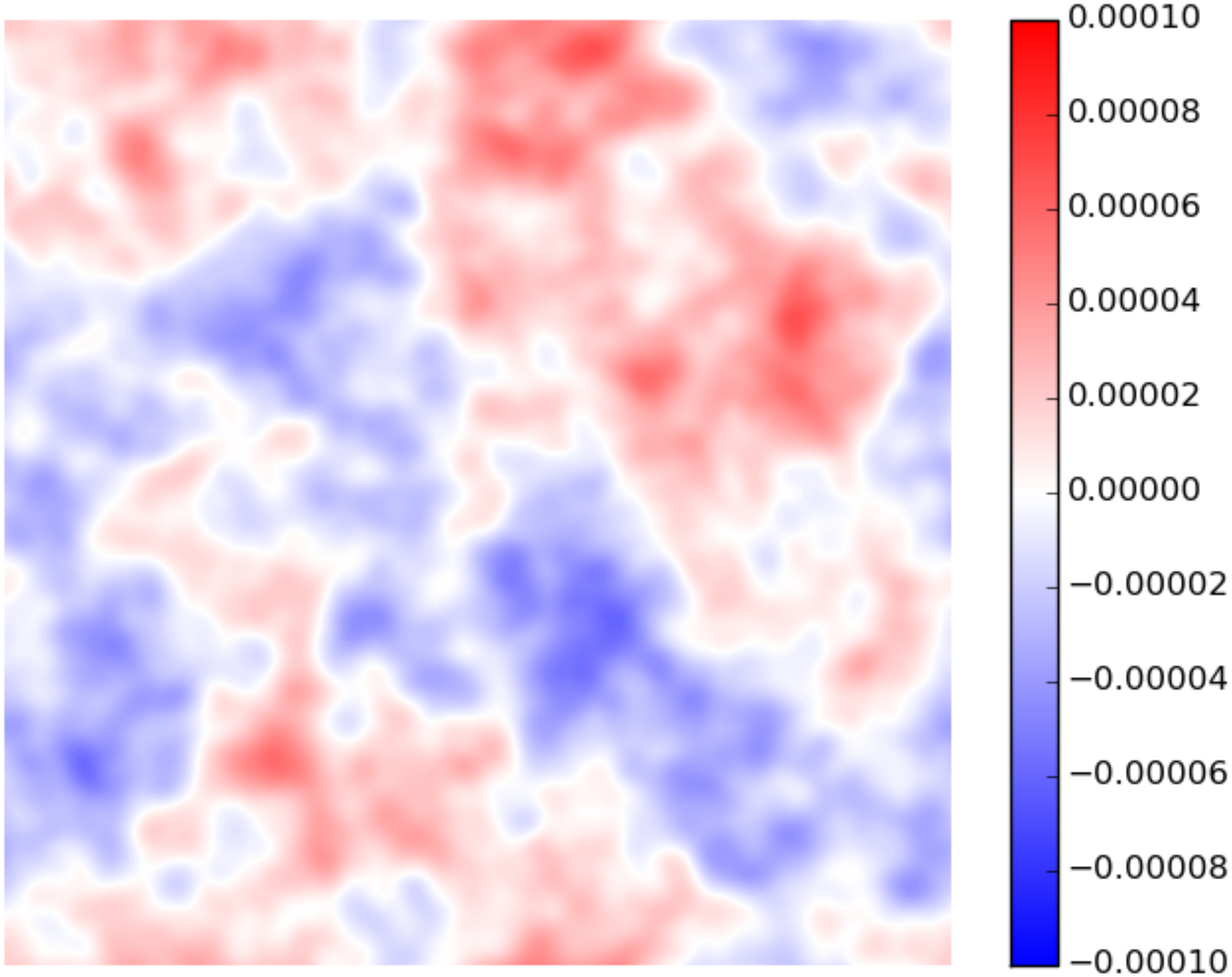
$$= \lim_{L\to\infty} \sum_{\vec{n}=-\infty}^{\infty} L^{-3} e^{i\frac{2\pi\vec{n}}{L}\cdot\vec{x}} \hat{s}_{\frac{2\pi\vec{n}}{L}},$$

$$\left\langle \hat{s}_{\frac{2\pi\vec{n}}{L}} \hat{s}^*_{\frac{2\pi\vec{n}'}{L}} \right\rangle = L^{-3}\delta_{\vec{n},\vec{m}} \underline{P_{\hat{s}}\left(\left|\frac{2\pi\vec{n}}{L}\right|\right)}$$

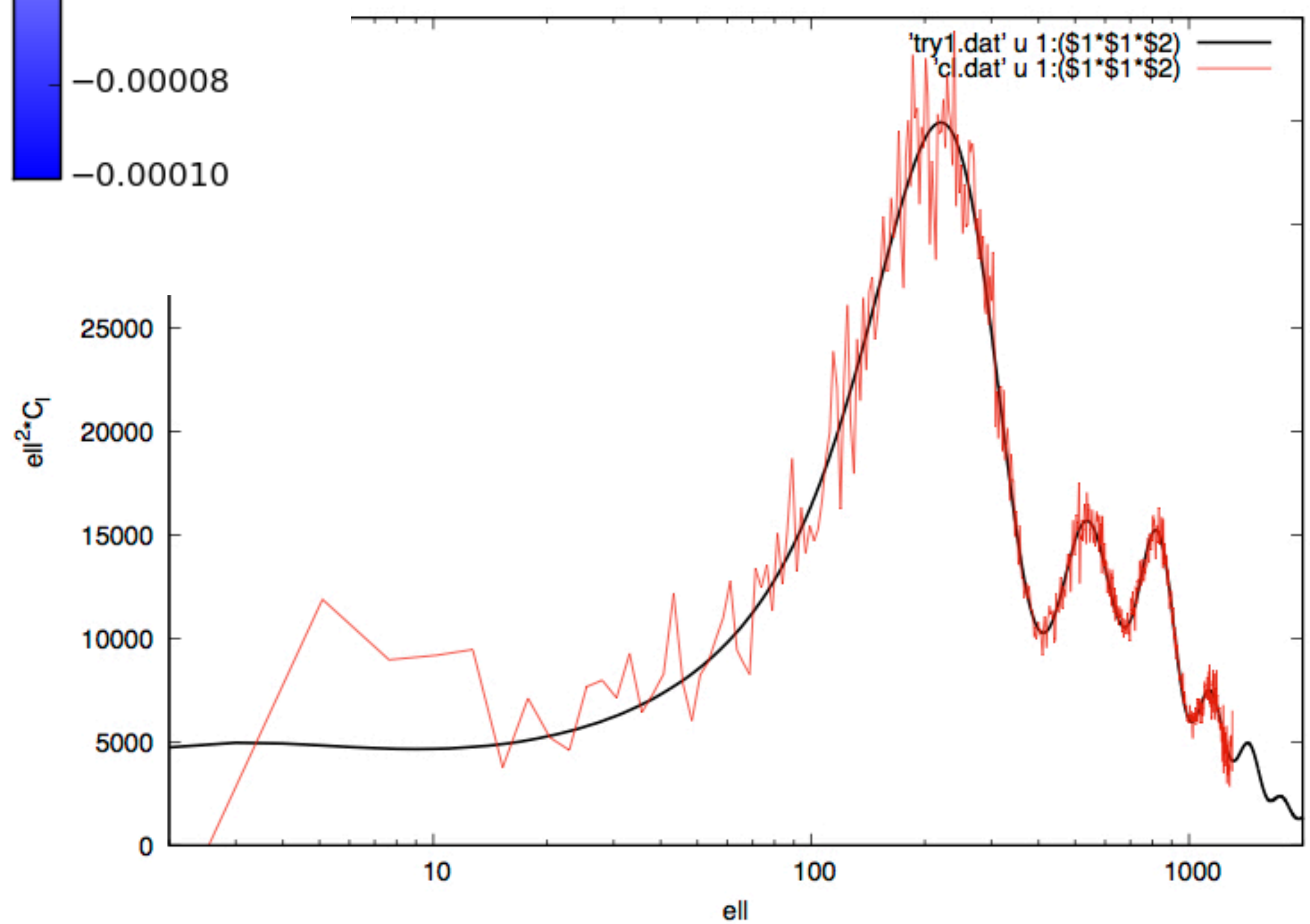
power spectrum only give us the info encoded in Amplitude

$$\hat{s}(\vec{k}) \sim \hat{A}(\vec{k}) \cancel{e^{i\hat{\phi}(\vec{k})}}$$

Loss info encoded in the phase!



[Pb] **lens_img3.py**



inflation

(inside of horizon)
growth function

$$P_{gal}(\vec{k}) = P_{ini}(k)T^2(k)D^2(a)[b + f\mu^2]^2$$

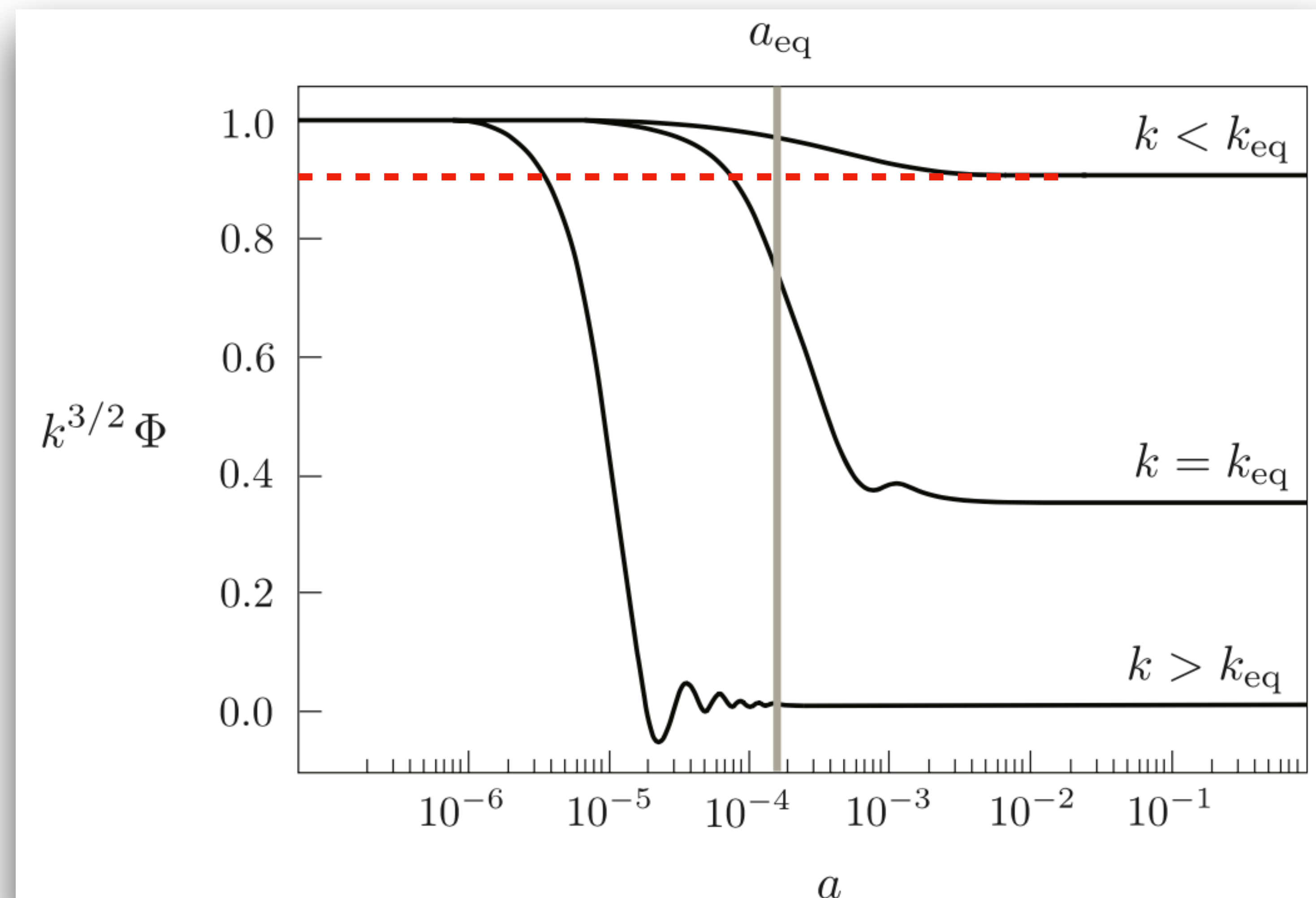
transfer function
(outside of horizon)

bias

RSD

anisotropic

isotropic



Perturbation statistics: correlation function

[from W. Percival]

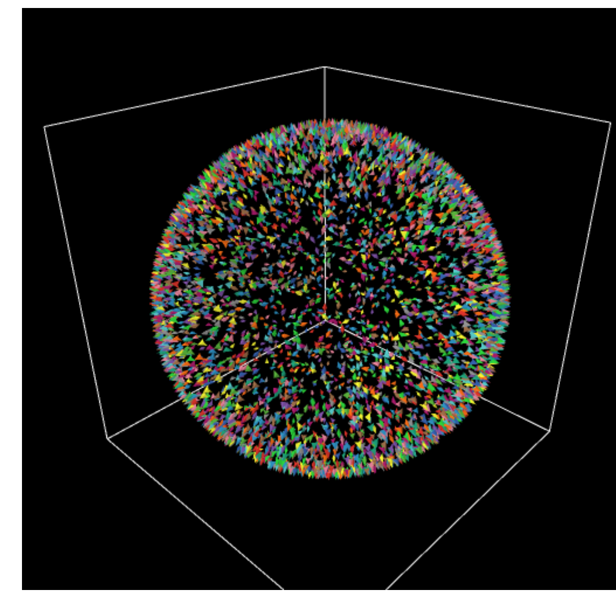
overdensity field $\delta \equiv \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$

definition of correlation function

$$\begin{aligned}\xi(\mathbf{x_1}, \mathbf{x_2}) &\equiv \langle \delta(\mathbf{x_1}) \delta(\mathbf{x_2}) \rangle \\ &= \xi(\mathbf{x_1} - \mathbf{x_2}) \\ &= \xi(|\mathbf{x_1} - \mathbf{x_2}|)\end{aligned}$$

from statistical homogeneity

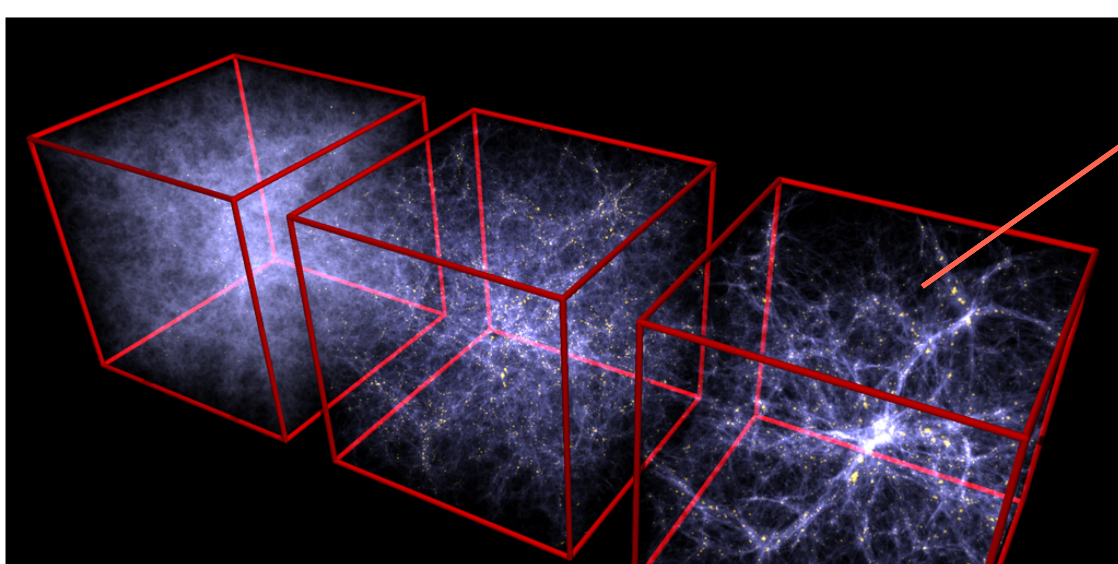
from statistical isotropy

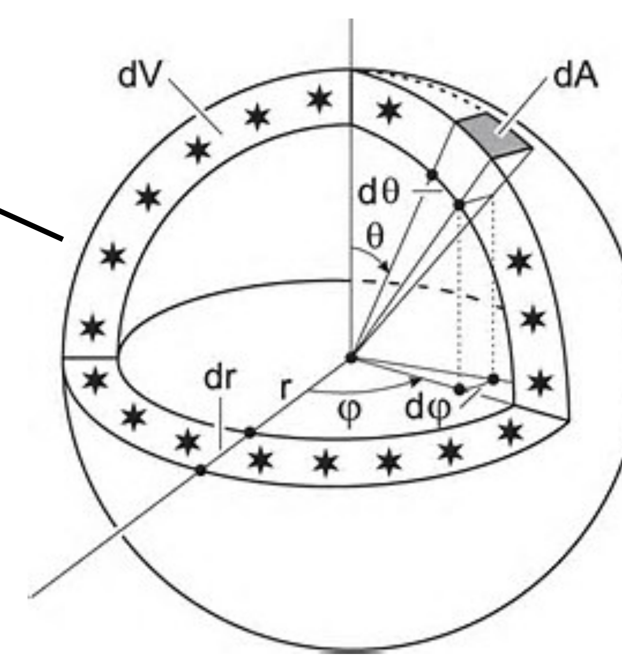


uniformly distributed random sample

can estimate correlation function using galaxy (DD) and random (RR) pair counts at separations ~r

$$1 + \xi(r) = \frac{\langle DD \rangle_r}{\langle RR \rangle_r}$$





Perturbation statistics: power spectrum

definition of
power spectrum

$$\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1 - \mathbf{k}_2)P(k_1)$$

power spectrum is the Fourier analogue of the correlation function

$$\begin{aligned} P(k) &\equiv \int \xi(r) e^{i\mathbf{k}.\mathbf{r}} d^3r \\ \xi(r) &= \int P(k) e^{-i\mathbf{k}.\mathbf{r}} \frac{d^3k}{(2\pi)^3} \end{aligned}$$

sometimes written in dimensionless form

$$\Delta^2(k) = \frac{k^3}{2\pi^2} P(k)$$

[from W. Percival]

Correlation function vs Power Spectrum

The power spectrum and correlation function contain the same information; accurate measurement of each give the same constraints on cosmological models.

Both power spectrum and correlation function can be measured relatively easily (and with amazing complexity)

The power spectrum has the advantage that different modes are uncorrelated (as a consequence of statistical homogeneity).

Models tend to focus on the power spectrum, so it is common for observations to do the same ...

