Cosmic Large-scale Structure Formations

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18 weeks

outline

Background (1 w)

- universe geometry and matter components (1 hr)
- Standard candle (SNIa) (0.5 hr)
- Standard ruler (BAO) (0.5 hr)

Linear perturbation (9 w)

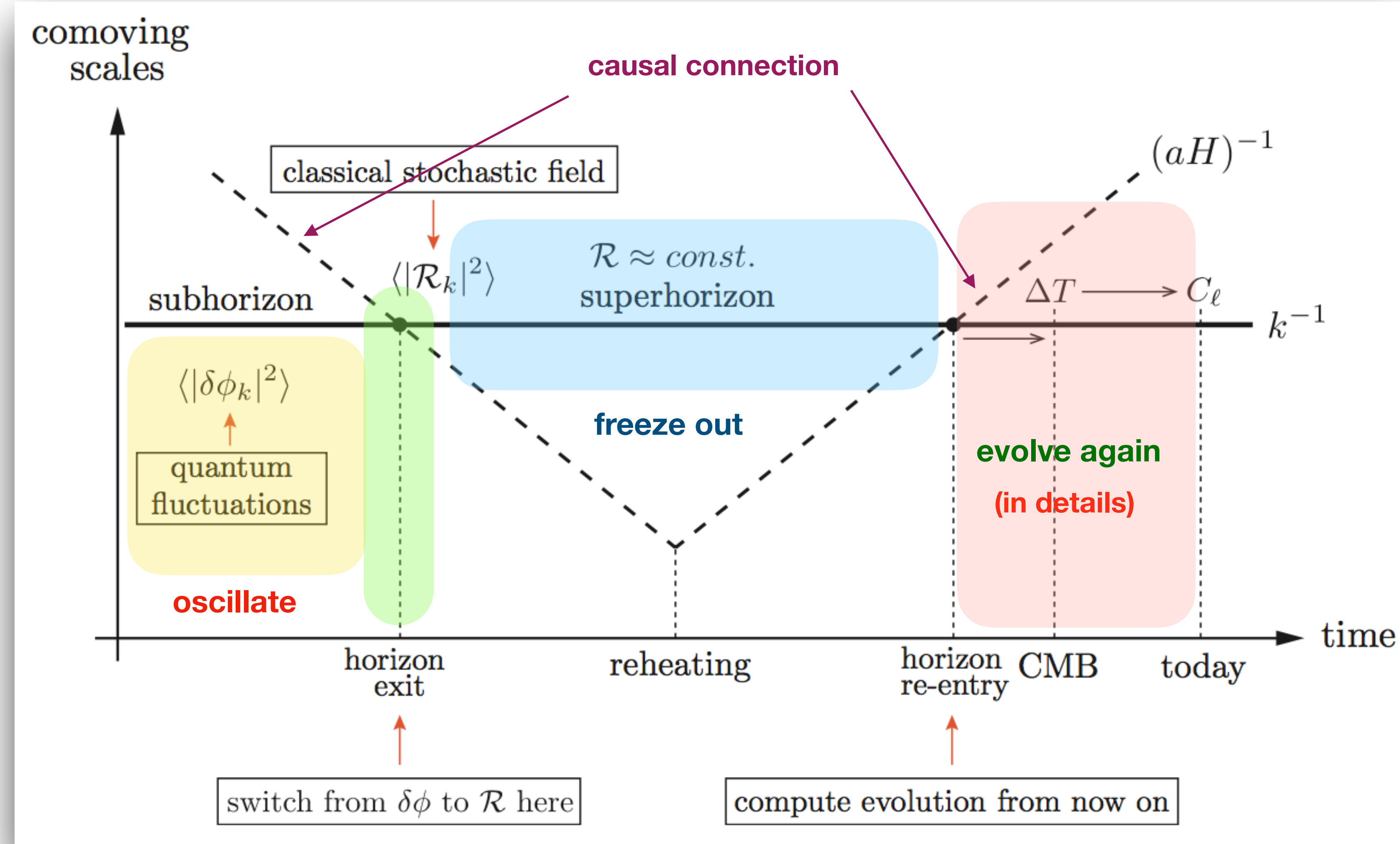
- relativistic treatment perturbation (2 hr)
- primordial power spectrum (2 hr)
- linear growth rate (2 hr)
- galaxy 2-pt correlation function (2 hr)
- Baryon Acoustic Oscillation (BAO) (2 hr)
- Redshift Space Distortion (RSD) (2 hr)
- Weak Lensing (2 hr)
- Einstein-Boltzmann codes (2 hr)

