Cosmic Large-scale Structure Formations

Bin HU

bhu@bnu.edu.cn

Astro@BNU

Office: 京师大厦9907

18 weeks

outline

Background (1 w)

- universe geometry and matter components (1 hr)
- Standard candle (SNIa) (0.5 hr)
- Standard ruler (BAO) (0.5 hr)

Linear perturbation (9 w)

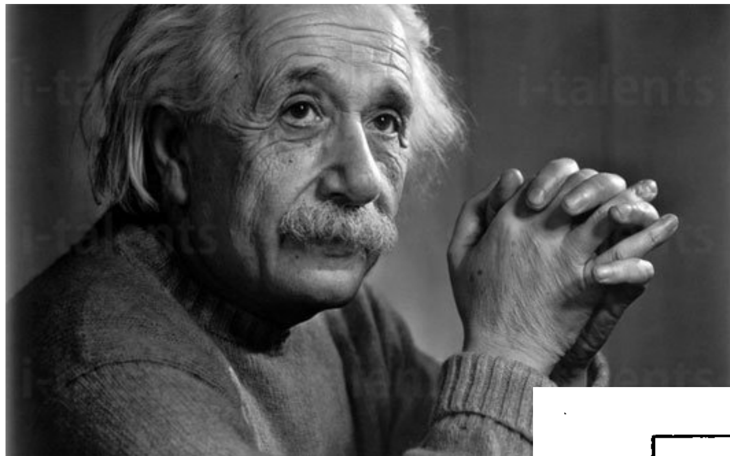
- relativistic treatment perturbation (2 hr)
- ~~primordial power spectrum (2 hr)~~
- linear growth rate (2 hr)
- galaxy 2-pt correlation function (2 hr)
- Baryon Acoustic Oscillation (BAO) (2 hr)
- Redshift Space Distortion (RSD) (2 hr)
- Weak Lensing (2 hr)
- Einstein-Boltzmann codes (2 hr)

Non-linear perturbation (6 w)

- Non-linear power spectrum (2 hr)
- halo model (2 hr)
- N-body simulation algorithms (2 hr)
- Press-Schechter (PS) halo mass function (2 hr)
- Extended-PS (EPS) halo mass function (2 hr)
- halo bias & halo density profile (2 hr)

Statistical analysis (2 w)

- Monte-Carlo Markov Chain sampler (2 hr)
- CosmoMC use (2 hr)



$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \longrightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

unstable to tiny pert.