[from W. Percival]

inflation

(inside of horizon) growth function

$$P_{gal}(\vec{k}) = P_{ini}(k)T^2(k)D^2(a)[b + f\mu^2]^2$$

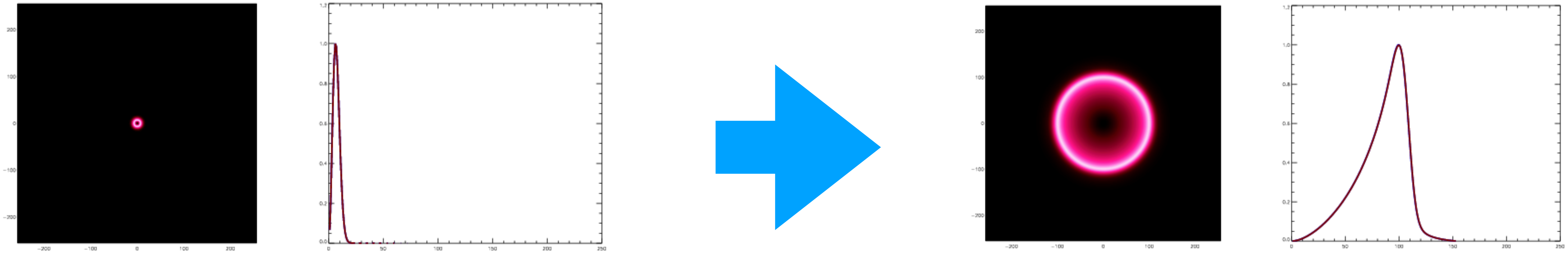
transfer function (outside of horizon)

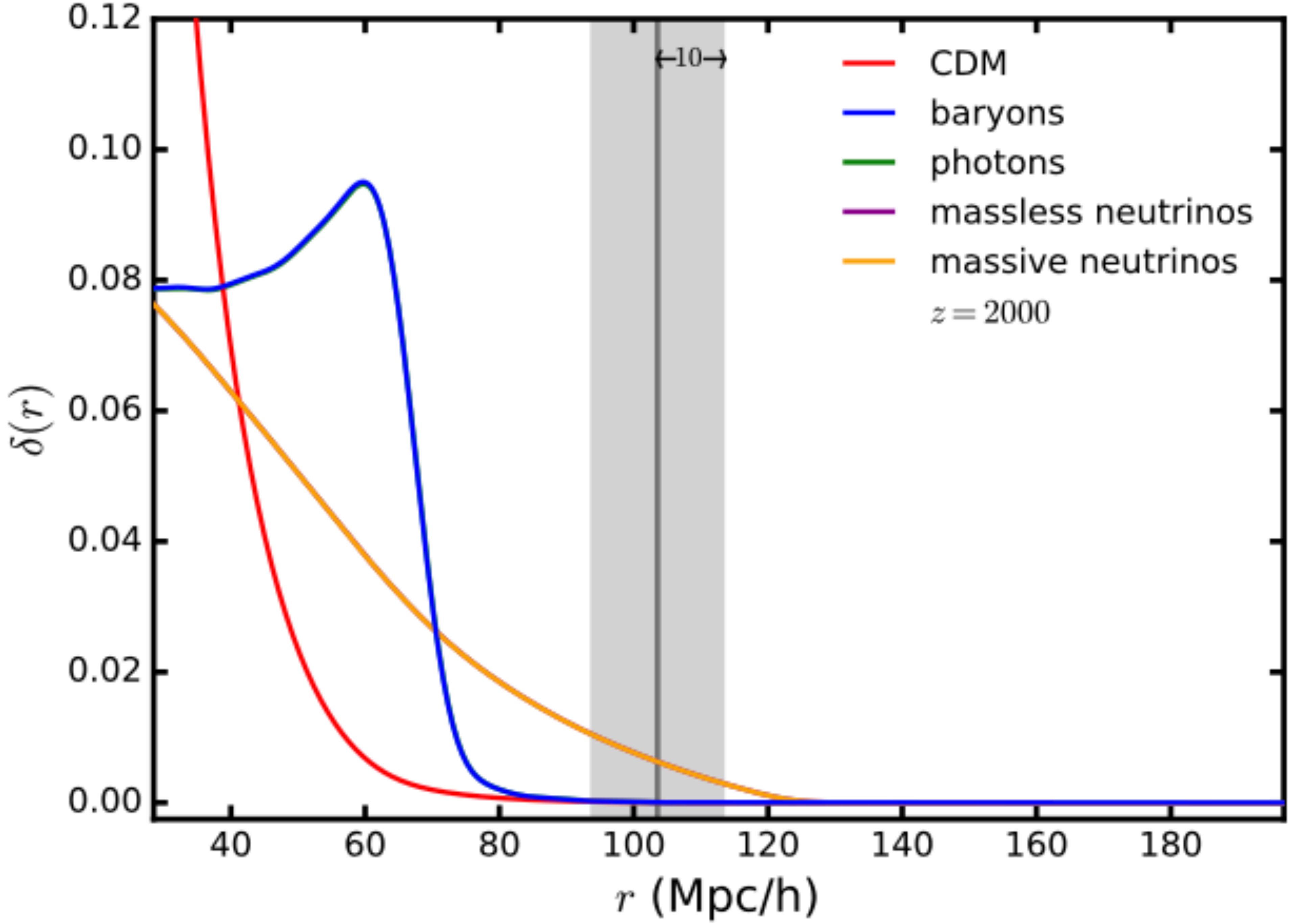
bias

RSD

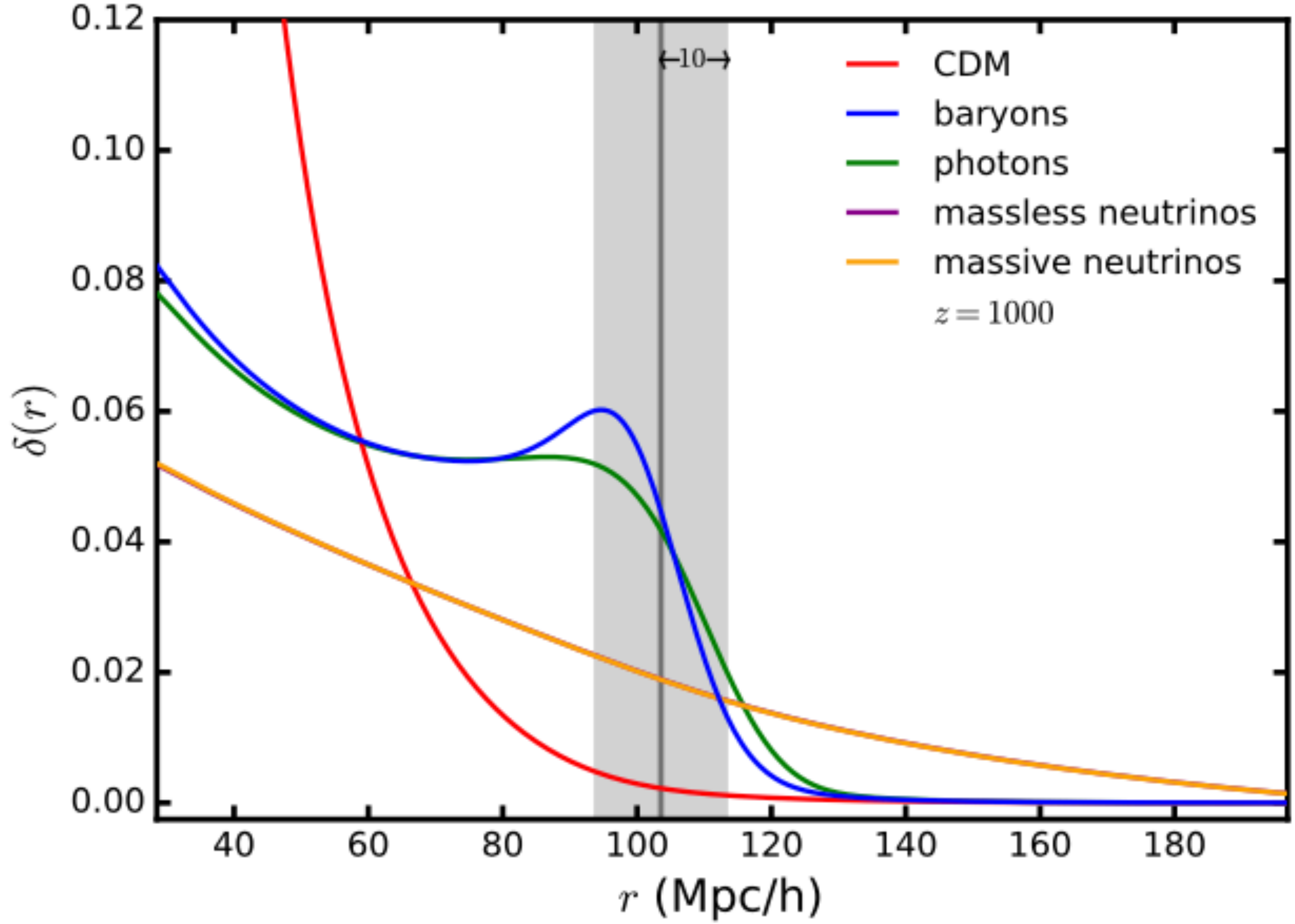
anisotropic

isotropic

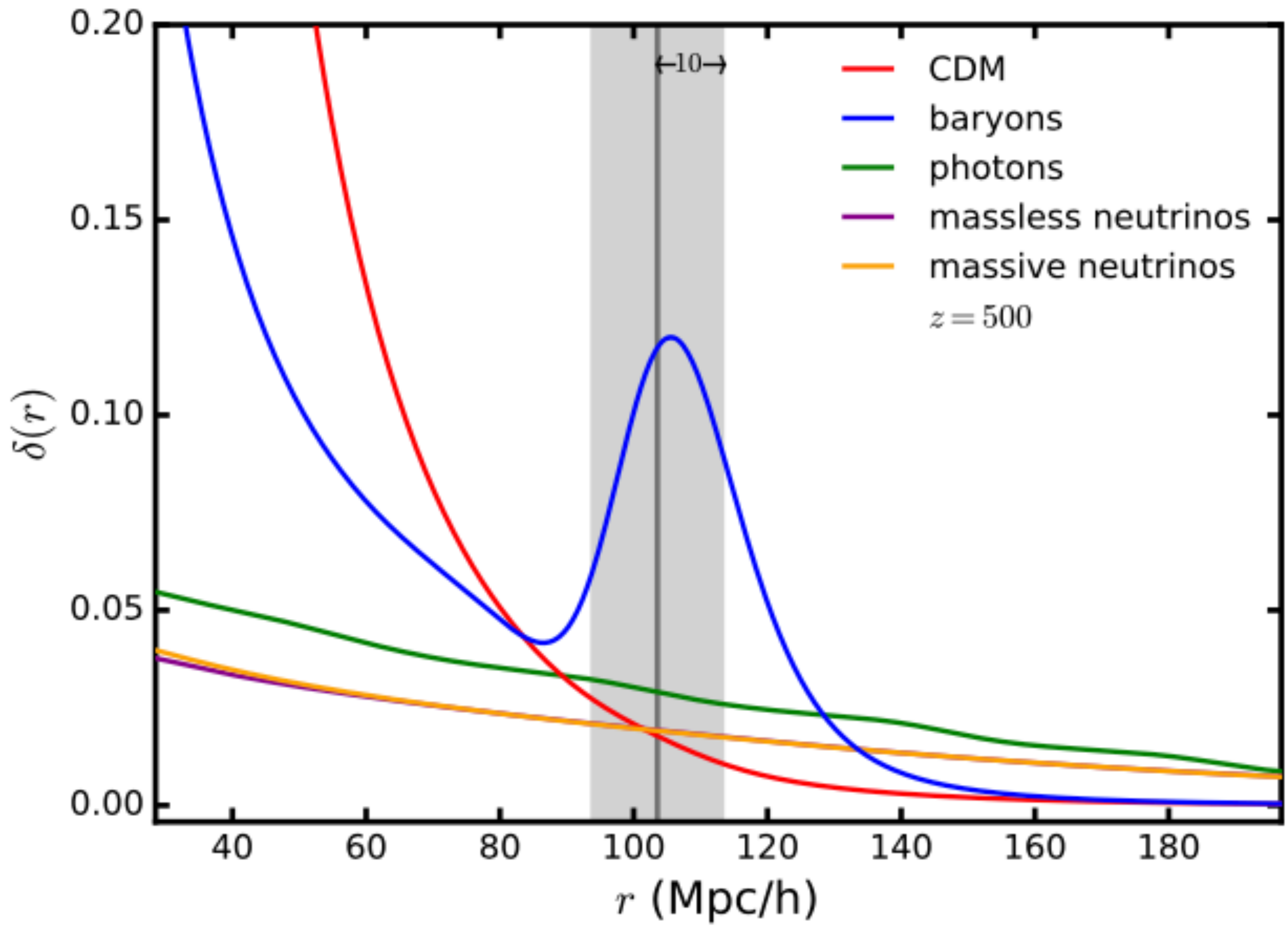




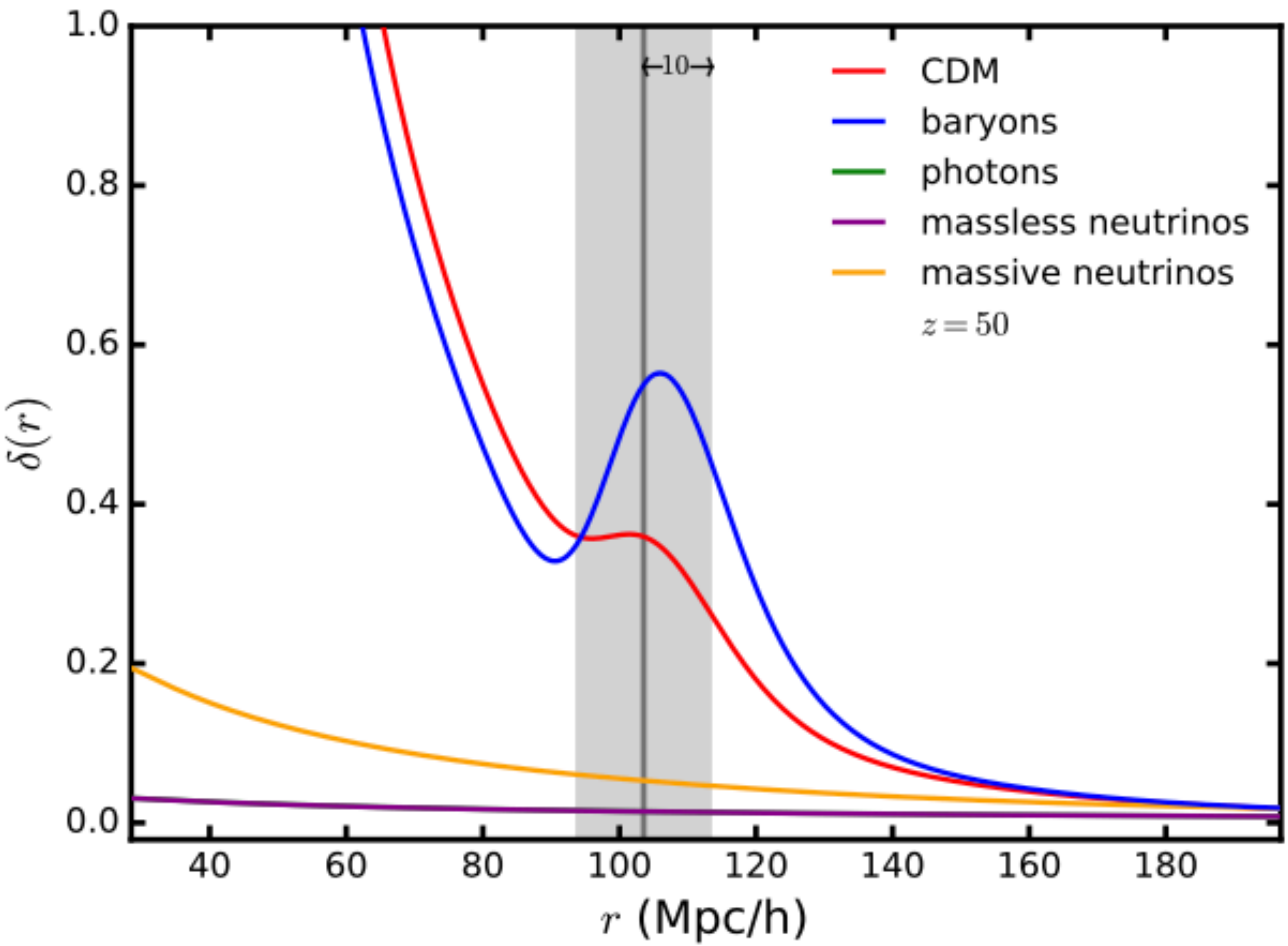
[by Shu-Xun Tian]



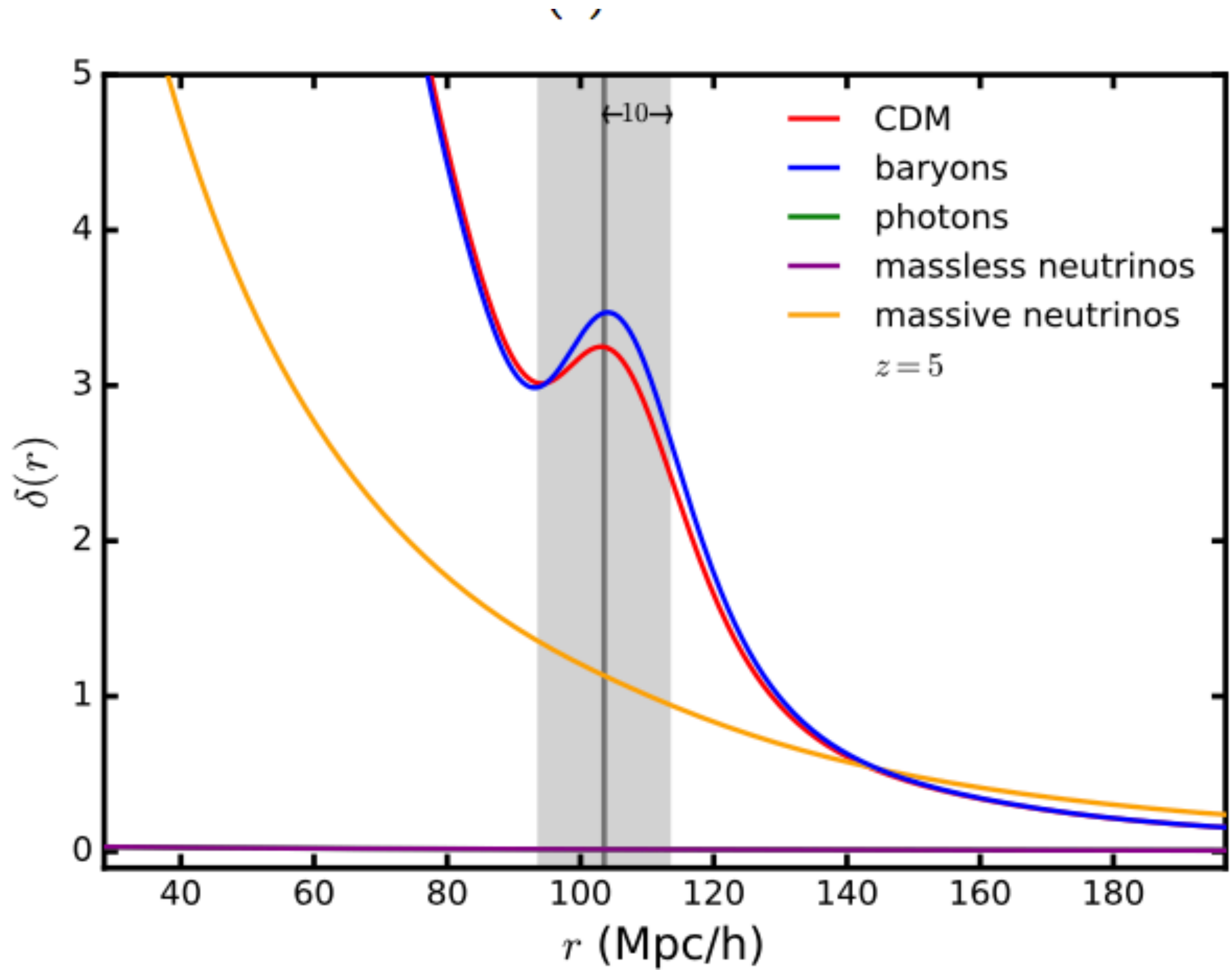
[by Shu-Xun Tian]