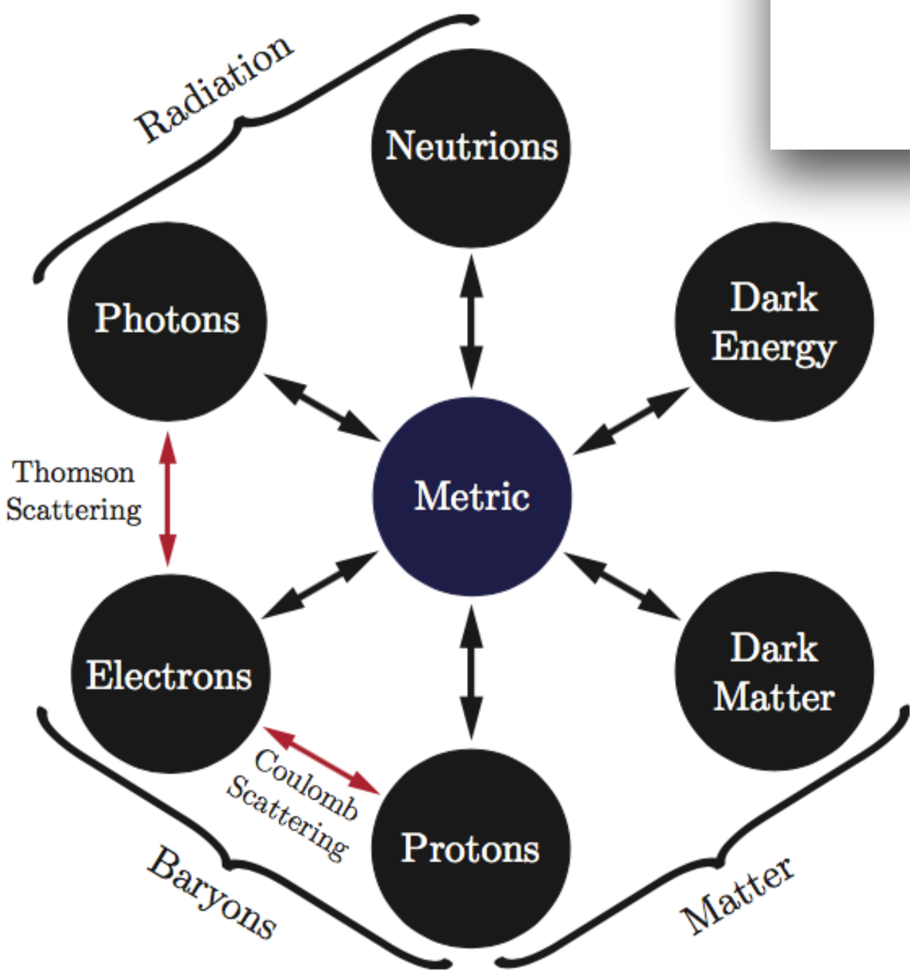
Non-linear perturbation (6 w)

- Non-linear power spectrum (2 hr)
- halo model (2 hr)
- N-body simulation algorithms (2 hr)
- Press-Schechter (PS) halo mass function (2 hr)
- Extended-PS (EPS) halo mass function (2 hr)
- halo bias & halo density profile (2 hr)

Statistical analysis (2 w)

- Monte-Carlo Markov Chain sampler (2 hr)
- CosmoMC use (2 hr)





in lecture2, we only discuss the gravitational sector. Now, we will study the matter sector

Gravitational potential

super-horizon

$$\mathcal{R} = -\Phi - \frac{2}{3(1+w)}\left(\frac{\Phi'}{\mathcal{H}} + \Phi\right)$$

gauge-inv curvature pert.

$$\begin{aligned} \nabla^2\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) &= 4\pi G a^2\,\delta\rho\,, \\ \Phi' + \mathcal{H}\Phi &= -4\pi G a^2(\bar\rho + \bar P)v \\ \Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi &= 4\pi G a^2\,\delta P\,. \end{aligned}$$

gravitational potential eq.

$$\boxed{\Phi'' + 3(1+w)\mathcal{H}\Phi' + wk^2\Phi = 0}$$ (deriv)

$$\delta = -\frac{2}{3}\frac{k^2}{\mathcal{H}^2}\Phi - \frac{2}{\mathcal{H}}\Phi' - 2\Phi \quad \Rightarrow \quad \delta \approx -2\Phi = const.$$

$$\Phi = const. \qquad \text{(superhorizon)}$$

density pert. is frozen on the super-horizon regime

loss causal connection

adiabatic IC

$$\delta\tau = \frac{\delta\rho_I}{\bar\rho'_I} = \frac{\delta\rho_J}{\bar\rho'_J} \qquad \text{for all species } I \text{ and } J$$

$$\delta_m = \frac{3}{4}\delta_r \approx -\frac{3}{2}\Phi_{\rm RD}$$ (deriv)