static universe: closed universe contained dust and cc

Einstein 1917

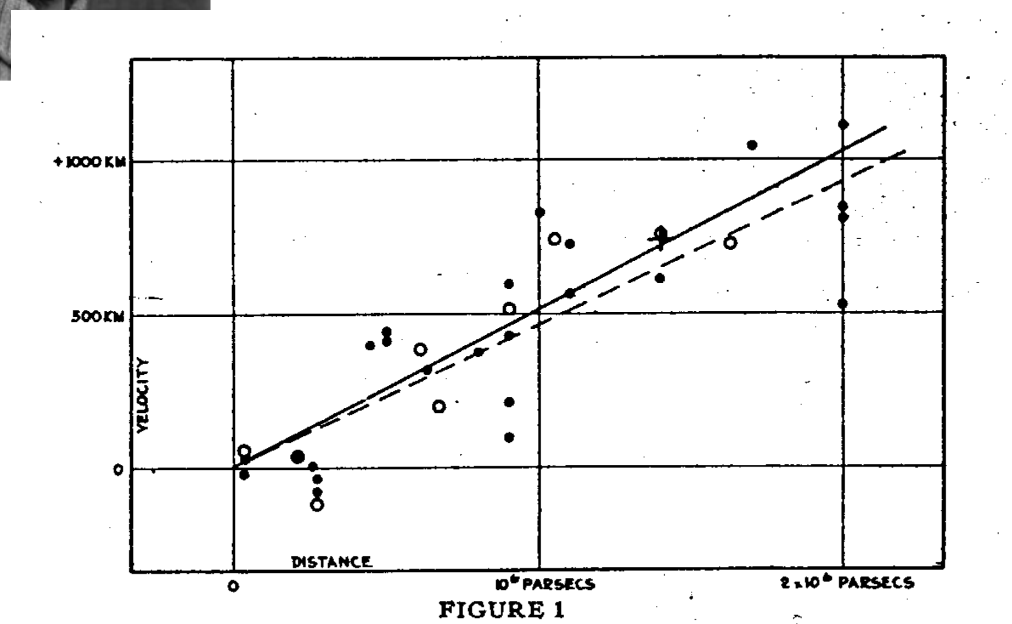


FIGURE 1

$$0 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3};$$
$$0 = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}.$$
$$\rho = \frac{\Lambda}{4\pi G}; \quad a = \sqrt{\frac{k}{\Lambda}}.$$

must be closed universe!