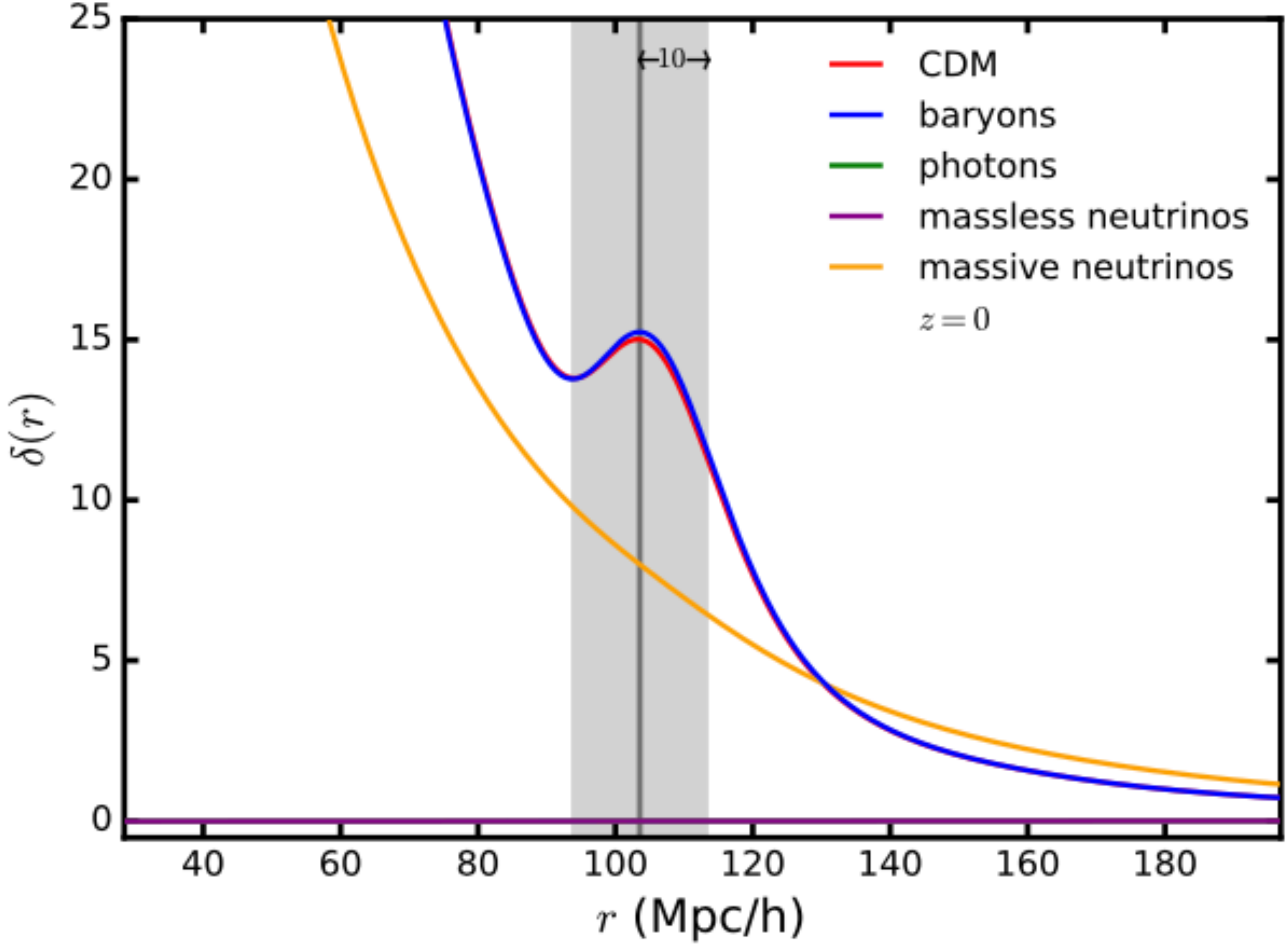
[by Shu-Xun Tian]



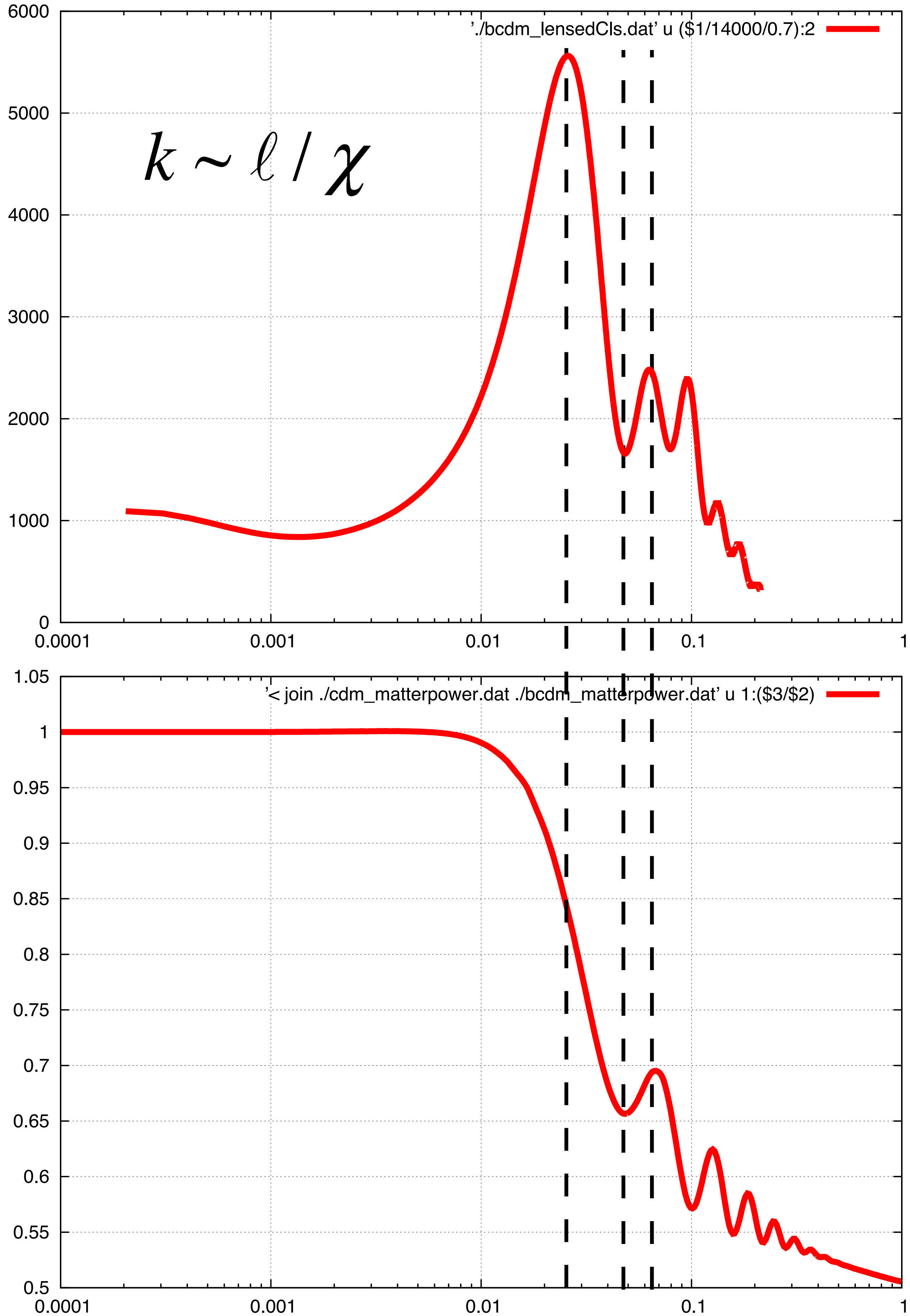
[by Shu-Xun Tian]

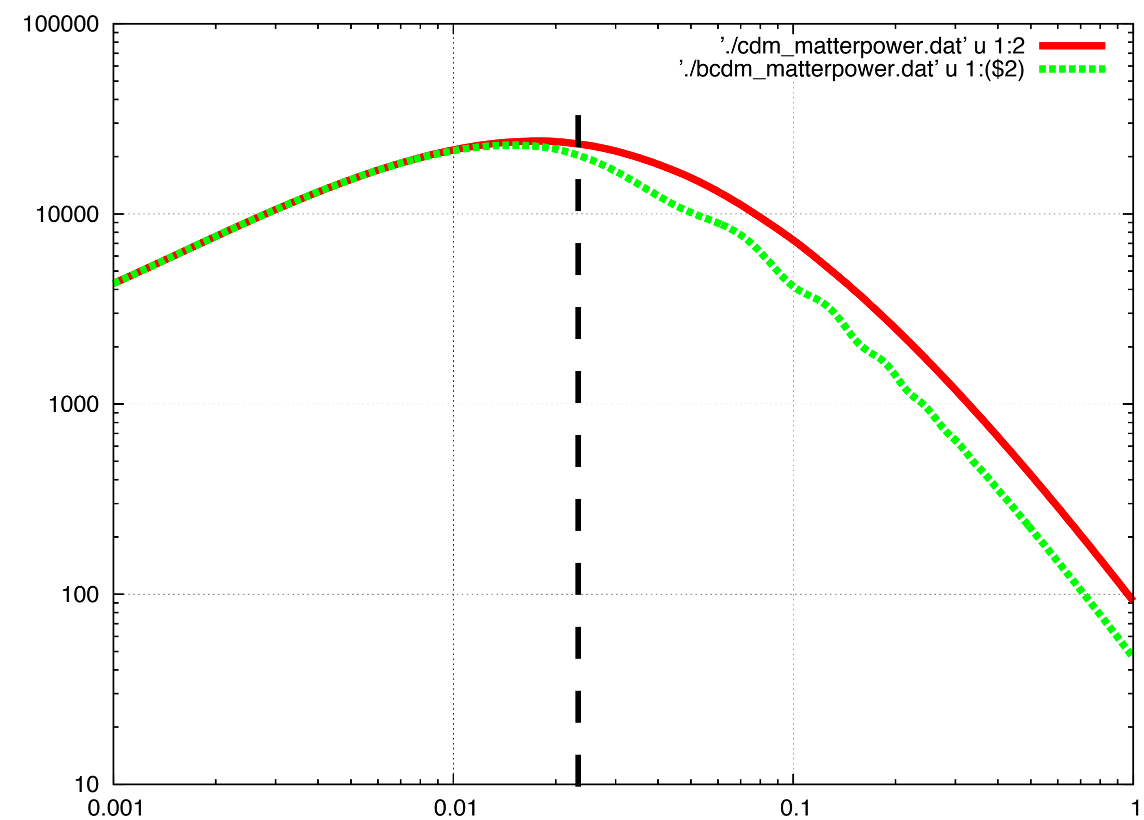


[by Shu-Xun Tian]



[by Shu-Xun Tian]


$k \sim \ell / \chi$
'./bcdm_lensedCls.dat' u ($1/14000/0.7):2
0
1000
2000
3000
4000
5000
6000
'< join ./cdm_matterpower.dat ./bcdm_matterpower.dat' u 1:($3/$2)
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
1.05
0.0001
0.001
0.01
0.1
1


'./cdm_matterpower.dat' u 1:2
'./bcdm_matterpower.dat' u 1:($2)
10
100
1000
10000
100000
0.001
0.01
0.1
1

inflation

(inside of horizon)
growth function

$$P_{gal}(\vec{k}) = P_{ini}(k)T^2(k)D^2(a)[b + f\mu^2]^2$$

transfer function
(outside of horizon)

bias

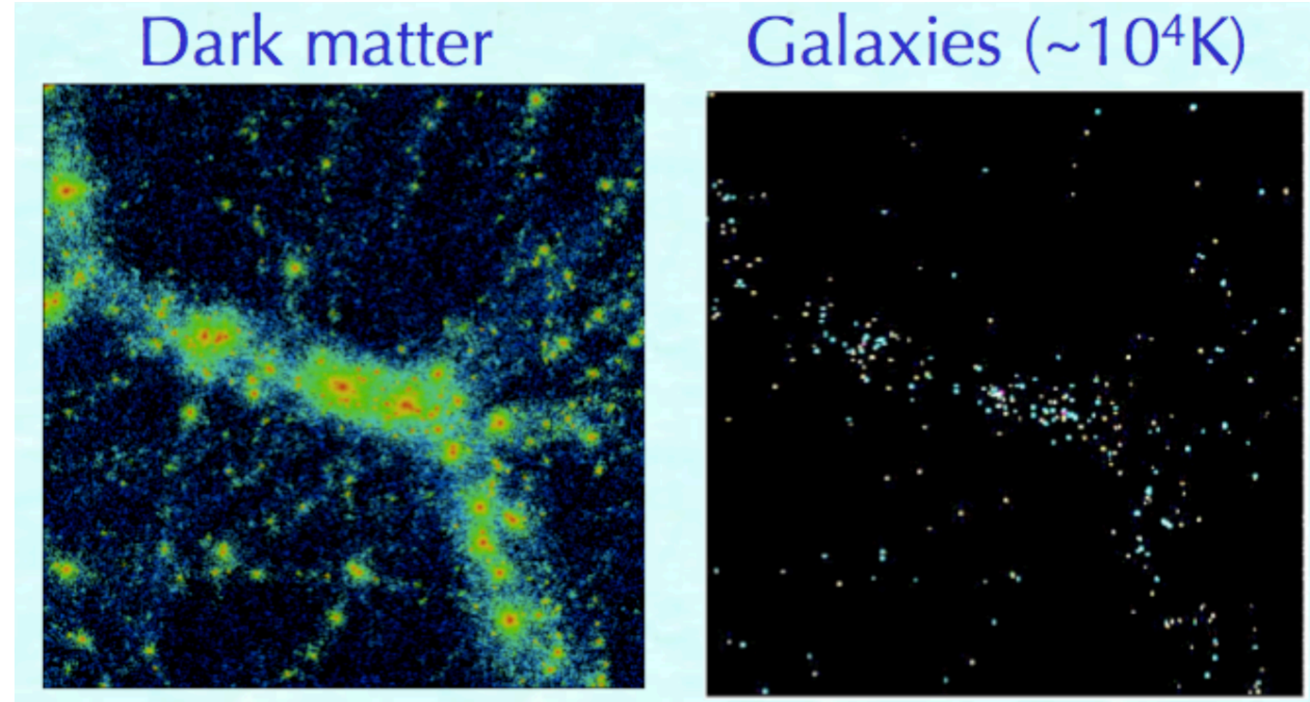
RSD
anisotropic

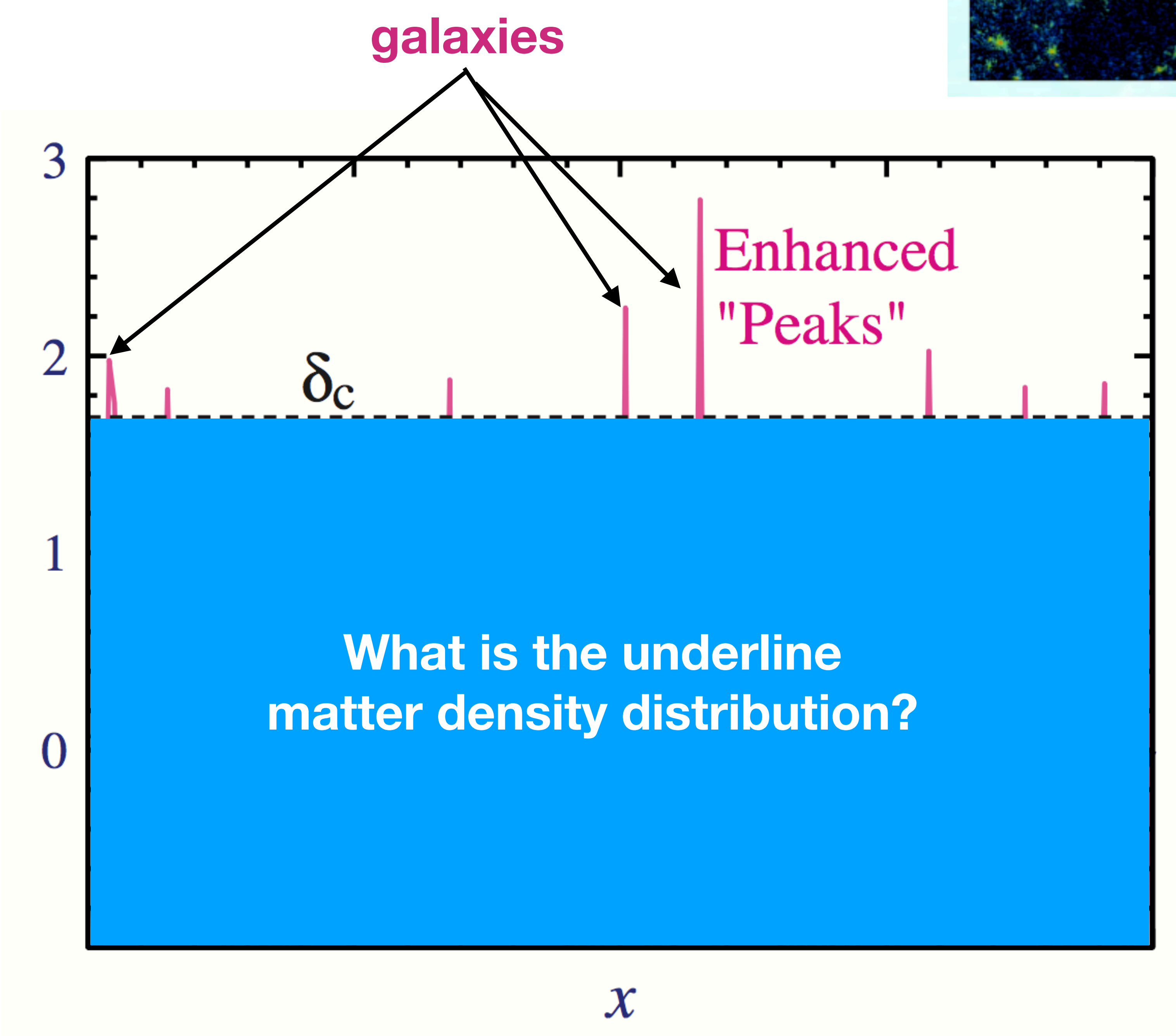
isotropic

peak-background split

galaxy: discrete distribution

matter: smoothed distribution





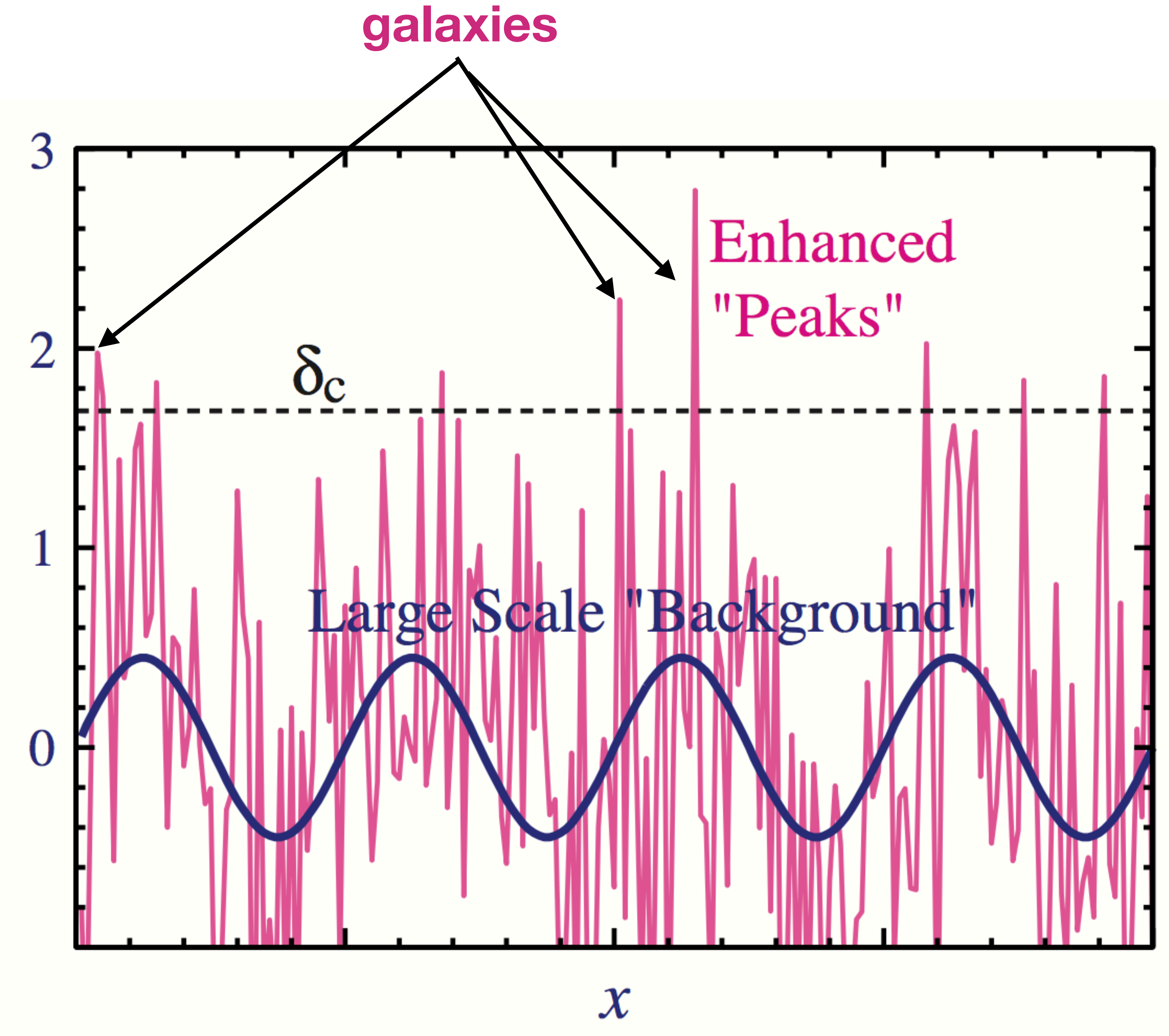
peak-background split

1. qualitatively, galaxy distribution can mimic underline matter distribution

2. quantitatively, they are not coincide!