Radiation-to-matter transition

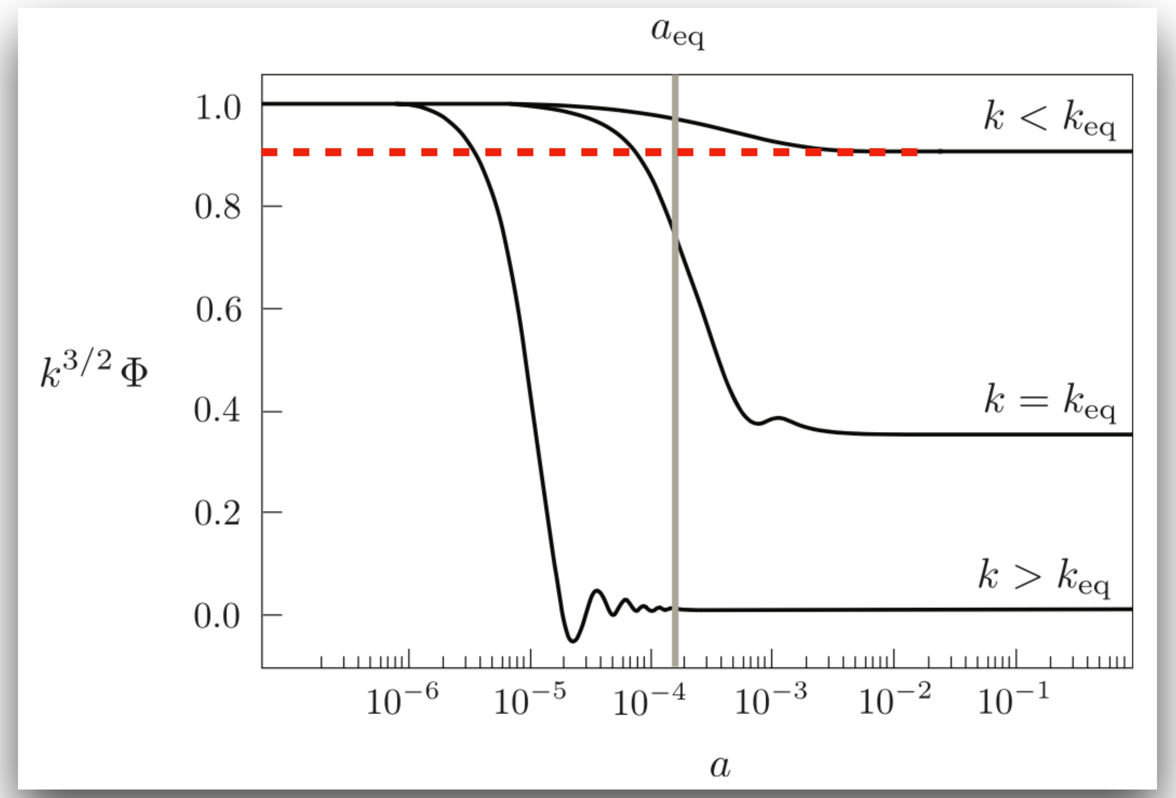
(Baumann lecture §4.3)

$\mathcal{R}$ is **always conserved** in the super-horizon regime,

Φ does not conserve if w is dynamical!

$$\boxed{\mathcal{R} = -\frac{5+3w}{3+3w}\Phi} \qquad \text{(superhorizon)}$$

$$\mathcal{R} = -\frac{3}{2}\Phi_{\rm RD} = -\frac{5}{3}\Phi_{\rm MD} \quad \Rightarrow \quad \Phi_{\rm MD} = \frac{9}{10}\Phi_{\rm RD}$$



sub-horizon evolution

RD w=1/3

$$\Phi'' + \frac{4}{\tau}\Phi' + \frac{k^2}{3}\Phi = 0 \qquad \Phi_{\boldsymbol{k}}(\tau) = A_{\boldsymbol{k}}\frac{j_1(x)}{x} + B_{\boldsymbol{k}}\frac{n_1(x)}{x} , \qquad x \equiv \frac{1}{\sqrt{3}}k\tau$$

$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x} = \frac{x}{3} + \mathcal{O}(x^3) ,$$

$$\cancel{n_1(x) = -\frac{\cos x}{x^2} - \frac{\sin x}{x} = -\frac{1}{x^2} + \mathcal{O}(x^0)}$$

growing mode $\Phi_k = 3A_k$ $\quad \Phi_{\boldsymbol{k}}(0) = -\frac{2}{3}\mathcal{R}_{\boldsymbol{k}}(0)$

decaying mode $B_{\boldsymbol{k}} \equiv 0$

$$\Phi_{\boldsymbol{k}}(\tau) = -2\mathcal{R}_{\boldsymbol{k}}(0)\left(\frac{\sin x - x\cos x}{x^3}\right) \qquad \text{(all scales)}$$

$$\Phi_{\boldsymbol{k}}(\tau) \approx -6\mathcal{R}_{\boldsymbol{k}}(0)\frac{\cos\left(\frac{1}{\sqrt{3}}k\tau\right)}{(k\tau)^2} \qquad \text{(subhorizon)}$$