$v=H_0d$

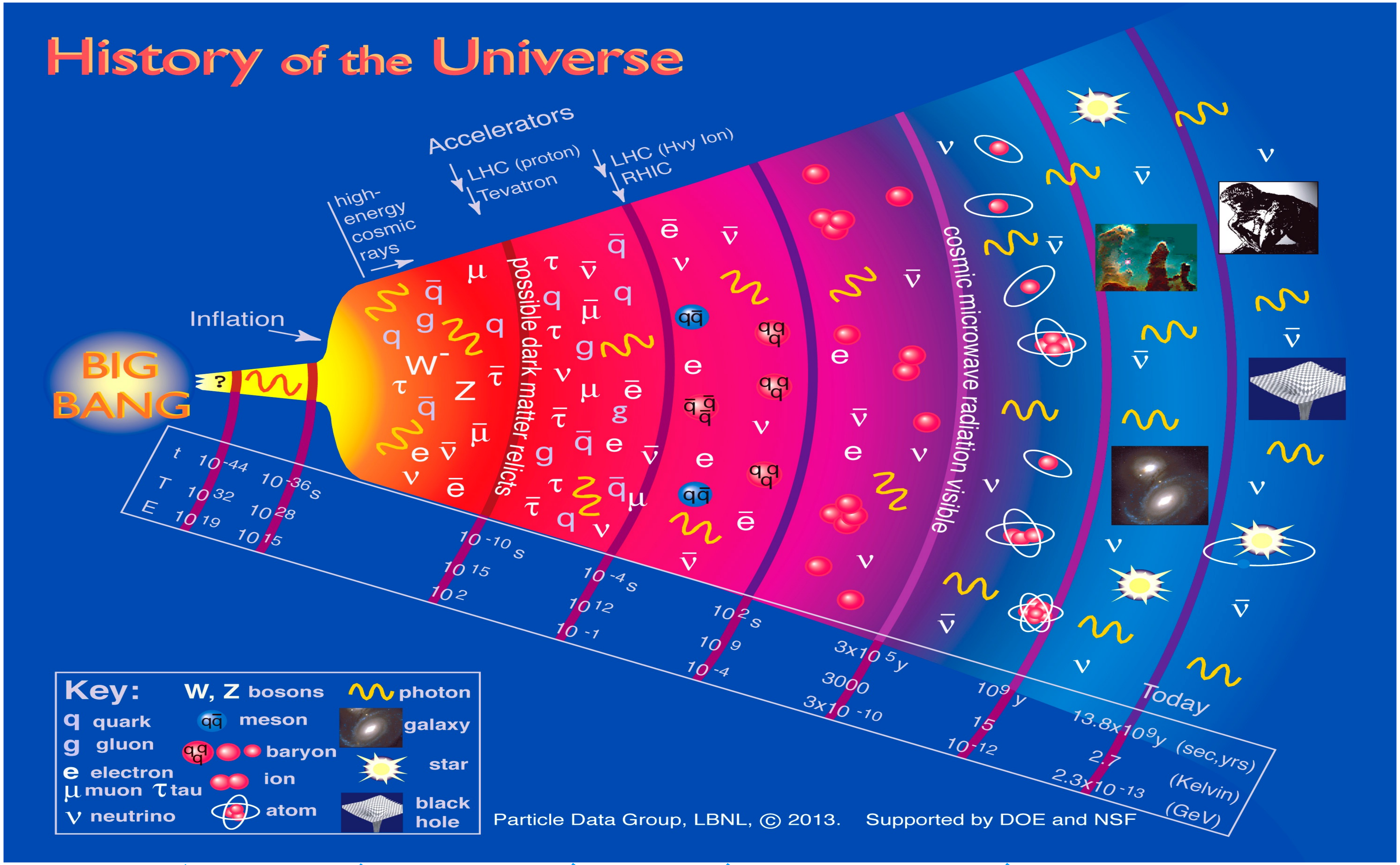
Hubble 1929

The further galaxy is, the faster escape from us $\longrightarrow$ universe is expanding

prediction： T_{CMB}=5K

measurement: T_{CMB}=2.7255K

Gamow 1948



T~1GeV

quark -> proton&neutron

T~1MeV

nucleosynthesis: create light element D, He, Li

T~1eV

recombination: $p^+ + e^- \to H$