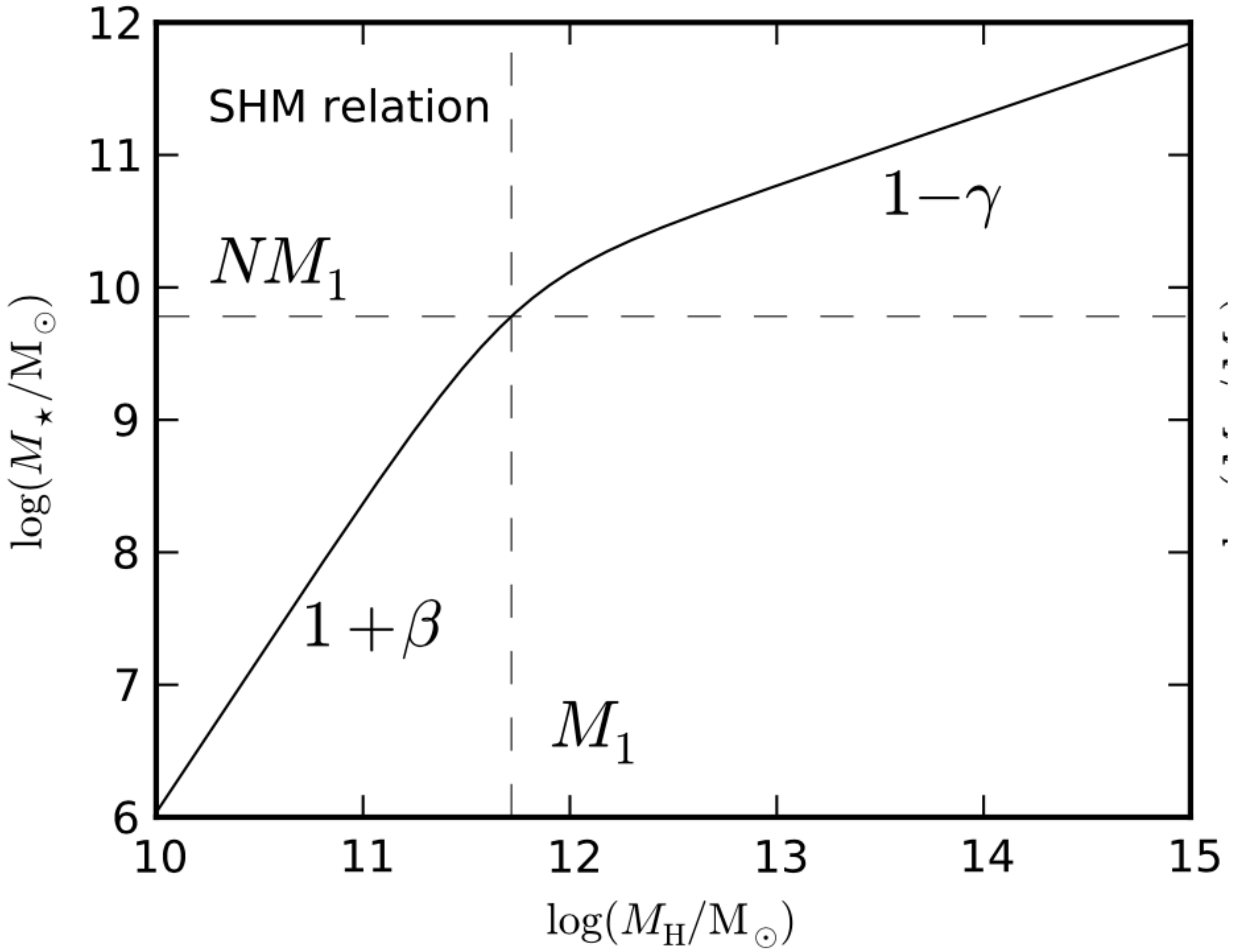
need introduce a bias factor!

$$\delta_g = b \cdot \delta_m$$



[from W. Hu]

stellar-halo mass relation



typically, single galaxy can only contribute 1%~10% mass to gravitational potential

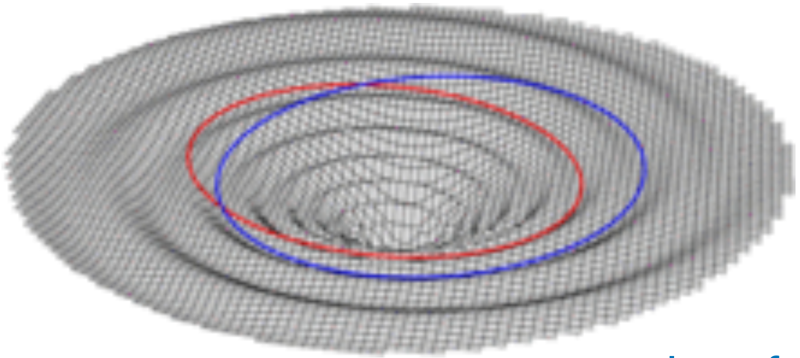
so, we can treat the single galaxy as a probe particle

$$M^{milkyway}_{halo} \sim 10^{12} M_\odot$$

$$M^{milkyway}_{stellar} \sim 10^{10} M_\odot$$

Q: if all the baryon is localised in galaxy, milkyway stellar mass shall be ~2.5E11

A: a large mount of the baryon (gas) is spread in Inter Galactic Medium.



Galaxies expected to be (almost) **unbiased** tracers of the cosmic **velocity field** (but not the density field).

The reason why galaxy density field is biased w.r.t. real matter density:

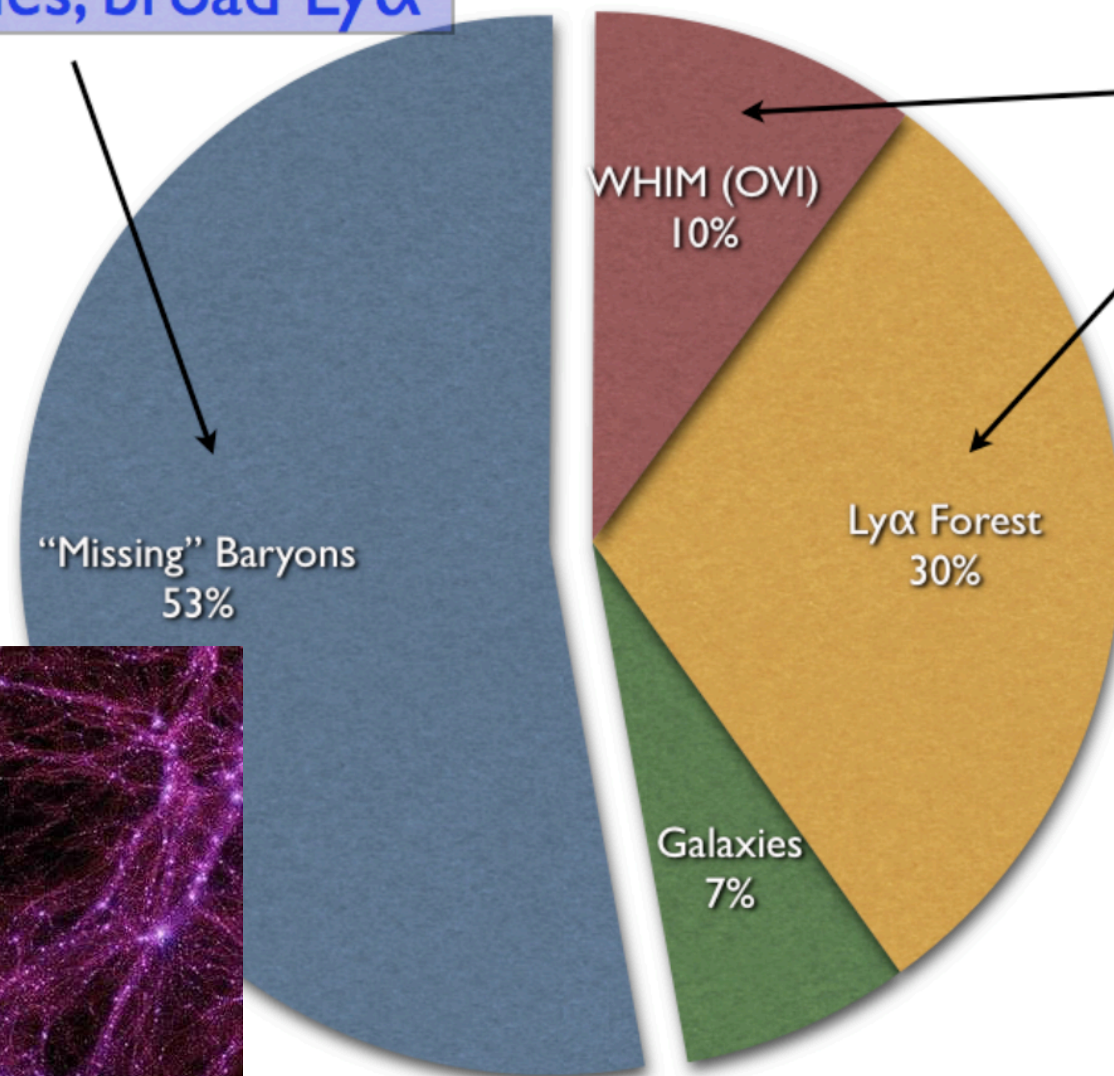
galaxy formation process, is not only driven by gravity, but also by complicated baryonic dominated mechanism, such as AGN feedback, SN explosion, etc. These process is very hard to model!

Once the galaxy is formed, its motion is only driven by the gravity, due to we can treat it as a test particle.

Motion of galaxies is independent of galaxy properties, galaxies act as test particles in flow of matter

Baryon Census (low-z)

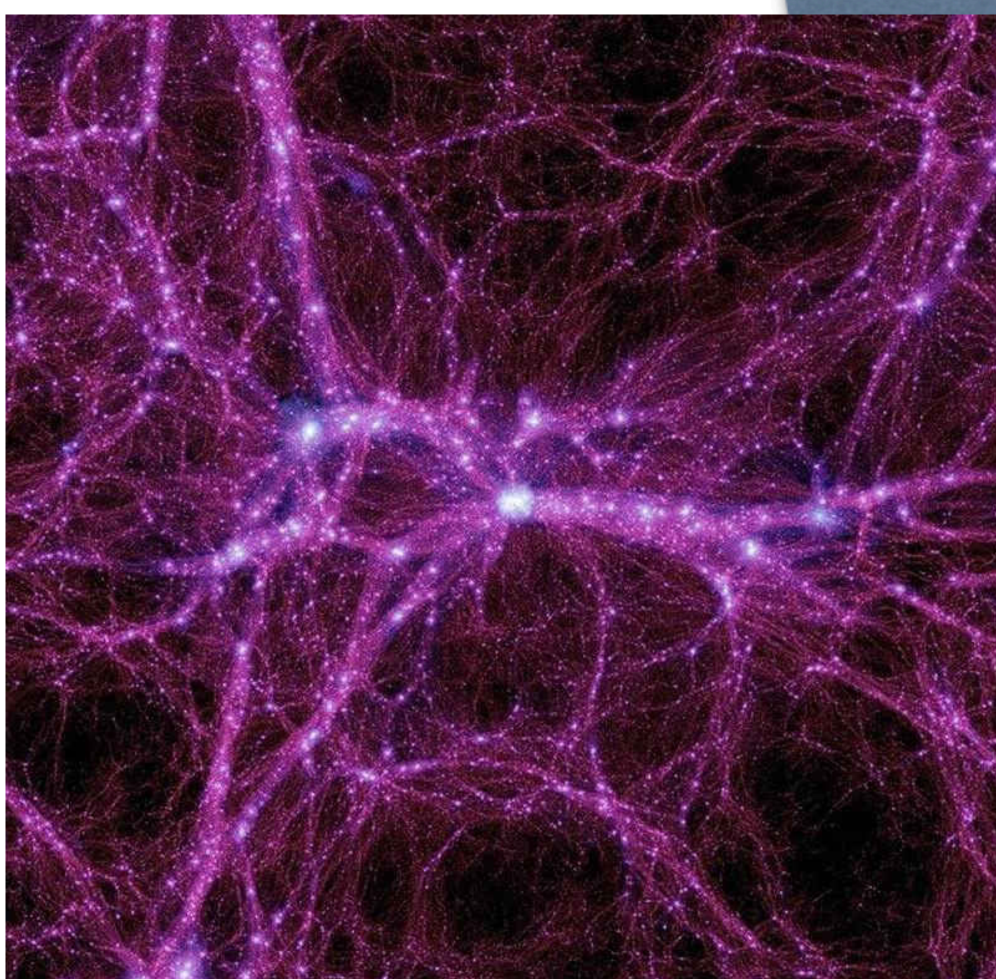
Probed by X-ray lines, broad Lyα

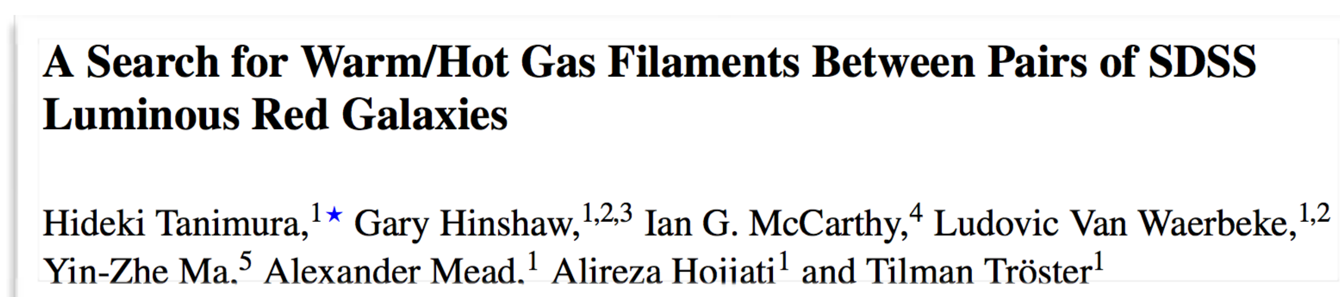


Both of these are uncertain

IGM Systematics:

- EUV radiation field
- Oxygen metallicity
- Ioniz corrections
- Cloud geometry





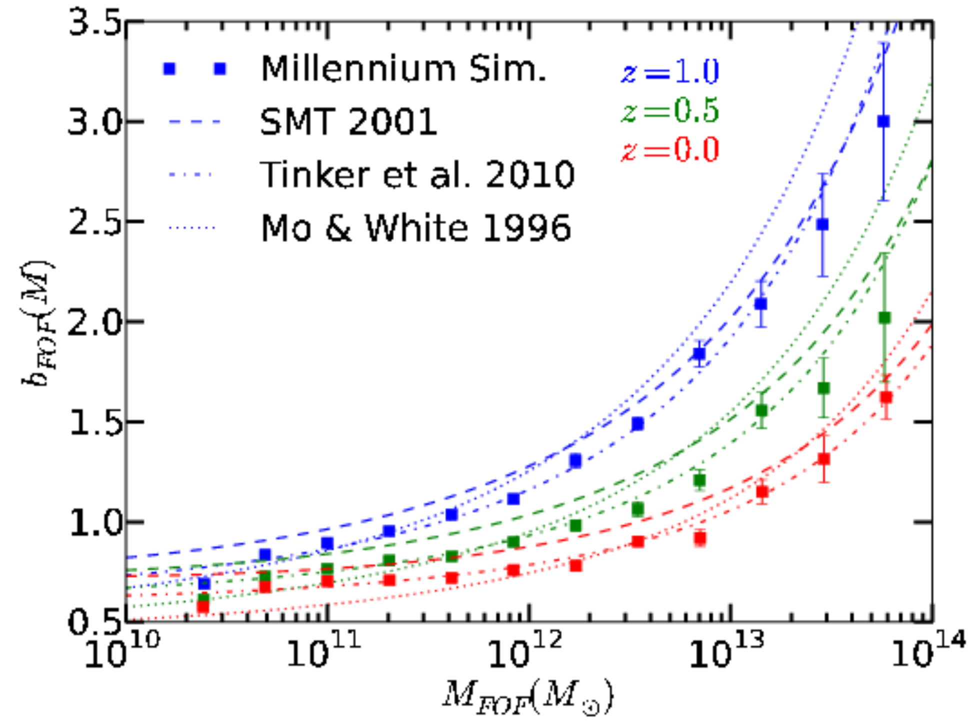
A Search for Warm/Hot Gas Filaments Between Pairs of SDSS Luminous Red Galaxies

Hideki Tanimura,[1★] Gary Hinshaw,[1,2,3] Ian G. McCarthy,[4] Ludovic Van Waerbeke,[1,2] Yin-Zhe Ma,[5] Alexander Mead,[1] Alireza Hojjati[1] and Tilman Tröster[1]

time evolution of the bias

$$\delta_b'' + \mathcal{H}\delta_b' = 4\pi G a^2 \boxed{(\bar{\rho}_b\delta_b + \bar{\rho}_c\delta_c)}$$
$$\delta_c'' + \mathcal{H}\delta_c' = 4\pi G a^2 \boxed{(\bar{\rho}_b\delta_b + \bar{\rho}_c\delta_c)}$$