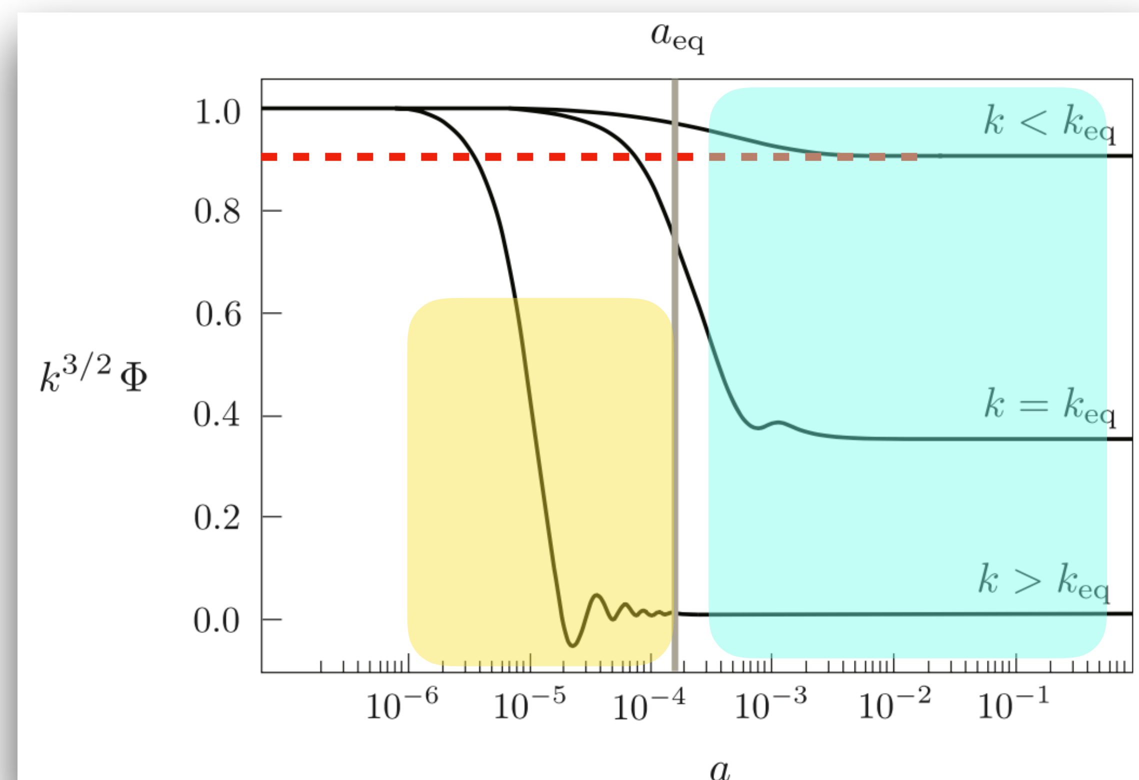
oscillating with frequency $\frac{1}{\sqrt{3}}k$,
amplitude decays as $\tau^{-2} \sim a^{-2}$

MD w=0

$$\Phi'' + \frac{6}{\tau}\Phi' = 0 ,$$

$$\Phi \propto \begin{cases} const. \\ \cancel{\tau^{-5} \propto a^{-5/2}} \end{cases}$$

in MD, gravitational potential is frozen on ALL scales!



Radiation

RD

$$\delta_r = -\frac{2}{3}(k\tau)^2\Phi - 2\tau\Phi' - 2\Phi \xrightarrow{\text{super-horizon}} \delta_r = -2\Phi \quad \text{(frozon)}$$

(Poisson eq.)

sub-horizon

$$\delta_r \approx \Delta_r = -\frac{2}{3}(k\tau)^2\Phi = 4\mathcal{R}(0)\cos\left(\frac{1}{\sqrt{3}}k\tau\right) \xleftarrow[\text{around 0}]{\text{harmonic oscillator}} \boxed{\delta_r'' - \frac{1}{3}\nabla^2\delta_r = 0}$$

MD

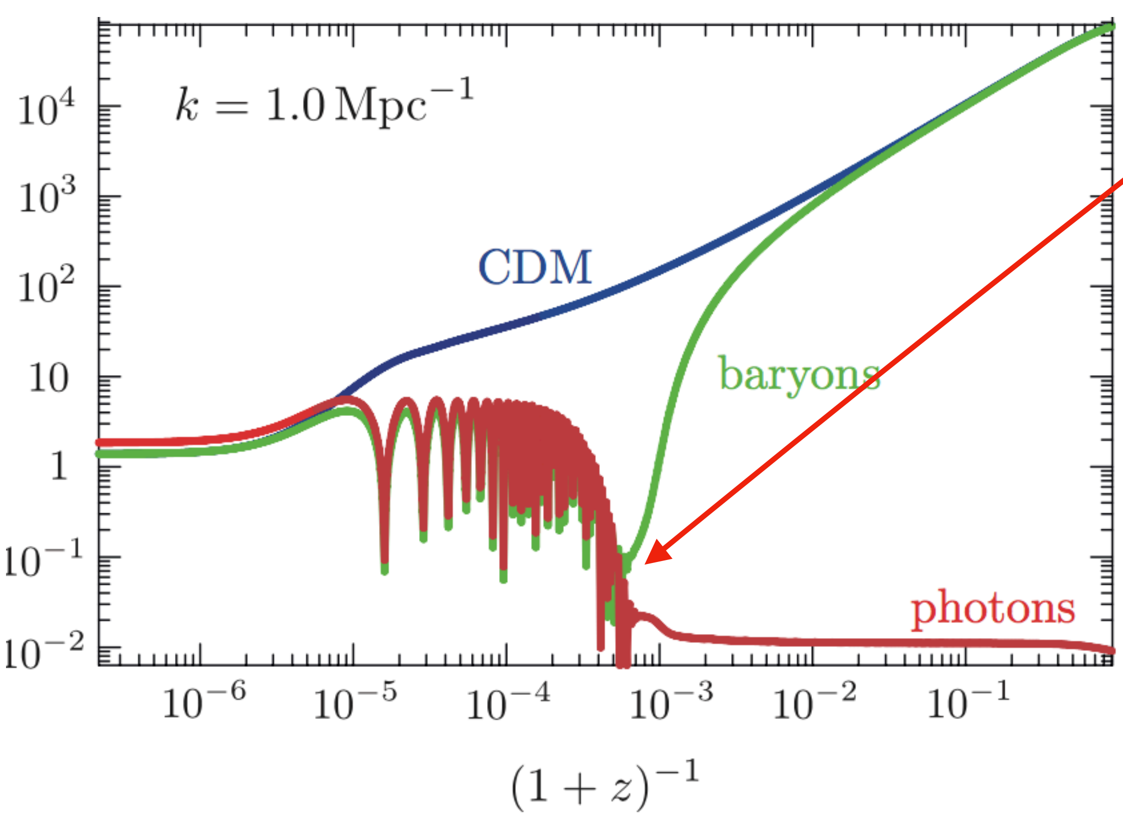
(continuity eq.) $\delta_r' = -\frac{4}{3}\nabla\cdot\boldsymbol{v}_r$