z~(100,1000)

dark ages

z~(6,10)

reionization

z<2

galaxy cluster

z<1

Dark energy

hot ⟶ cold

GR is a classical theory, does not involve any quantum phenomenon (no $\hbar$)

A typical Schwarzschild black hole radius: $\frac{2GM}{c^2}$

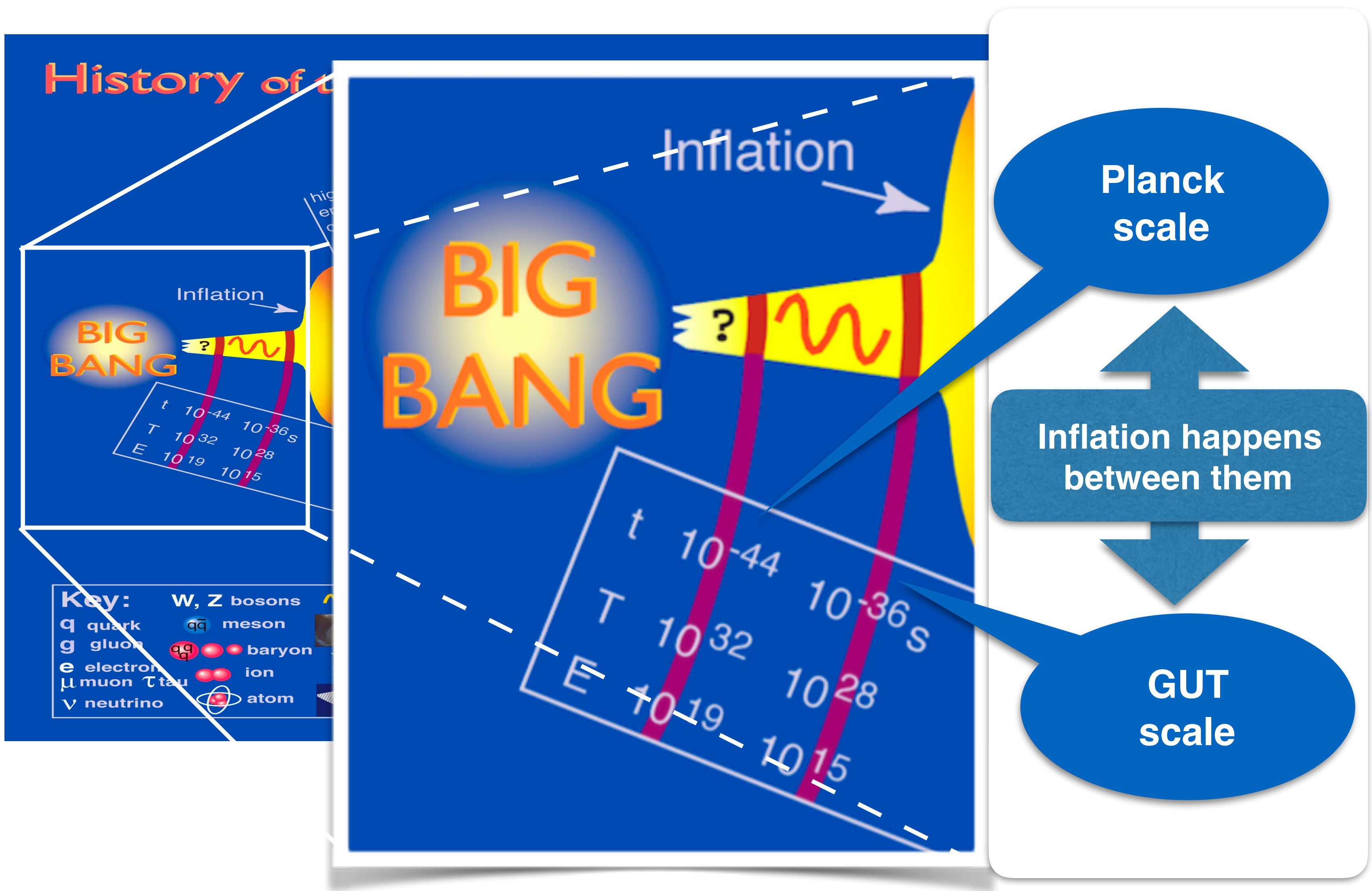
$$G_{\mu\nu}(x) + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}(x)$$