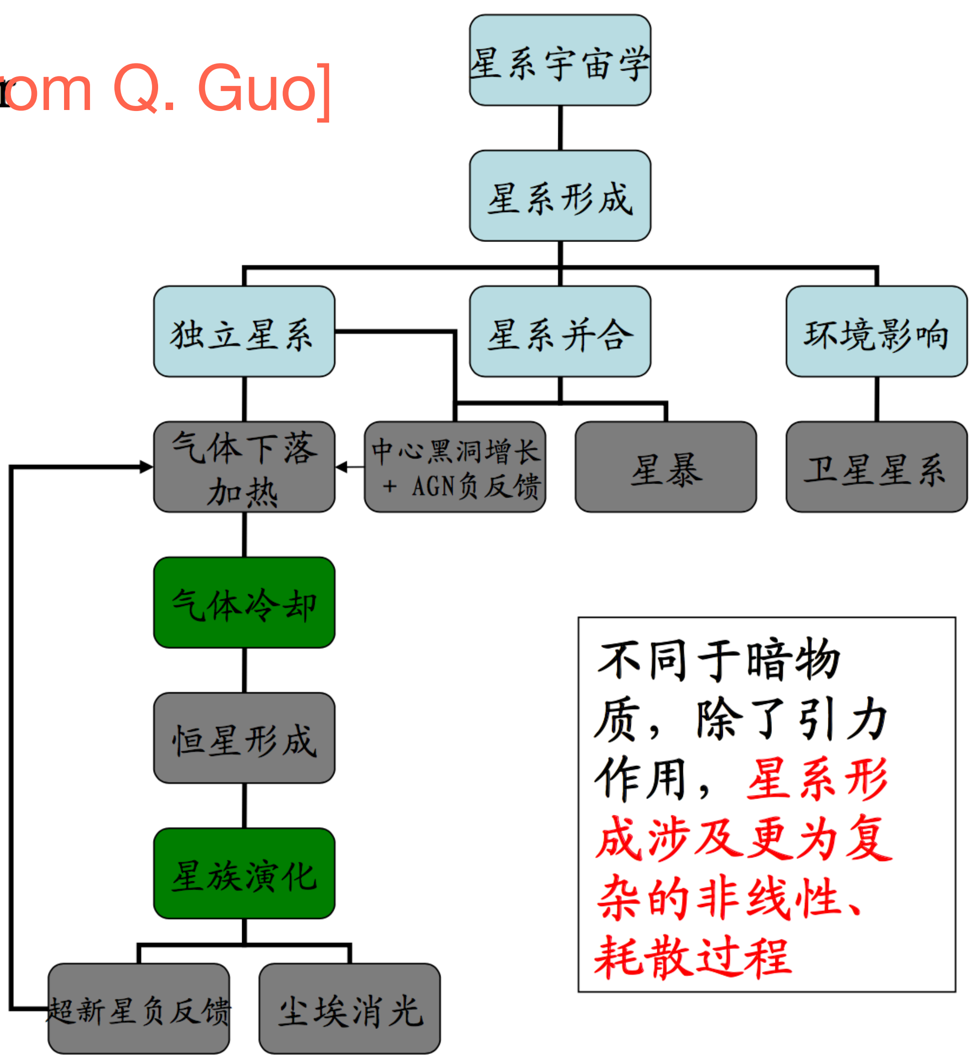
→ **same source (gravitational potential)**



for linear bias, from high-z to low-z, b(z) --> unity

semi-analytic galaxy formation model

[from Q. Guo]



不同于暗物质，除了引力作用，星系形成涉及更为复杂的非线性、耗散过程

complicated interaction, can not calculate analytically need simulation! (expensive!)

N-body simulation (only CDM, purely gravity)

↓

Add hot/cold gas, bh, star, etc. on top of N-body

↓

measure galaxy bias

inflation

(inside of horizon)
growth function

$$P_{gal}(\vec{k}) = P_{ini}(k)T^2(k)D^2(a)[b + f\mu^2]^2$$

transfer function
(outside of horizon)

bias

RSD
anisotropic

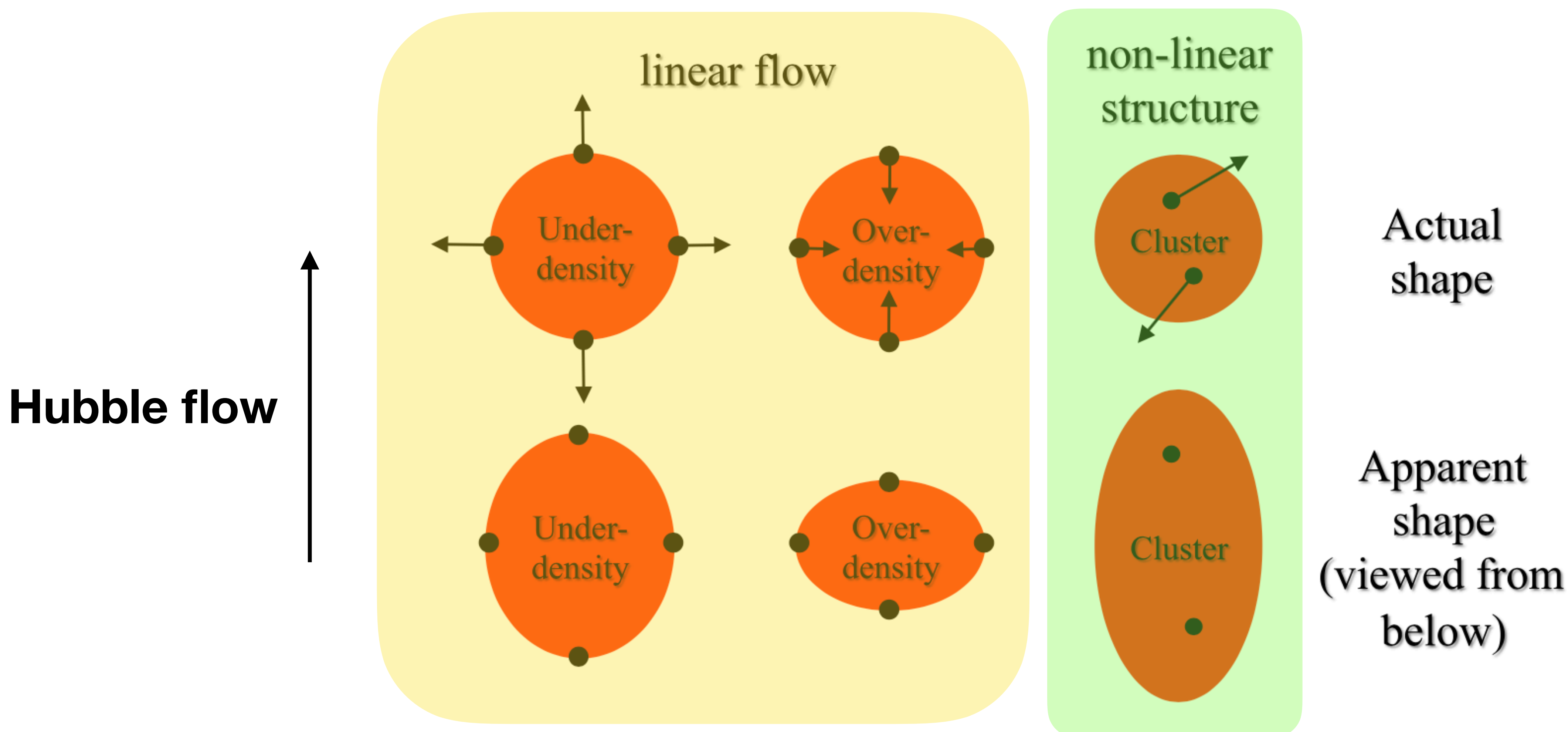
isotropic

Redshift Space Distortion

Redshift measures a combination of "Hubble recession" and "peculiar velocity".

$$v_{\rm obs} = Hr + v_{\rm pec} \quad \Rightarrow \quad \chi_{\rm obs} = \chi_{\rm true} + \frac{v_{\rm pec}}{aH}$$

two type of peculiar velocity (coherent or random)



Kaiser effect **Finger of God**

[from W. Percival]

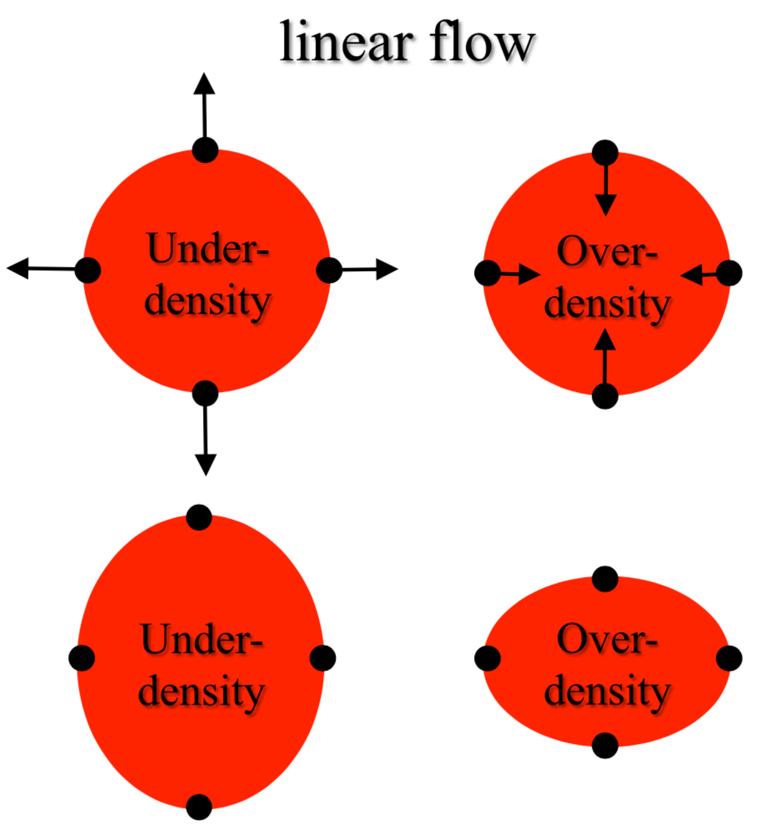
Kaiser effect

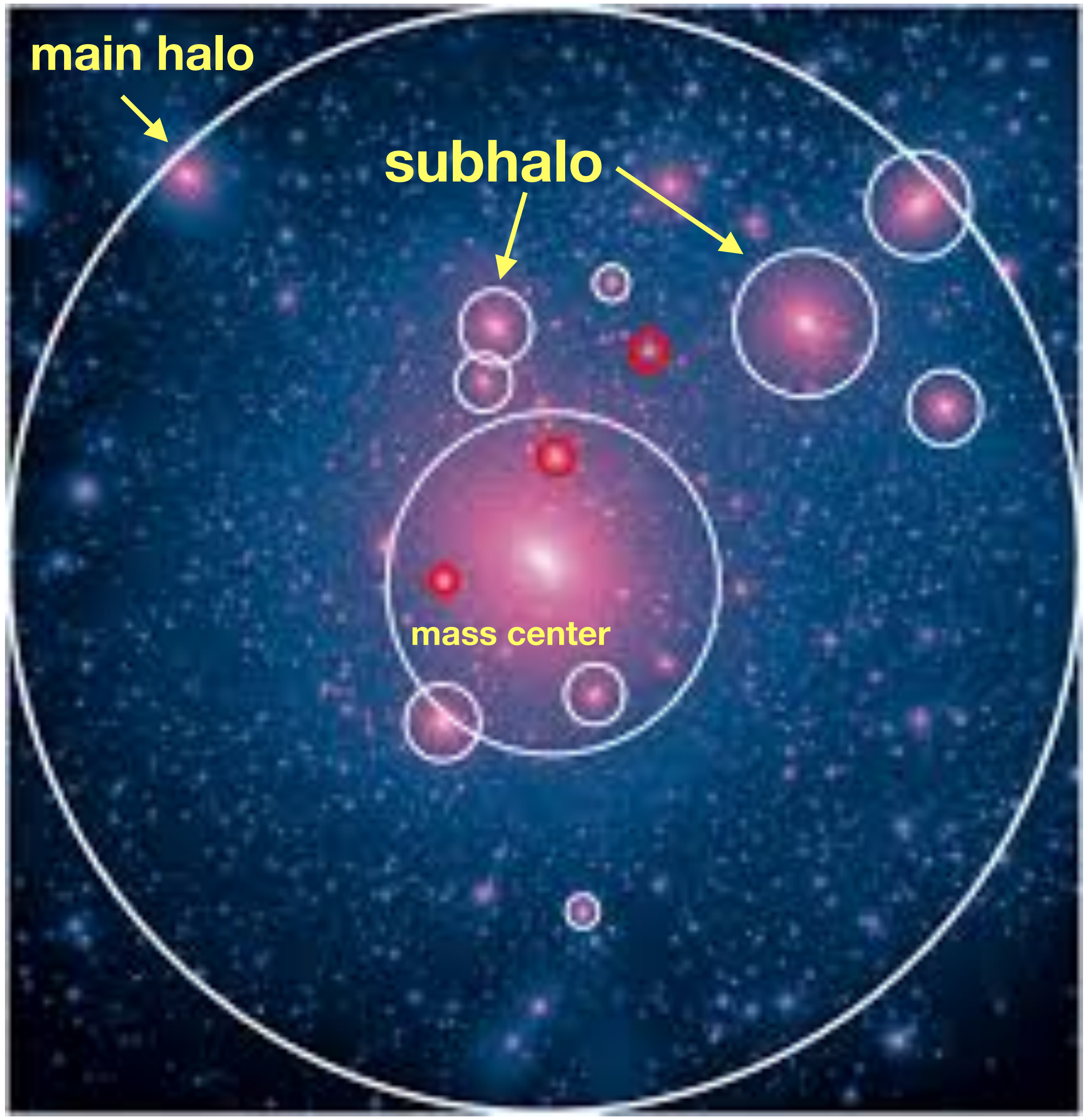
between halos

super-cluster infall

The Kaiser Effect describes the peculiar velocities of galaxies bound to a central mass as they **undergo infall**.
This differs from the Fingers-of-God in that the peculiar velocities are **coherent**, not random, towards the central mass

This effect can only be detected on **large scales**





DM fluid

$$\dot{\vec{u}} + 2H\vec{u}(\vec{x}) = \frac{\vec{g}}{a} \qquad \vec{g}(t,\vec{x}) = -\frac{\vec{\nabla}\Phi(t,\vec{x})}{a}$$

$$\nabla^2\Phi(t,\vec{x}) = 4\pi G a^2 \bar{\rho}\delta_m(t,\vec{x})$$

in MD epoch $\quad \delta(t,\vec{x}) = D(t)\bullet\delta(t_i,\vec{x}) \qquad \Phi(t,\vec{k}) = -4\pi G a^2\bar{\rho}D(t)\dfrac{\delta(t_i,\vec{k})}{k^2}$

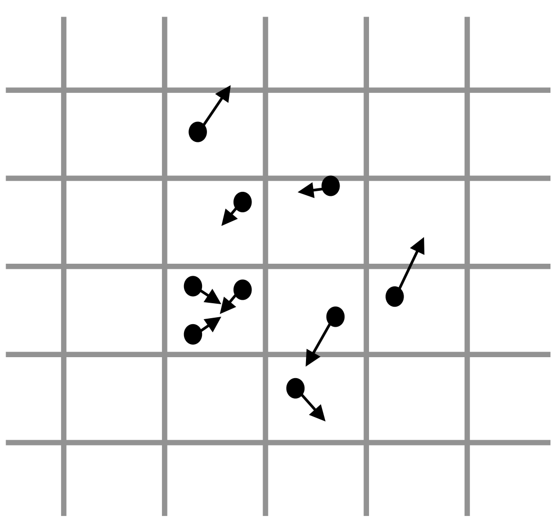
$$\vec{g}(t,\vec{k}) = -i\vec{k}\frac{\Phi(t,\vec{k})}{a} = i4\pi G a\bar{\rho}D(t)\frac{\vec{k}}{k^2}\delta(t_i,\vec{k})$$

$$\rho(t,\vec{x}) = \bar{\rho}(1+\delta(t,\vec{x}))$$

Eulerian coordinate

$$\bar{\rho}(1+\delta(t,\vec{x}))d^3x = \bar{\rho}d^3x'$$

Lagrangian coordinate



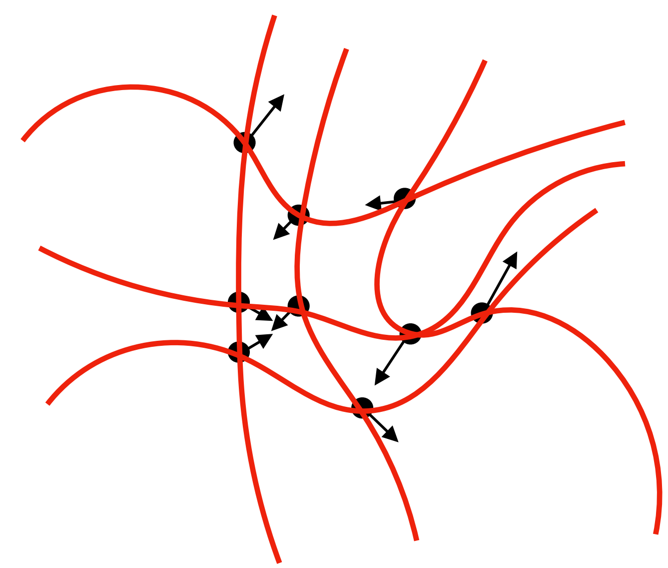
$$1+\delta(t,\vec{x}) = \left|\frac{\partial x^{i\prime}}{\partial x^j}\right|$$

solve the above, we get

$$\vec{x}' = \vec{x} + D(t)\frac{\vec{\nabla}}{\nabla^2}\delta(t_i,\vec{x})$$

$$\vec{v} = \dot{D}(t)\frac{\vec{\nabla}}{\nabla^2}\delta(t_i,\vec{x})$$

$\dot{\vec{x}} = 0$



$\vec{v}(t) \equiv \dot{\vec{x}}'(t)$

We know, in the Eulerian frame, the density field satisfy

$$\dot{\vec{u}} + 2H\vec{u}(\vec{x}) = \frac{\vec{g}}{a} \qquad \vec{g}(t,\vec{x}) = -\frac{\vec{\nabla}\Phi(t,\vec{x})}{a}$$

$$\nabla^2\Phi(t,\vec{x}) = 4\pi G a^2 \bar{\rho}\delta_m(t,\vec{x})$$

in MD epoch $\qquad \delta(t,\vec{x}) = D(t)\bullet\delta(t_i,\vec{x}) \qquad \Phi(t,\vec{k}) = -4\pi G a^2 \bar{\rho} D(t)\frac{\delta(t_i,\vec{k})}{k^2}$