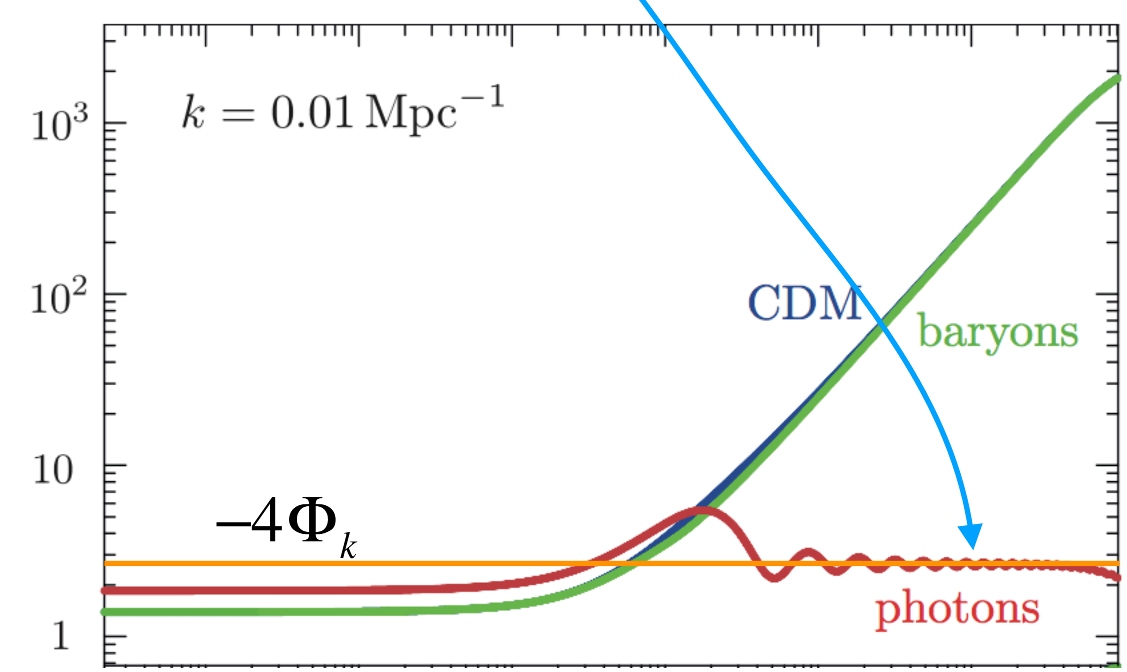
(Euler eq.) $\boldsymbol{v}_r' = -\frac{1}{4}\nabla\delta_r - \nabla\Phi$