uncertainty principle: $\delta P \bullet \delta\lambda \sim \hbar$

the inertial energy of particle with mass M: $E = Mc^2$

Planck Mass $M_* \sim \sqrt{\hbar / G} \sim 10^{19}\, GeV$

when the system energy approaches Planck mass, we need to quantise gravity!

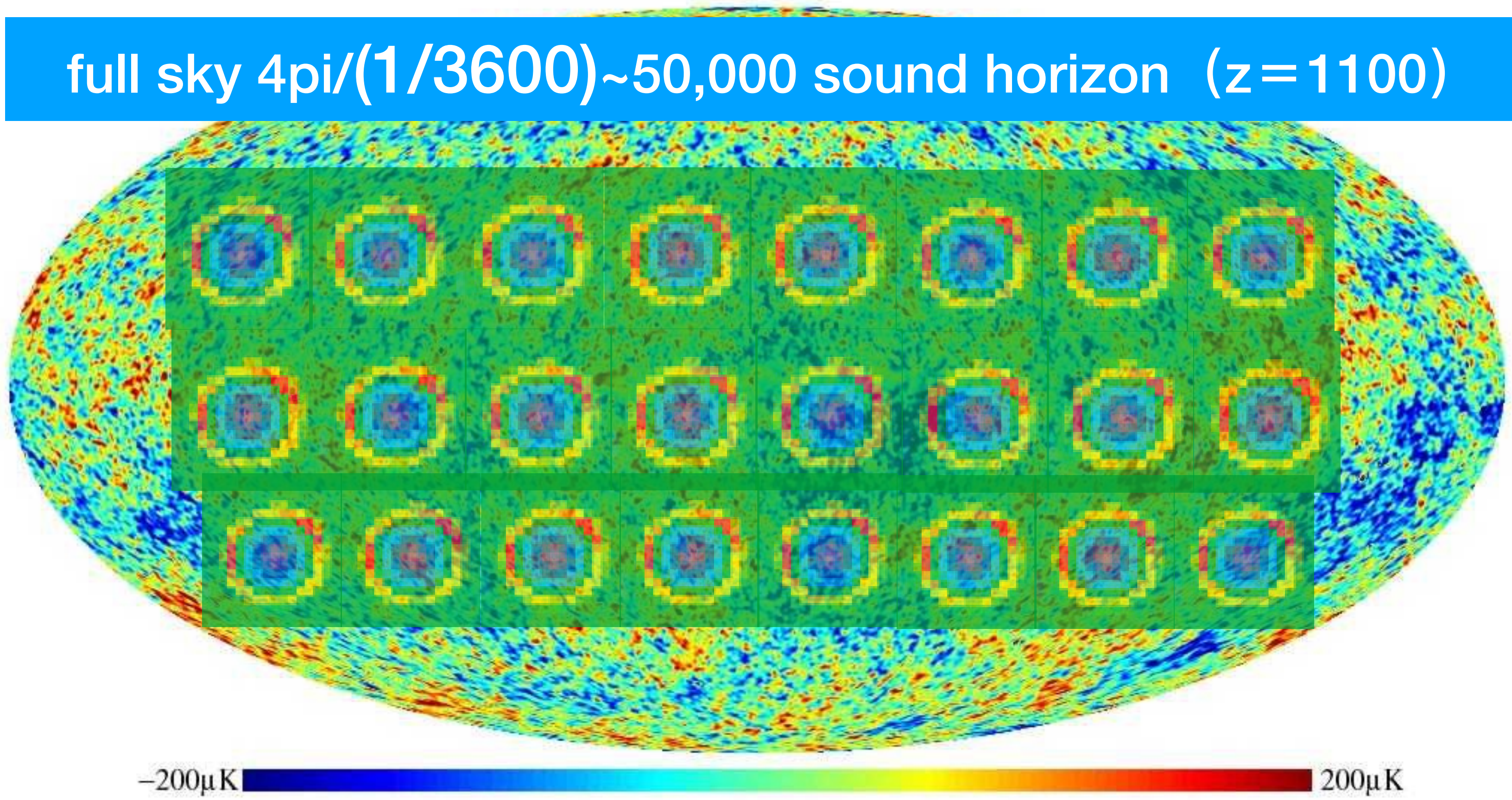


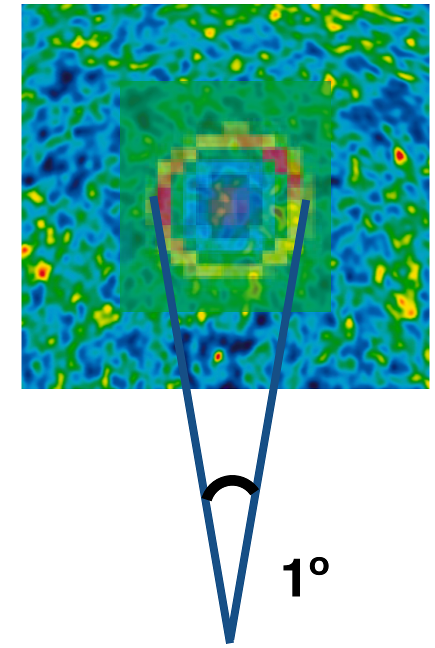
From t=0 to 10^{-44} s (Planck time), cosmic energy scale is above 10^{19} GeV (Planck energy)

why do we need inflation?