$$\boxed{\ddot{D} + 2H\dot{D} - 4\pi G\bar{\rho}D = 0}$$

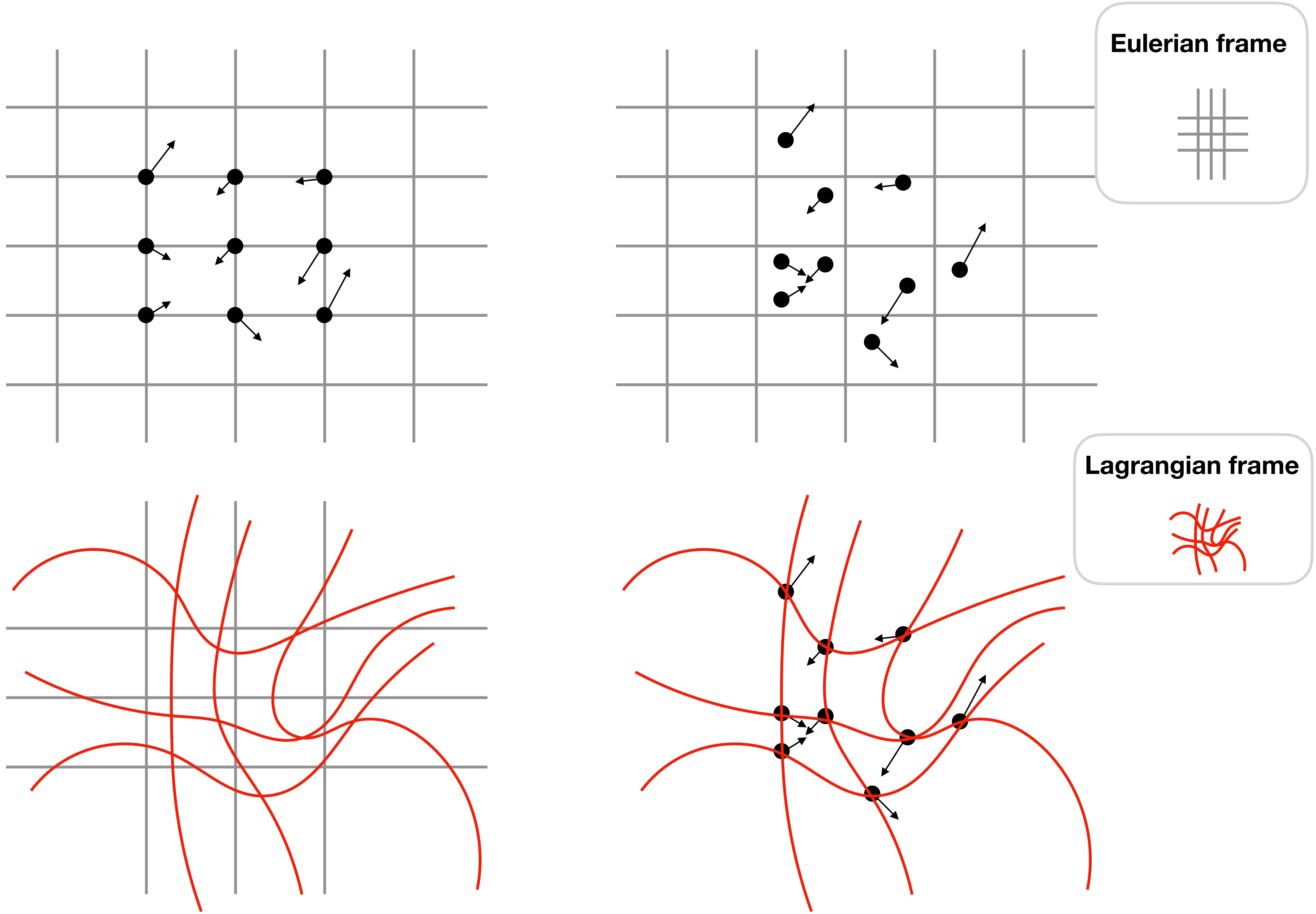
You can prove that the velocity field in the Lagrangian frame $\vec{v}(t) \equiv \dot{\vec{x}}'(t)$

is identical to the fluid velocity field in the Eulerian frame $\vec{u}(t,\vec{x})$

$$\vec{v} = \dot{D}(t)\frac{\vec{\nabla}}{\nabla^2}\delta(t_i,\vec{x})$$

$$\vec{u}(t,\vec{x}) = \vec{v}(t)$$

under Zeldovich approximation, each individual particle travels straight line!




Eulerian observer
Lagrangian observer

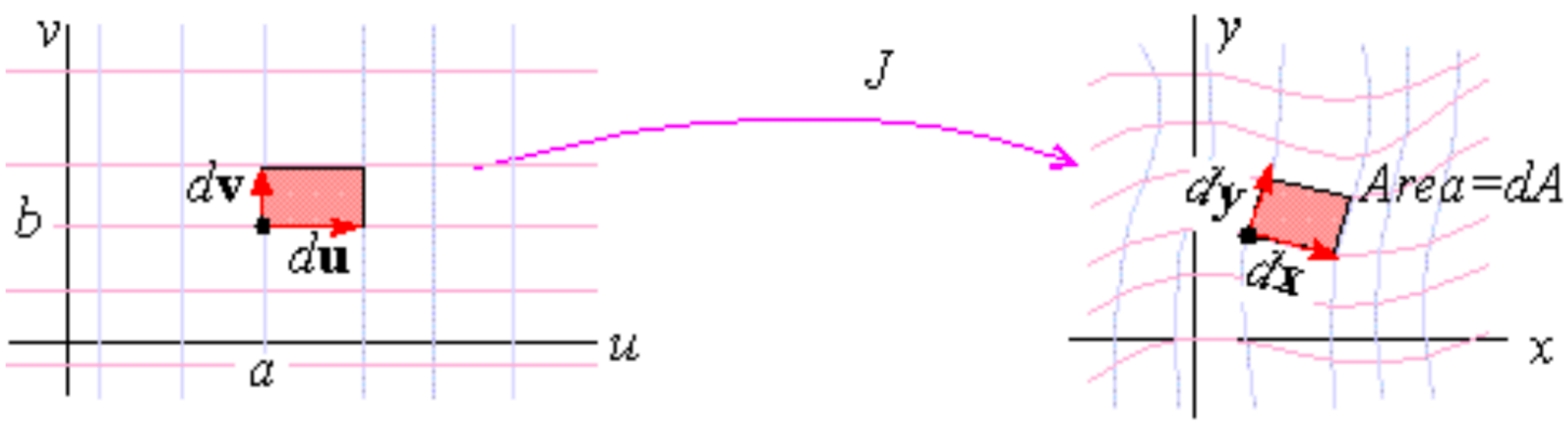
The Area Differential

Jacobi determinant in the coordinate transformation

Let $T(u,v)$ be a smooth coordinate transformation with Jacobian $J(u,v)$, and let R be the rectangle spanned by $\mathbf{du} = \langle du, 0\rangle$ and $\mathbf{dv} = \langle 0, dv\rangle$. If du and dv are sufficiently close to 0, then $T(R)$ is approximately the same as the parallelogram spanned by

$$\begin{aligned} d\mathbf{x} &= J(u,v)\,\mathbf{du} = \langle x_u du, y_u du, 0\rangle \\ d\mathbf{y} &= J(u,v)\,\mathbf{dv} = \langle x_v dv, y_v dv, 0\rangle \end{aligned}$$

If we let dA denote the area of the parallelogram spanned by $d\mathbf{x}$ and $d\mathbf{y}$, then dA approximates the area of $T(R)$ for du and dv sufficiently close to 0.



The cross product of $d\mathbf{x}$ and $d\mathbf{y}$ is given by

$$d\mathbf{x} \times d\mathbf{y} = \left\langle 0, 0, \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} \right\rangle dudv$$

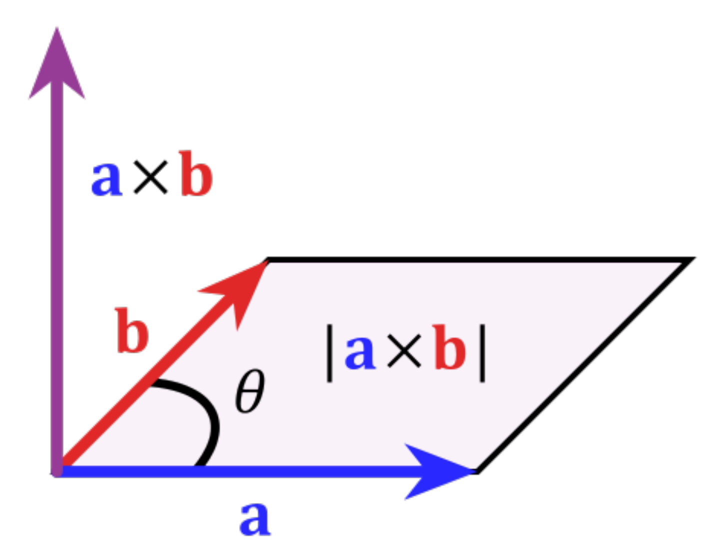
from which it follows that

$$dA = \|d\mathbf{x} \times d\mathbf{y}\| = |x_u y_v - x_v y_u|\, dudv \tag{2}$$

Consequently, the *area differential* dA is given by

$$dA = \left| \frac{\partial(x,y)}{\partial(u,v)} \right| dudv \tag{3}$$

That is, the area of a small region in the uv-plane is scaled by the Jacobian determinant to approximate areas of small images in the xy-plane.



density field at Eulerian coordinate $\vec{x}$

$$\vec{x}' = \vec{x} + D(t)\frac{\vec{\nabla}}{\nabla^2}\delta(t_i,\vec{x})$$

$$\rho(\vec{x})d\vec{x} = \bar{\rho}d\vec{x}'$$

$$\vec{x} = \vec{x}' - D(t)\frac{\vec{\nabla}'}{\nabla'^2}\delta(t_i,\vec{x}')$$

$$\rho(\vec{x}) = \bar{\rho}\left|\frac{\partial\vec{x}'}{\partial\vec{x}}\right| = \bar{\rho}\left|\frac{\partial\vec{x}}{\partial\vec{x}'}\right|^{-1} = \frac{\bar{\rho}}{\left|\delta_{ij} - D(t)\Psi_{ij}\right|}$$

$$\Psi_{ij} \equiv \frac{\partial^2\delta(t_i,\vec{x}')}{\partial x_i'\partial x_j'}$$

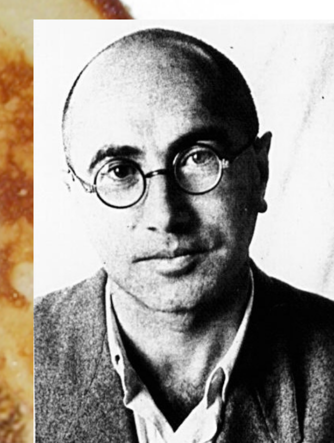
tidal shear tensor

If eigenvalues are $\lambda_1 < \lambda_2 < \lambda_3$

$$\rho(\vec{x}) = \frac{\bar{\rho}}{(1-D\lambda_1)(1-D\lambda_2)(1-D\lambda_3)}$$

The over density region will first collapse to a pancake along the λ_3 axis

Zeldovich Pancake



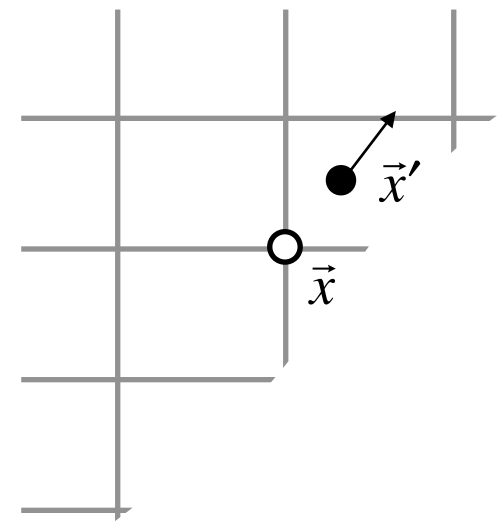
$$\vec{x}' = \vec{x} + D(t)\frac{\vec{\nabla}}{\nabla^2}\delta(t_i,\vec{x})$$

$$\underset{\uparrow}{\Delta\vec{x}} \equiv \vec{x}' - \vec{x} = D(t)\frac{\vec{\nabla}}{\nabla^2}\delta(t_i,\vec{x})$$

displacement field

$$\vec{v} = \dot{D}(t)\frac{\vec{\nabla}}{\nabla^2}\delta(t_i,\vec{x})$$

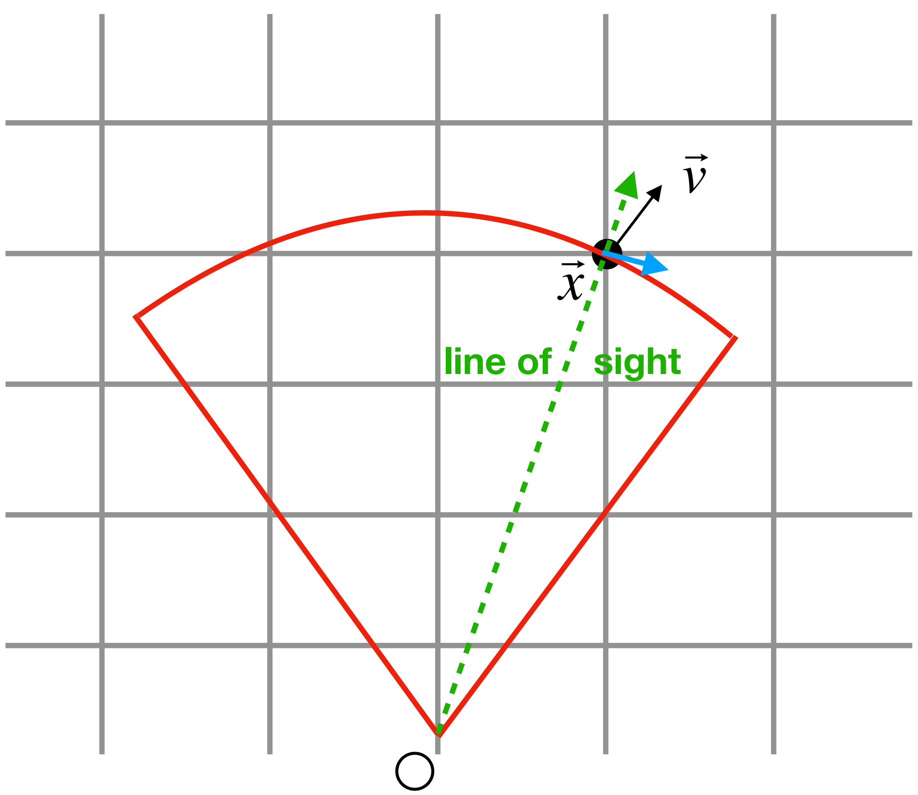
$$\vec{v} = Hf\,\Delta\vec{x}$$



growth rate: $f \equiv \dfrac{d\log D}{d\log a} \approx \Omega_m^{\,0.55}; (\Lambda CDM)$

a galaxy at $\vec{x}$ in real space, corresponds to $\vec{s}$ in redshift space

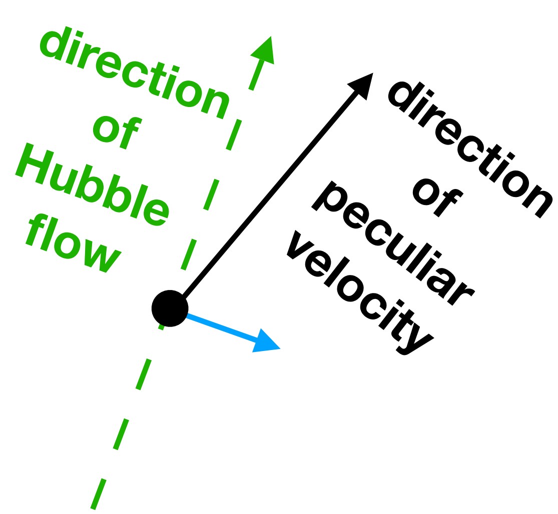
$$\boxed{\vec{s} = \vec{x} + \frac{\hat{\vec{x}}\bullet\vec{v}}{H}\hat{\vec{x}}}$$



$$\vec{v}_{obs} = H\vec{x}_{true} + \vec{v}_p$$

$$\vec{v}_{obs} = H\vec{x}_{obs} = H\vec{s}$$

only the LoS component contribute to the redshift measurement



$$\boxed{\vec{s} = \vec{x} + \frac{\hat{\vec{x}} \bullet \vec{v}}{H}\hat{\vec{x}}}$$

Follow the above prescription

$$\rho(\vec{x})d\vec{x} = \rho(\vec{s})d\vec{s}$$

$$(1+\delta^x(\vec{x}))d\vec{x} = (1+\delta^s(\vec{s}))d\vec{s}$$

$$\vec{v} = \dot{D}(t)\frac{\vec{\nabla}}{\nabla^2}\delta(t_i,\vec{x})$$

$$\vec{v} = Hf \bullet D(t)\frac{\vec{\nabla}}{\nabla^2}\delta(t_i,\vec{x})$$

$$\delta^s(t,\vec{s}) = \frac{1+\delta^x(t,\vec{x}) - |J|}{|J|} \qquad |J| \equiv \left|\frac{\partial s^i}{\partial x^j}\right|$$

$$\vec{s} = \vec{x} + f(t)D(t)\frac{-i(\hat{\vec{x}} \bullet \vec{k})}{-k^2}\delta(t_i,\vec{k})\hat{\vec{x}}$$

$$J = \left\{\frac{\partial s_i}{\partial x_j}\right\} = \left\{\delta_{ij} + f(t)D(t)\frac{-i(\hat{\vec{x}} \bullet \vec{k})}{-k^2}(-ik_j)\delta(t_{ini},\vec{k})\hat{\vec{x}}_i\right\}$$

$$\hat{k} \bullet \hat{x} = \mu$$

inclination angle between wave mode direction $\hat{k}$ and LoS direction $\hat{x}$

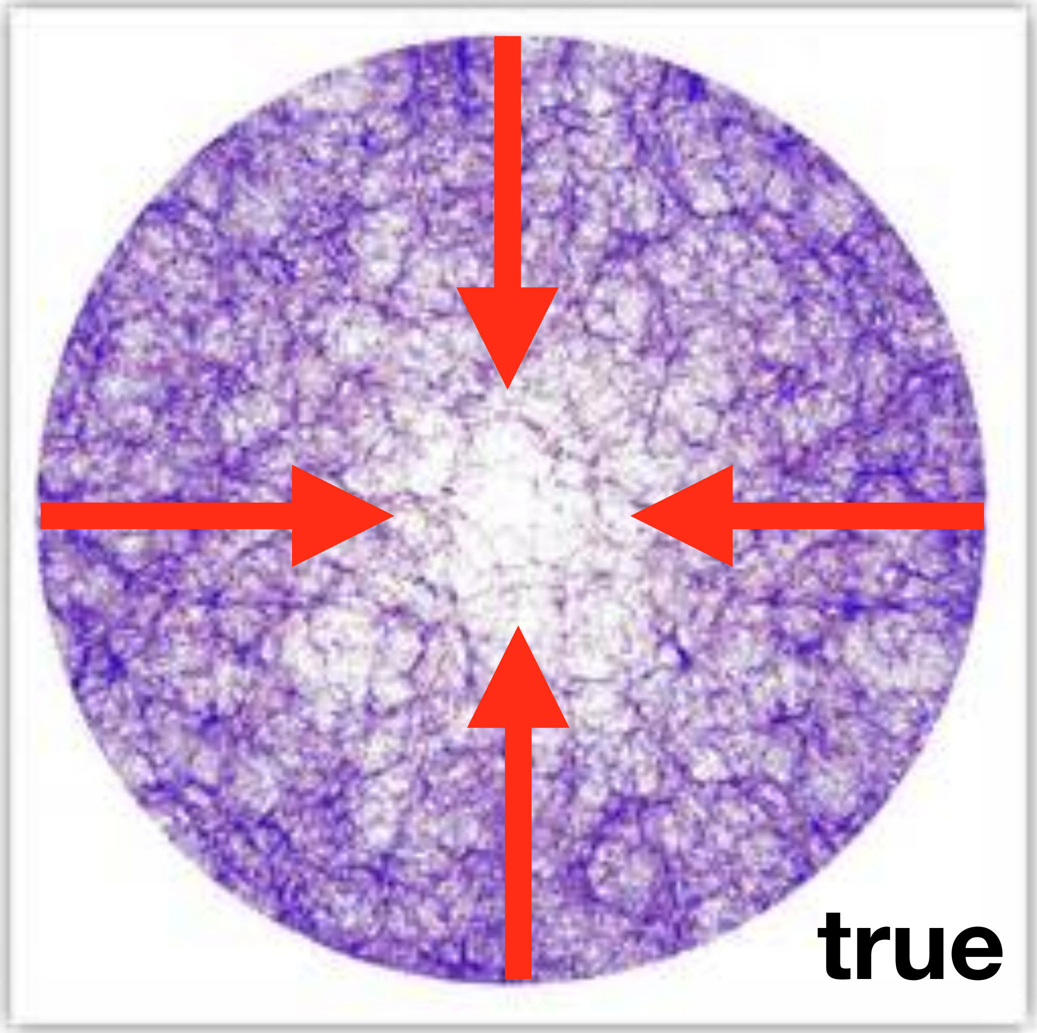
$$Det|J| = 1 + f\mu^2 D\delta(t_i,\vec{k}) = 1 + f\mu^2\delta^x(t,\vec{k})$$