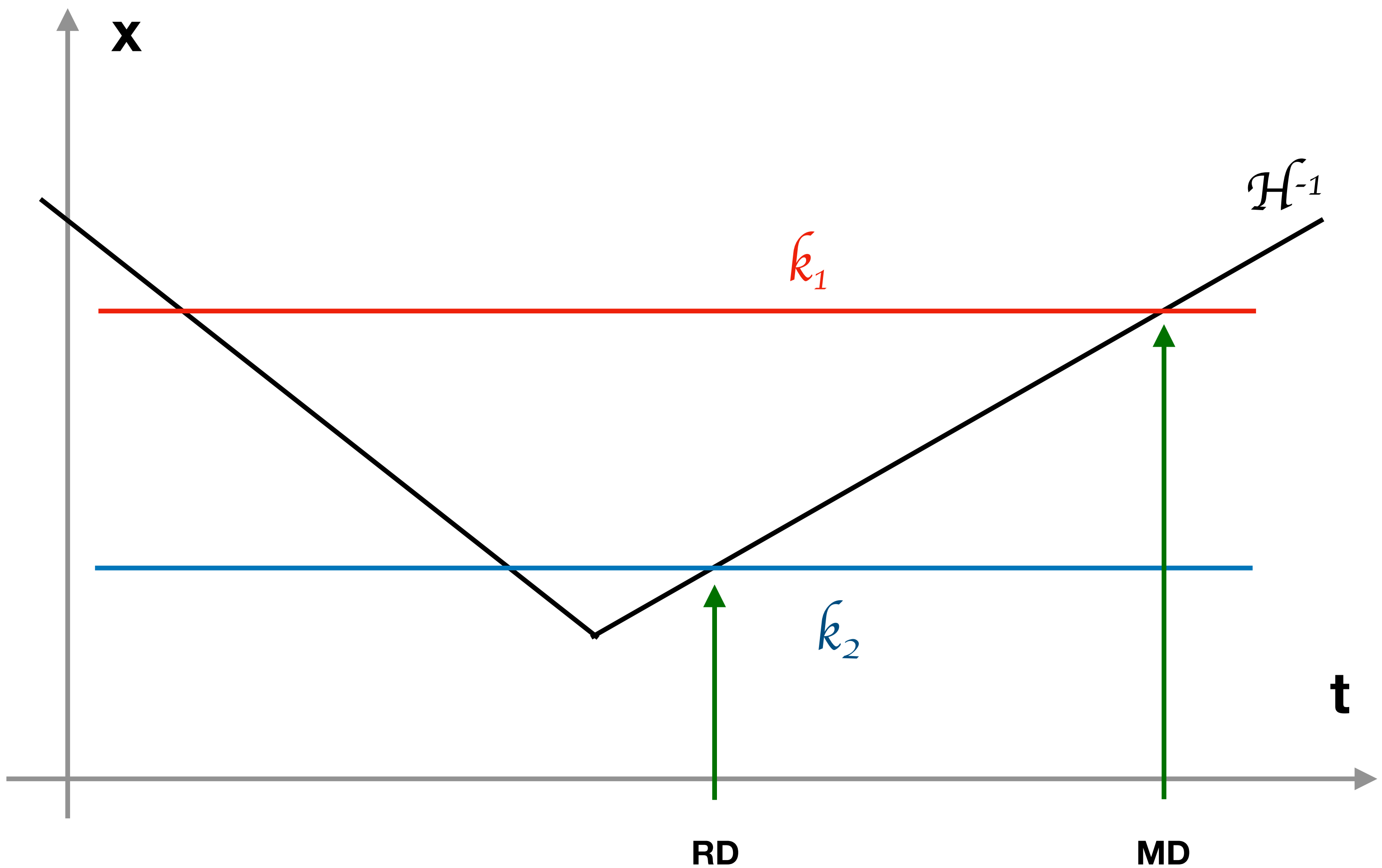
$$\boxed{\delta_r'' - \frac{1}{3}\nabla^2\delta_r = \frac{4}{3}\nabla^2\Phi} = const.$$

harmonic oscillator with a constant driving force

harmonic oscillator with a shifted equivalent point $\delta_r = -4\Phi_{\rm MD}(k)$






x
$\mathcal{H}^{-1}$
k_1
k_2
t
RD
MD

Dark Matter

(continuity eq.) $\delta_m' = -\nabla\cdot \boldsymbol{v}_m$

(Euler eq.) $\boldsymbol{v}_m' = -\mathcal{H}\boldsymbol{v}_m - \nabla\Phi$

$$\delta_m'' + \mathcal{H}\delta_m' = \nabla^2\Phi$$

Φ shall response to the **total** density pert.

However, in RD, δ_r oscillate around 0 rapidly

the time averaged potential shall only feels DM pert.

$$\delta_m'' + \mathcal{H}\delta_m' - 4\pi G a^2 \bar{\rho}_m \delta_m \approx 0$$

$$\boxed{\frac{d^2\delta_m}{dy^2} + \frac{2+3y}{2y(1+y)}\frac{d\delta_m}{dy} - \frac{3}{2y(1+y)}\delta_m = 0}$$

$$y \equiv \frac{a}{a_{\rm eq}}$$

[Pb1.]