$1\deg^2 \sim (\pi/180)^2 \sim 1/3600$

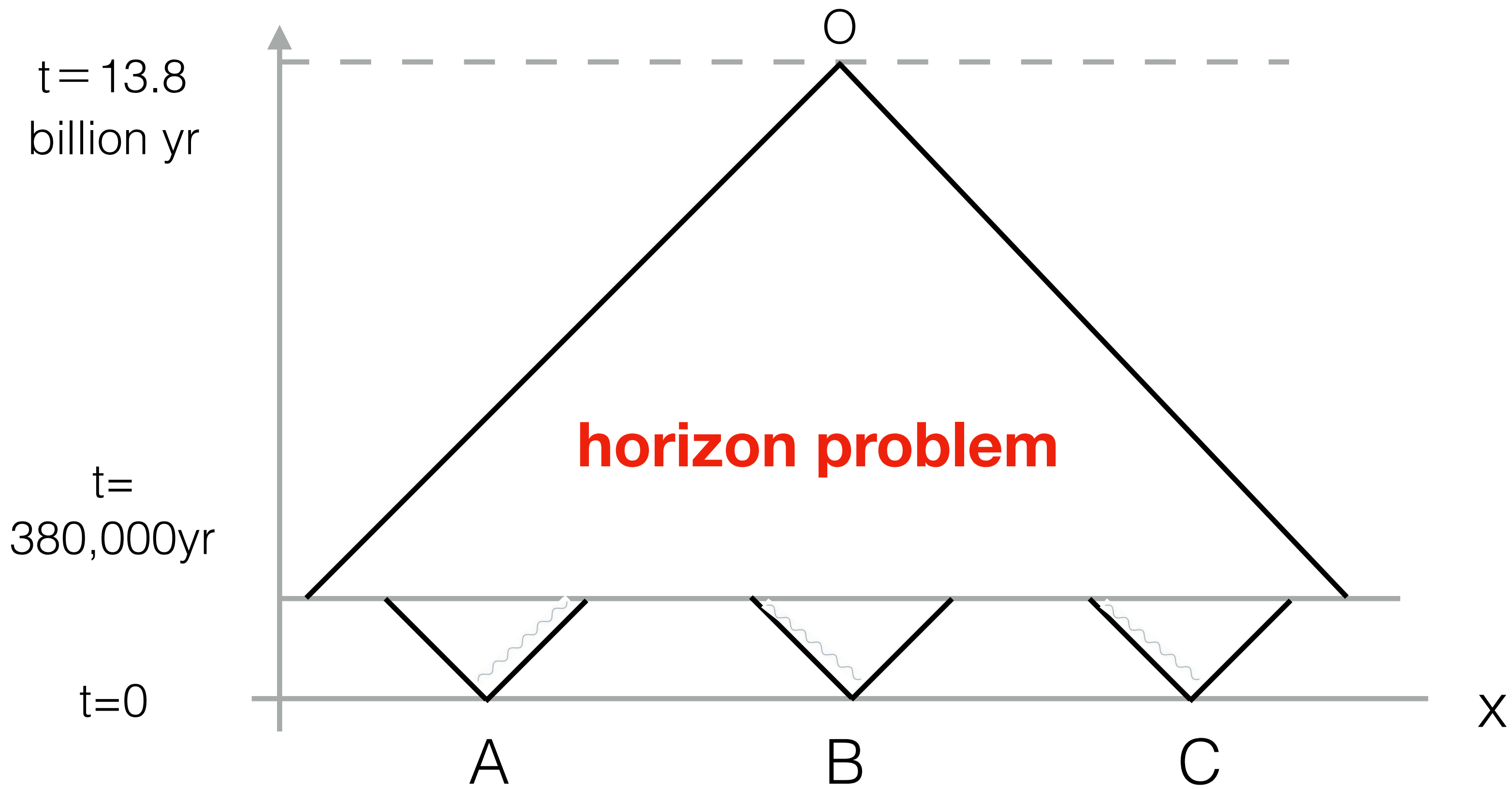
full sky 4pi/(1/3600)~50,000 sound horizon（z=1100)





A photon from t=0，with velocity c/3,
via 380,000yr can travel：
38×10^4 / 3 lyr ～3×10^4 pc

A photon from t＝0，with velocity c,
via 13.8 billion yr, can travel：
138×10^8 lyr～5×10^9 pc