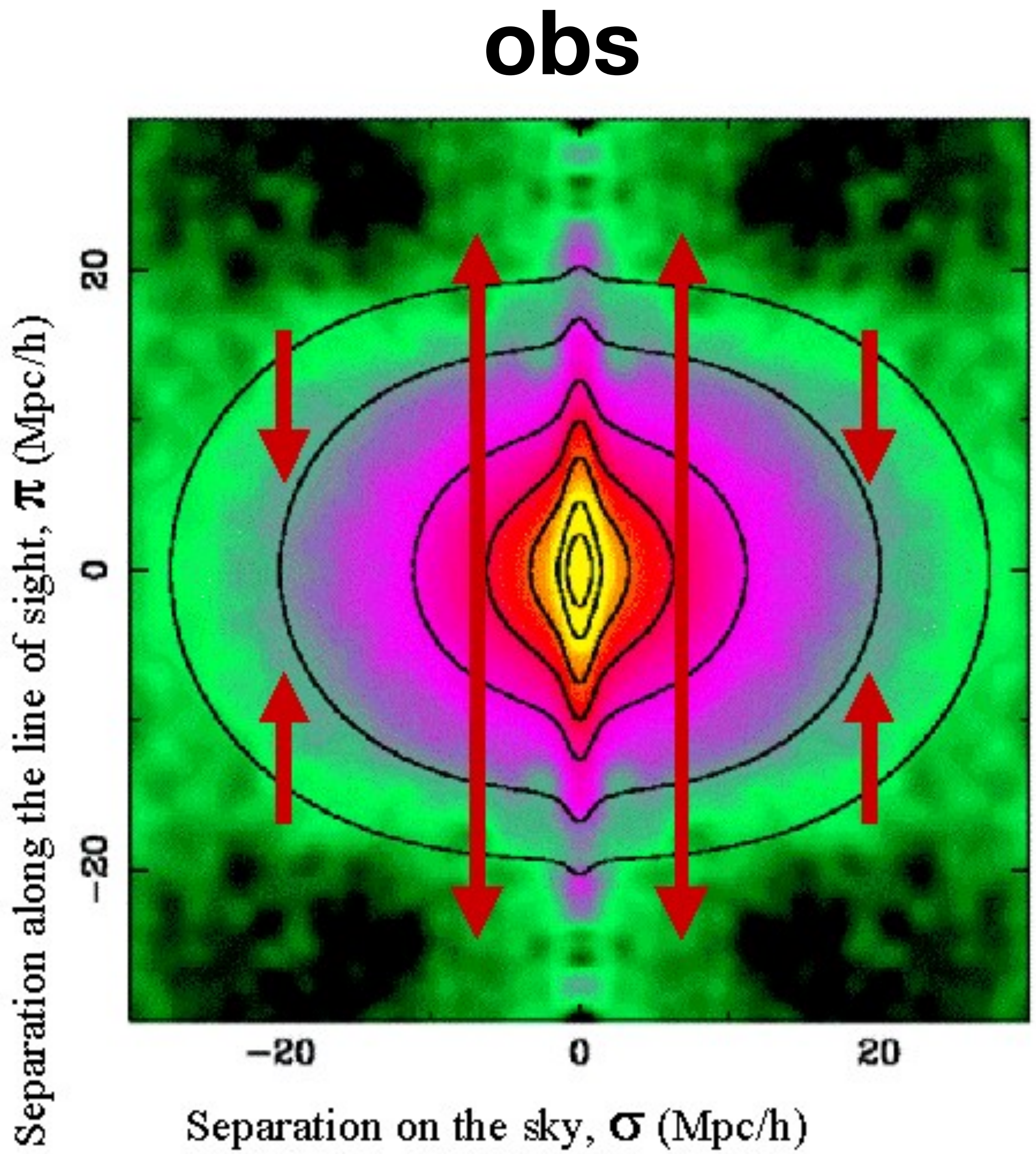
isotropic

$$\boxed{\delta^s(t,\vec{k}) = \delta^x(t,\vec{k})(1+f\mu^2)}$$

anisotropic

Kaiser formula (Kaiser, 1987, MNRAS, 227, 1)


true


obs
Separation along the line of sight, π (Mpc/h)
Separation on the sky, σ (Mpc/h)
20
0
−20
−20
0
20

what do linear z-space distortions measure?

linear scales,

$$\delta_g^s(\mu) = \delta_g + \mu^2\theta$$

$$P_g^s(\mu) = \langle|\delta_g + \mu^2\theta|^2\rangle$$

$$= P_{gg} + 2\mu^2 P_{g\theta} + \mu^4 P_{\theta\theta}$$

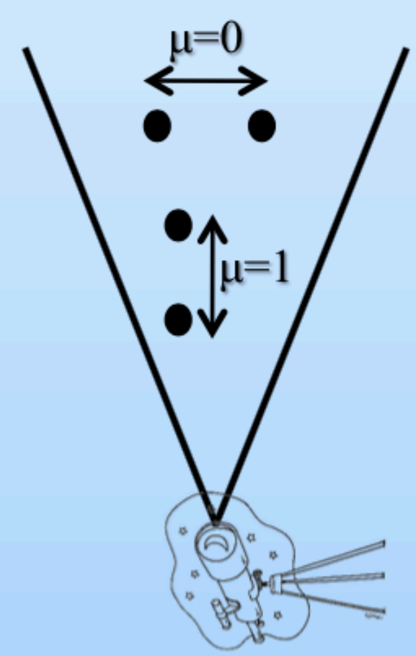
Galaxy-galaxy power

Velocity-velocity power

Galaxy-velocity divergence cross power

$$\mu = cos(\alpha)$$

$$\theta = \nabla\cdot\mathbf{u}$$



In linear regime,

$$\theta = -f\delta(\text{mass}), \quad f \equiv \frac{d\ln G}{d\ln a}$$

Linear growth rate

so amplitude of power spectrum constrains

$$(\sigma_8^s)^2 = [b\sigma_8(\text{mass}) + \mu^2 f\sigma_8(\text{mass})]^2$$

[from W. Percival]

Kaiser 1987, MNRAS, 227, 1

FoG

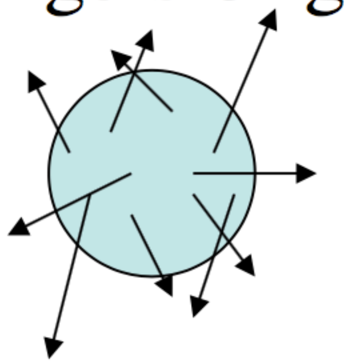
within a halo

The Fingers-of-God effect is attributed to **random** velocity dispersions in **galaxy clusters** that deviate a galaxy's velocity from pure Hubble flow, stretching out a cluster in redshift space.

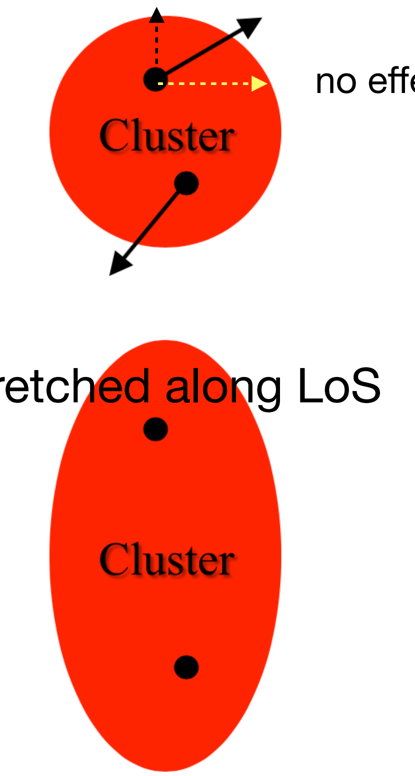
small-scale

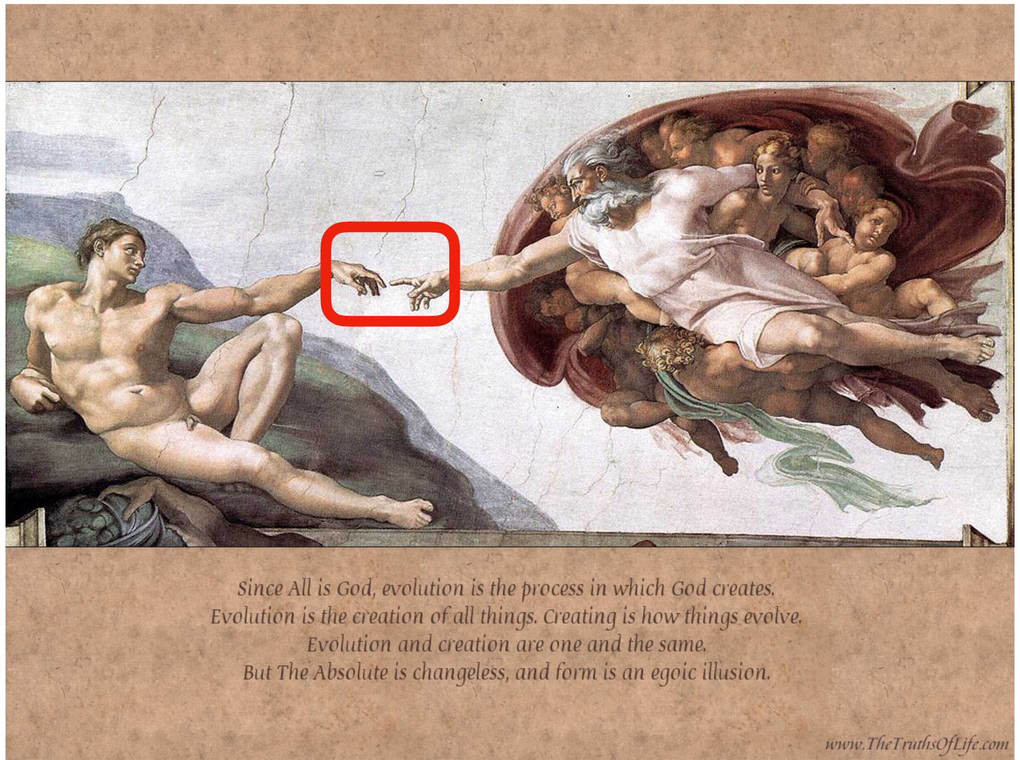
Random (thermal) motion

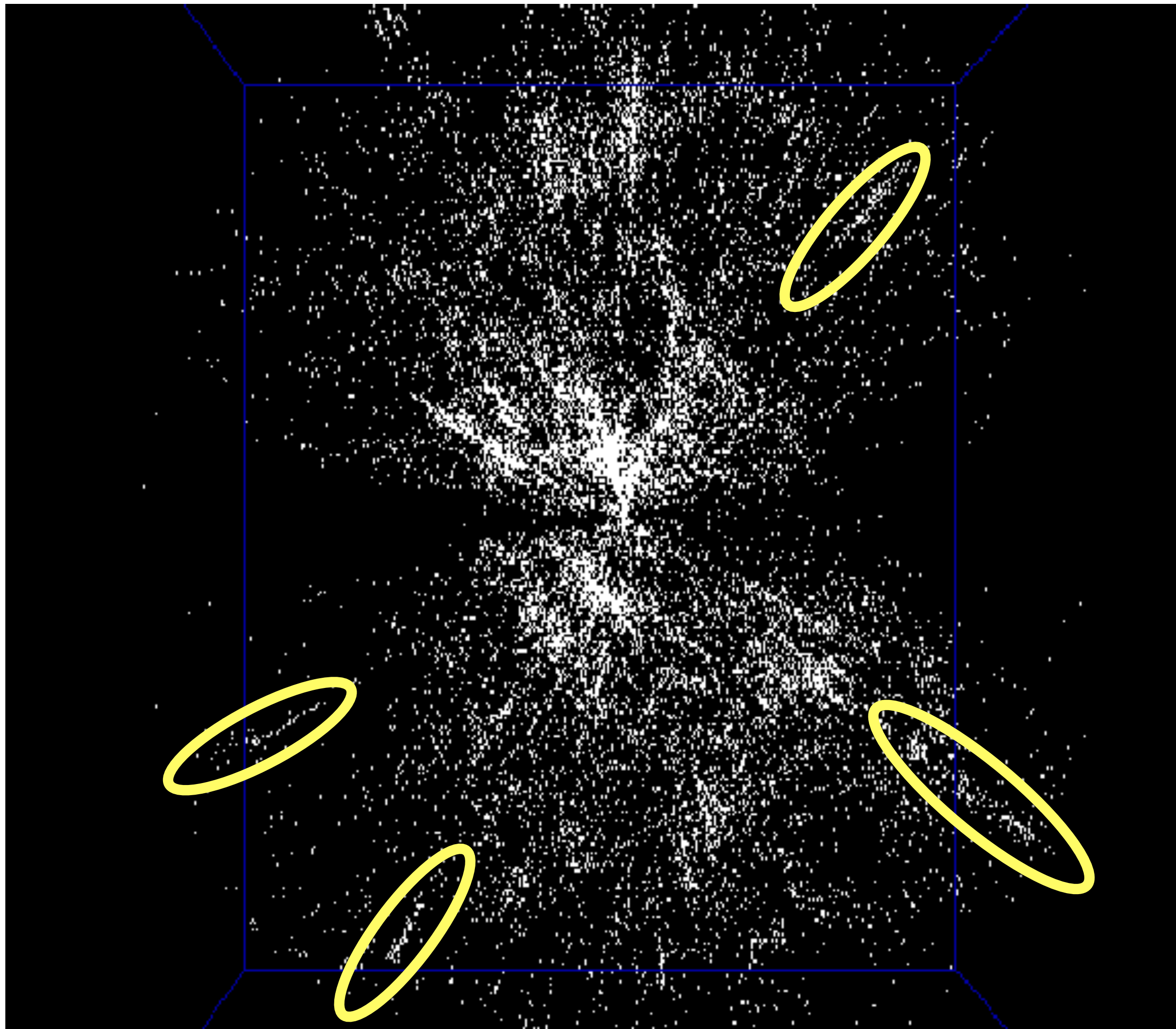
(fingers-of-god)



non-linear structure







Fingers-of-god

- So far we have neglected the motion of particles/ galaxies inside "virialized" dark matter halos.
- These give rise to fingers-of-god which suppress power at high *k*.
- Peacock (1992) 1st modeled this as Gaussian "noise" so that
  - $P^s(k, \mu) = P^r(k)\,[b+f\mu^2]^2\,\mathrm{Exp}[-k^2\mu^2\sigma^2]$
- Sometimes see this written as $P_{\delta\delta}+P_{\delta\theta}+P_{\theta\theta}$ times Gaussians or Lorentzians.
  - Beware: no more general than linear theory!

[from M. White]

Redshift space distortions

At large distances (distant observer approximation), redshift-space distortions affect the power spectrum through:

$$P_s = P_r(1 + \beta\mu^2)^2(1 + k^2\mu^2\sigma_p^2/2)^{-1}$$

Large-scale distortion can also be written in terms of $\beta = f/b$

On small scales, galaxies lose all knowledge of initial position. If pairwise velocity dispersion has an exponential distribution (superposition of Gaussians), then we get this damping term for the power spectrum.

[from W. Percival]

Legendre expansion

Rather than deal with a 2D function we frequently expand the angular dependence in a series of Legendre polynomials.
The Rayleigh expansion of the plane-wave related the moments in *k*-space and *r*-space:

$$\Delta^2(k,\hat{k}\cdot\hat{z}) \equiv \frac{k^3 P(k,\mu)}{2\pi^2} = \sum_\ell \Delta^2_\ell(k) L_\ell(\mu)$$

$$\xi(r,\hat{r}\cdot\hat{z}) \equiv \sum_\ell \xi_\ell(r) L_\ell(\hat{r}\cdot\hat{z}) \quad , \quad \xi_\ell(r) = i^\ell \int \frac{dk}{k}\, \Delta^2_\ell(k) j_\ell(kr)$$

If we use recurrence relations between j_l we can write ξ_l in terms of integrals of ξ times powers of *r*. e.g.

$$\int \frac{dk}{k}\Delta^2_2(k) j_2(kr) = \frac{3}{s^2}\int_0^s s^2 ds\ \xi(s) - \xi(s) = \bar{\xi}(< s) - \xi(s)$$

[from M. White]

Legendre expansion

Note that the ratios of the moments is independent of *k* but not of *r*.

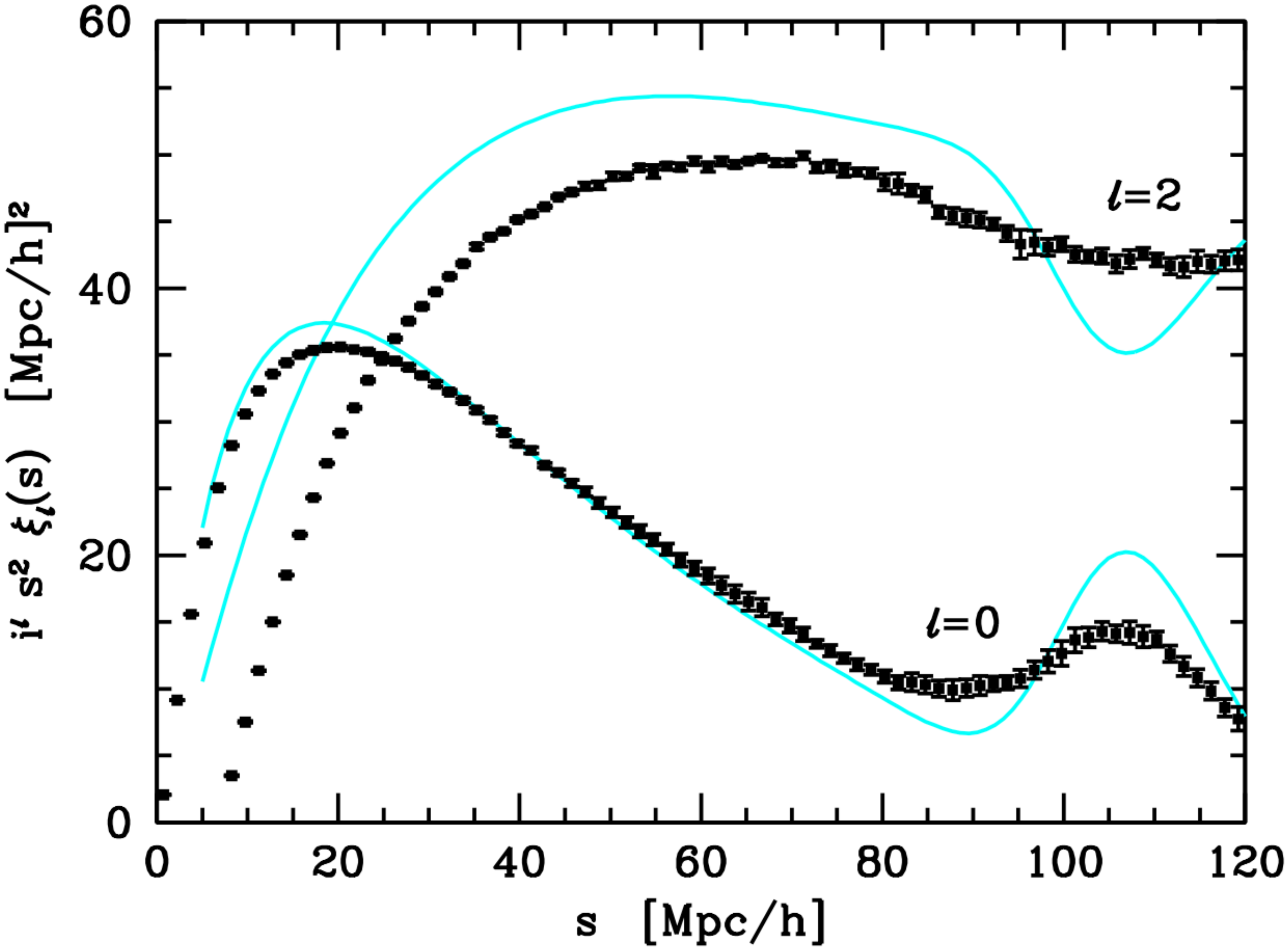
The Kaiser formula involved only terms up to μ^4, so on large scales ($k\sigma \ll 1$) this series truncates quite quickly.

$$\begin{pmatrix} \Delta_0^2(k) \\ \Delta_2^2(k) \\ \Delta_4^2(k) \end{pmatrix} = \Delta^2(k) \begin{pmatrix} b^2 + \frac{2}{3}bf + \frac{1}{5}f^2 \\ \frac{4}{3}bf + \frac{4}{7}f^2 \\ \frac{8}{35}f^2 \end{pmatrix}$$

Typically only measure (well) ***l***=0, 2.