ref: Baumann lecture §5.2.3

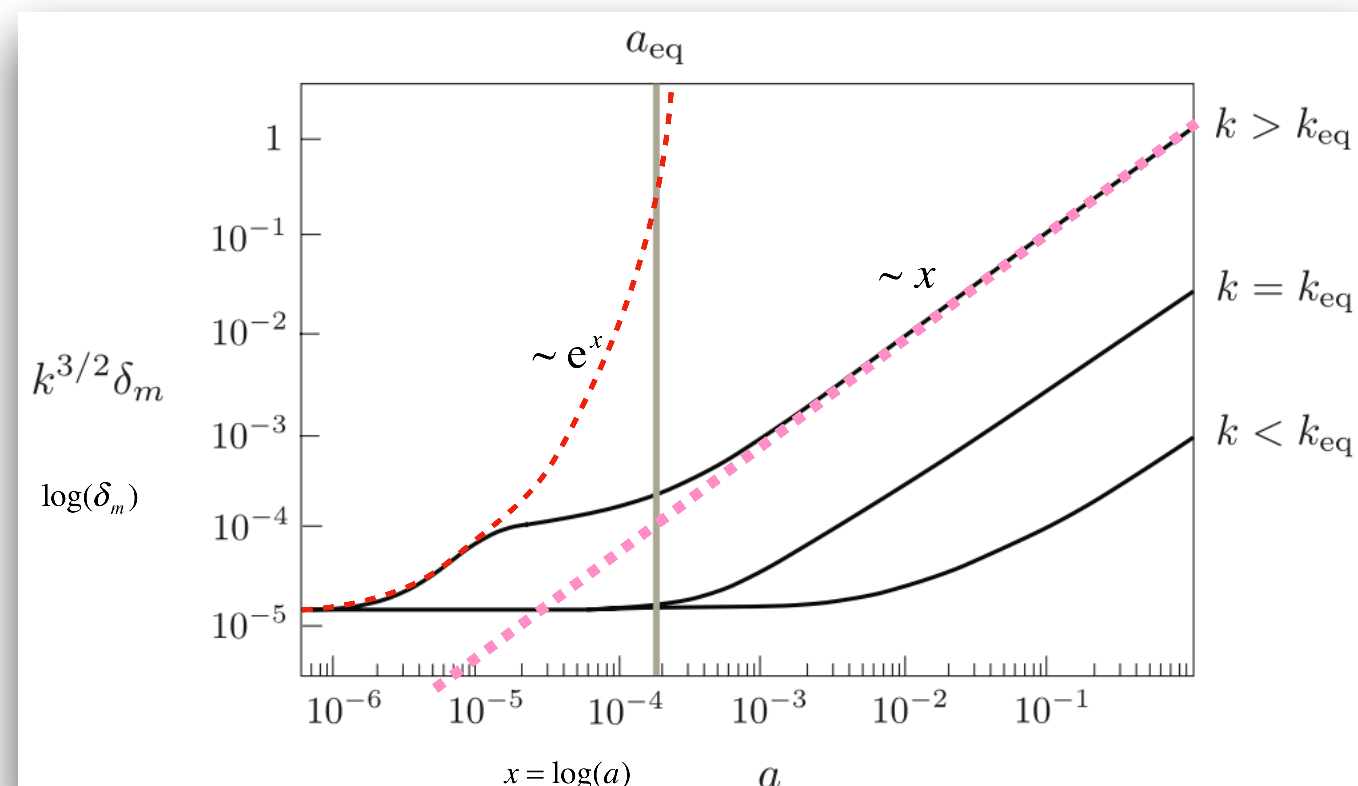
$$\delta_m \propto \begin{cases} 2+3y \\ (2+3y)\ln\left(\dfrac{\sqrt{1+y}+1}{\sqrt{1+y}-1}\right) - 6\sqrt{1+y} \end{cases}$$

RD $y \ll 1$

$\delta_m \sim \log(a)$

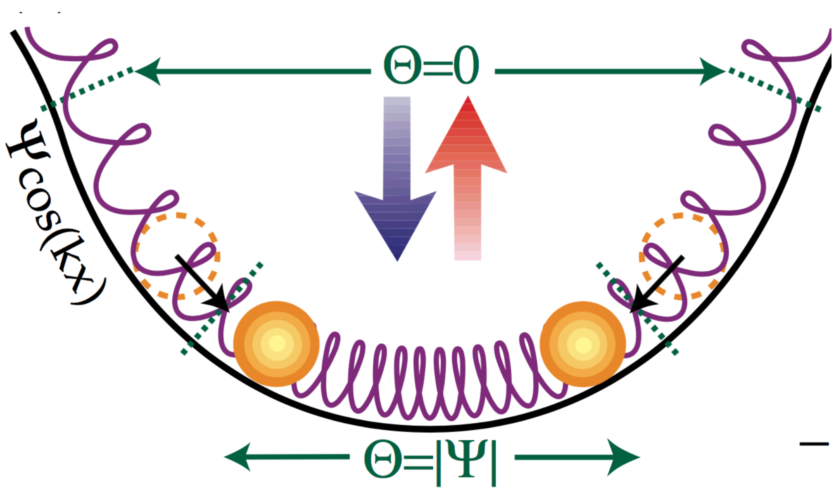
MD $y \gg 1$

$\delta_m \sim a$



Baryon

Before z_{rec}~1100, baryon (electron) is tightly coupled with photon via Compton scattering