[from M. White]

Kaiser is not particularly accurate



[from M. White]

Alcock-Paczynski Test

AP (I)

Alcock & Paczynski (1979),
An evolution free test for non-zero cosmological constant,
Nature **281**, **358**

A pure geometric probe of the cosmic expansion history,

by measuring shapes of objects which are known to be isotropic.

If we adopt an incorrect cosmology to measure these objects,

they appear stretched/elongated in the line-of-sight (LOS) direction.

An evolution free test for non-zero cosmological constant

Charles Alcock

The Institute for Advanced Study, Princeton, New Jersey 08450

Bohdan Paczyński*

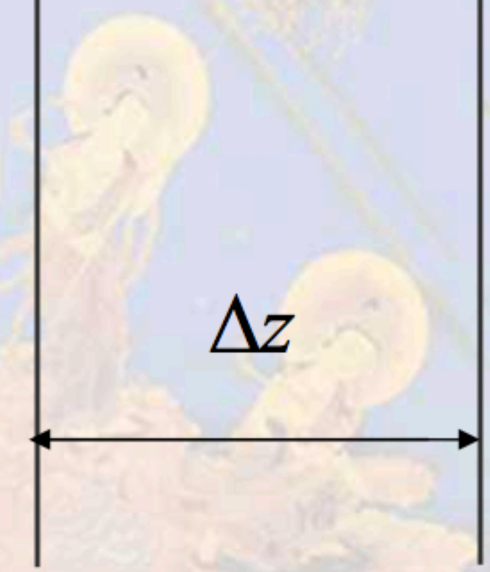
Department of Astronomy, University of California at Berkeley, Berkeley, California 94720 and Princeton University Observatory, Princeton, New Jersey 08540

The cosmological constant has recently been questioned because of difficulties in fitting the standard $\Lambda = 0$ cosmological models to observational data[1,2]. We propose here a cosmological test that is a sensitive estimator of Λ. This test is unusual in that it involves no correction for evolutionary effects. We present here the idealised conception of the method, and hint at the statistical problem that its realisation entails.

[from X.D. Li]

AP (II)

Considering some objects in the Universe which is known to be isotropic.
We are measuring its redshift span Δz and angular size $\Delta\theta$:



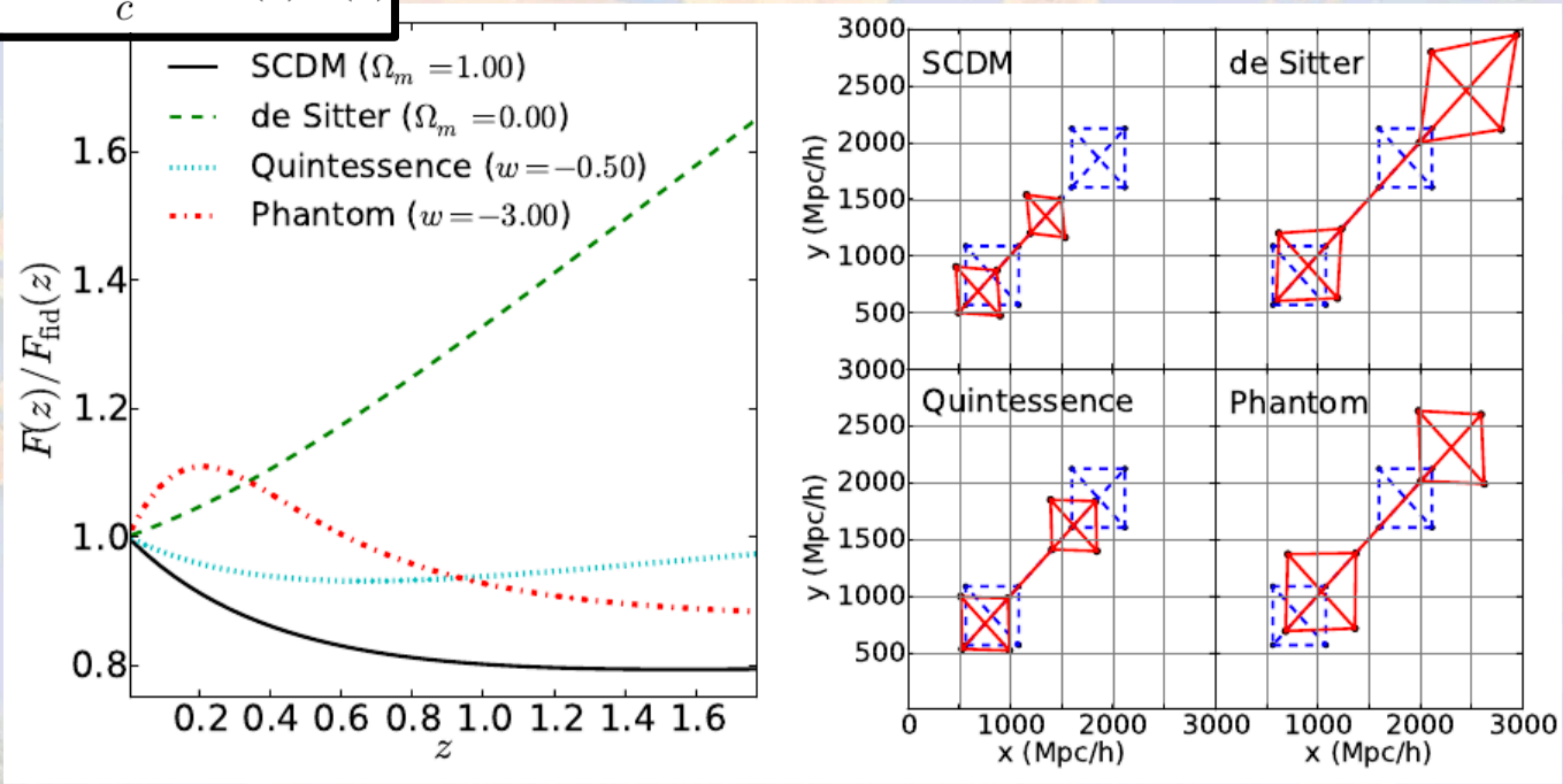
Adopting a certain cosmology we calculate its sizes in the radial and tangential directions:

$$\Delta r_{\parallel} = \frac{c}{H}\Delta z, \quad \Delta r_{\perp} = (1+z)D_A(z)\Delta\theta$$

[from X.D. Li]

AP (III)

$$\boxed{F(z) \equiv \frac{(1+z)}{c} D_A(z) H(z)}$$

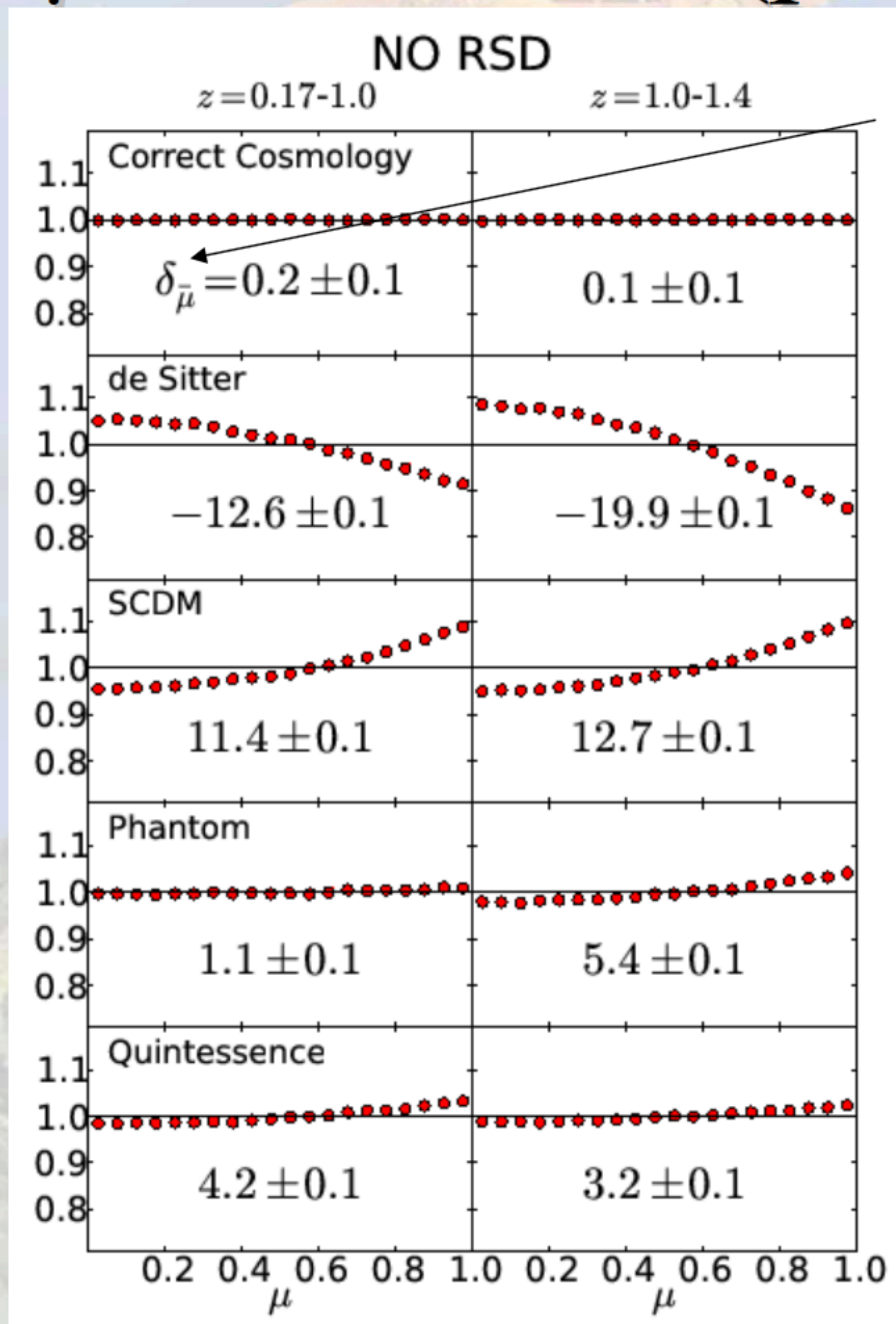


- Wrong cosmologies adopted to calculate $r(z)$ → Anisotropy
- Small/Large $F(z)$ → compression/stretch along LOS
- Note the **cosmological dependence** and **redshift dependence**!

[from X.D. Li]

μ from Mock (pure AP)

Degenerate with RSD!

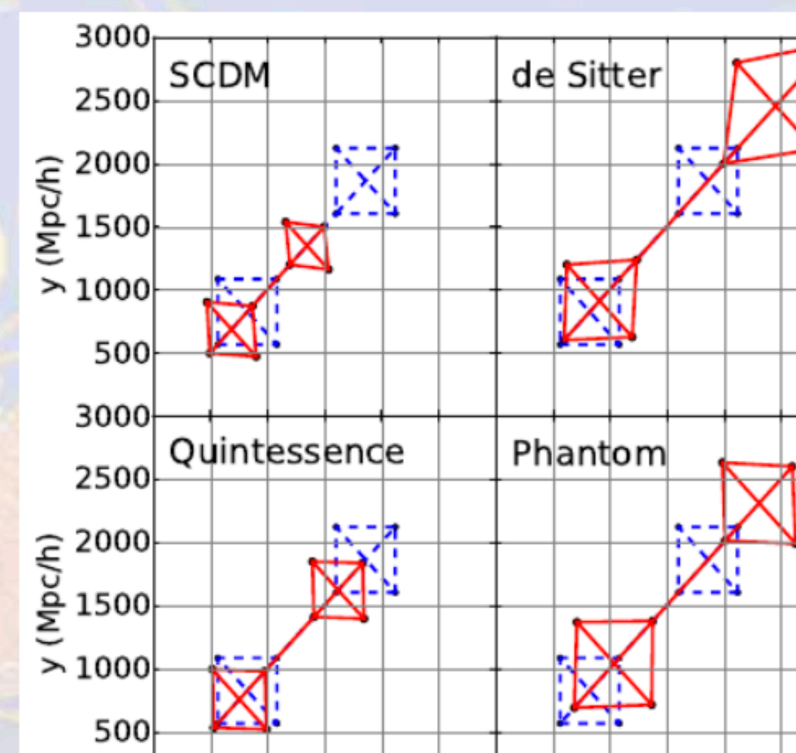


$$\delta_\mu \equiv (\bar{\mu} - 0.5) \times 10^3$$

- Correct Cosmology: uniform
- Incorrect Cosmologies:
  - Deviated from uniform (δ_μ=0) at **234σ, 160σ, 50σ, 42σ**
  - Redshift dependence detected at:

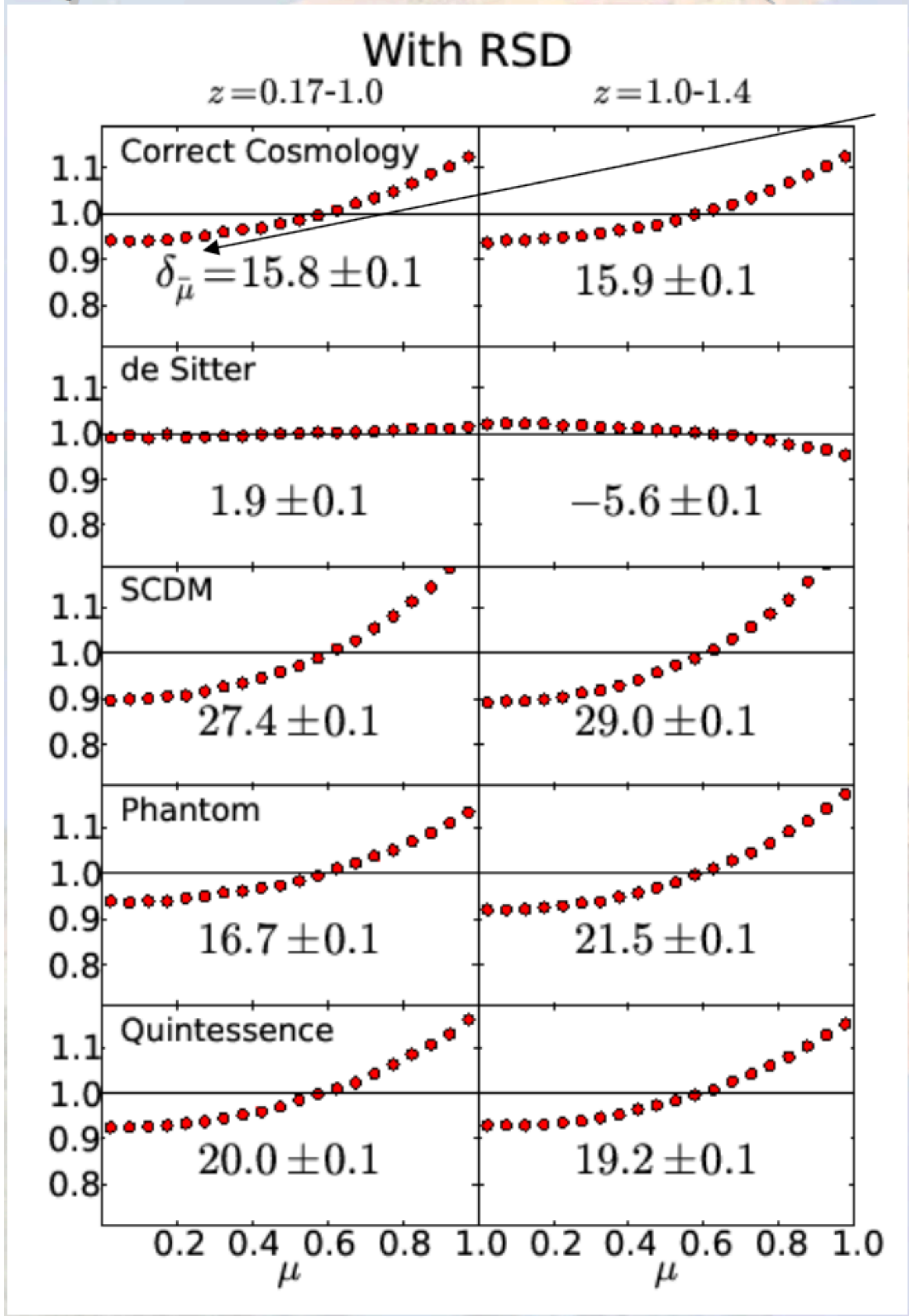
48σ, 8.1σ,

29σ, 7.0σ



[from X.D. Li]

μ from Mock (AP+RSD)



Degenerate with RSD!

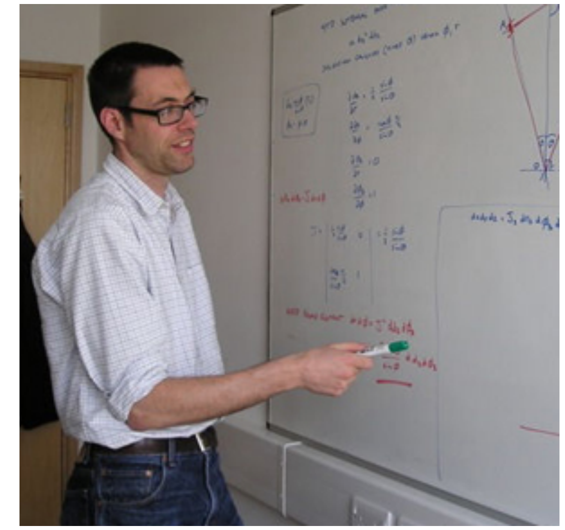
$$\delta_\mu \equiv (\bar{\mu} - 0.5) \times 10^3$$

- Deviation from uniform:
  - $\delta_\mu > 0$ at >40σ: RSD overwhelms AP!
- But, redshift-dependence of μ...
  - Correct cosmology: **< 1σ**
  - Incorrect cosmologies:

    40σ, 10.4σ, 33σ, 5.5σ
- *Effect of RSD is large but its redshift dependence is small*

[from X.D. Li]

FurtherReading:



Large Scale Structure Observations

https://arxiv.org/abs/1312.5490

Will Percival @ ICG, Portsmouth

BAO

http://mwhite.berkeley.edu/BAO/bao_iucca.pdf

RSD

http://mwhite.berkeley.edu/Talks/SantaFe12_RSD.pdf

Martin White @ UC Berkeley