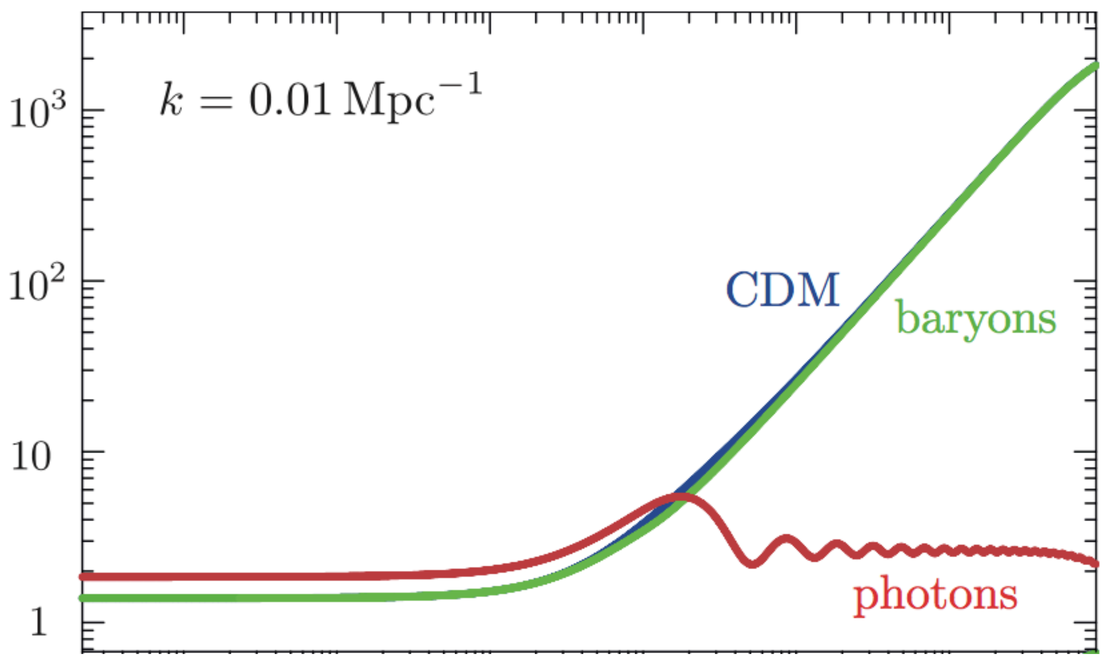


$$\delta_\gamma = \frac{4}{3}\delta_b \qquad \boldsymbol{v}_\gamma = \boldsymbol{v}_b$$

photon pressure support the oscillation on the small scale

$$\delta_b \sim oscillate \qquad \delta_c \sim grow \qquad \delta_b \ll \delta_c$$

– **After** decoupling, baryon behaves as non-relativistic matter
(the same as CDM)



$$\delta_b'' + \mathcal{H}\delta_b' = 4\pi G a^2 \boxed{(\bar\rho_b\delta_b + \bar\rho_c\delta_c)}$$
$$\delta_c'' + \mathcal{H}\delta_c' = 4\pi G a^2 \boxed{(\bar\rho_b\delta_b + \bar\rho_c\delta_c)}$$

$\longrightarrow$ **same source (gravitational potential)**

We shall expect they behave more like each other!

$$D \equiv \delta_b - \delta_c$$

$$D'' + \frac{2}{\tau}D' = 0 \quad \Rightarrow \quad D \propto \begin{cases} const. \\ \tau^{-1} \end{cases}$$

$$\delta_m'' + \frac{2}{\tau}\delta_m' - \frac{6}{\tau^2}\delta_m = 0 \quad \Rightarrow \quad \delta_m \propto \begin{cases} \tau^2 \\ \tau^{-3} \end{cases}$$

$$\frac{\delta_b}{\delta_c} = \frac{\bar\rho_m\delta_m + \bar\rho_c D}{\bar\rho_m\delta_m - \bar\rho_b D} \to \frac{\delta_m}{\delta_m} = 1\ ,$$

The same mechanism for the linear bias evolution

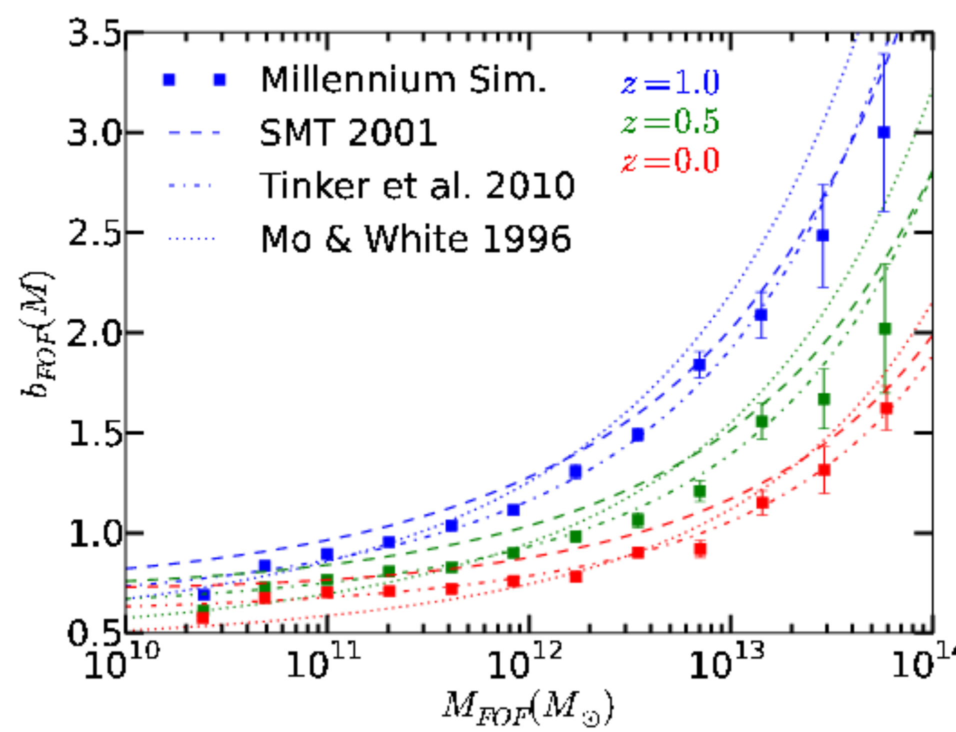
$$\delta_{tracer} = b \bullet \delta_m$$

we observe luminous objects, e.g. galaxies, not matter density per. se.

How many population in NK?

night map

day map



for linear bias, from high-z to low-z, b(z) --> unity

Further reading

- **Baumann lecture note/**Chapter 5