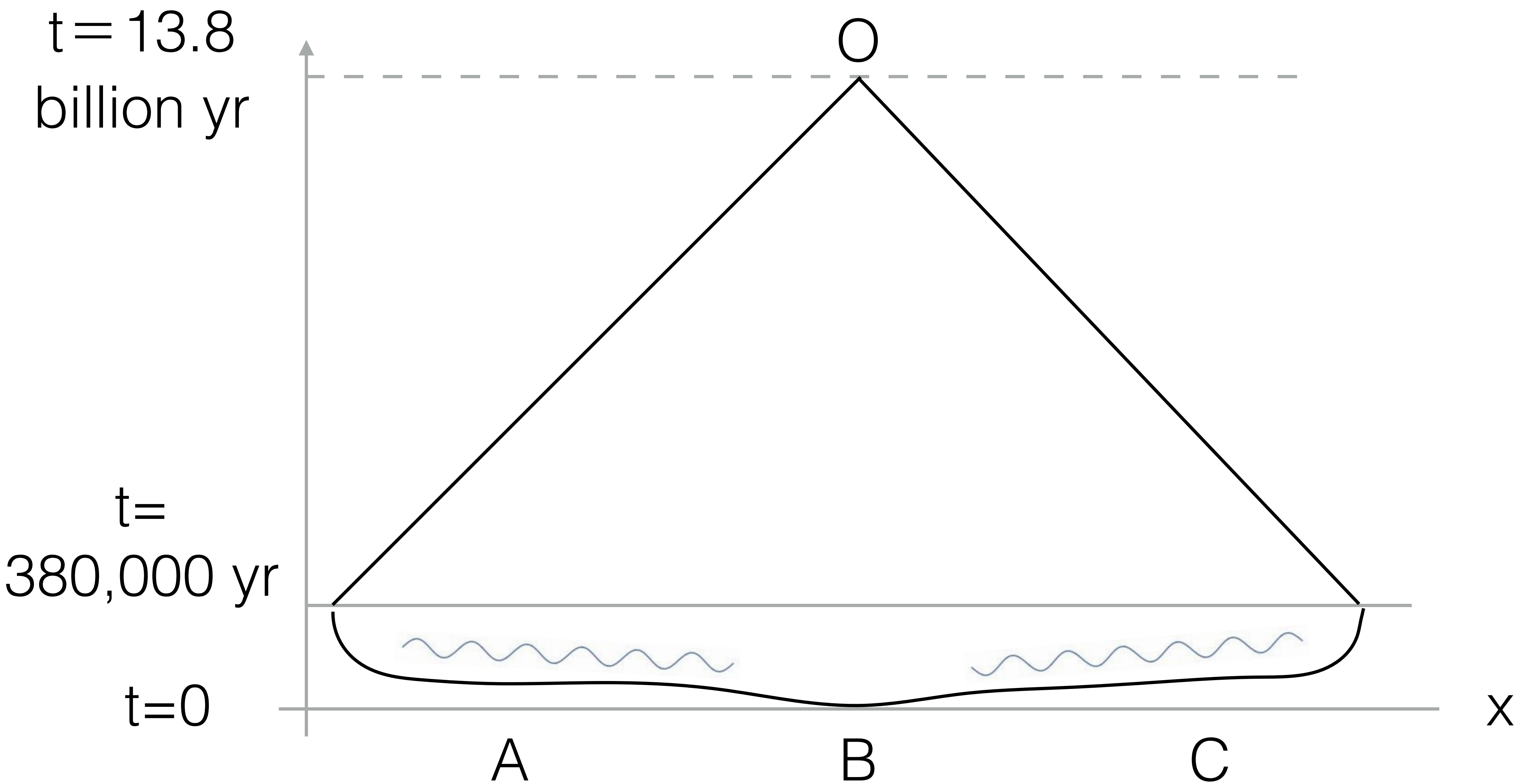
remove the co-moving factor
$a_{z=0}/a_{z=1100}$~1000

ratio：5×10^9 / 3×10^4 / 10^3～140

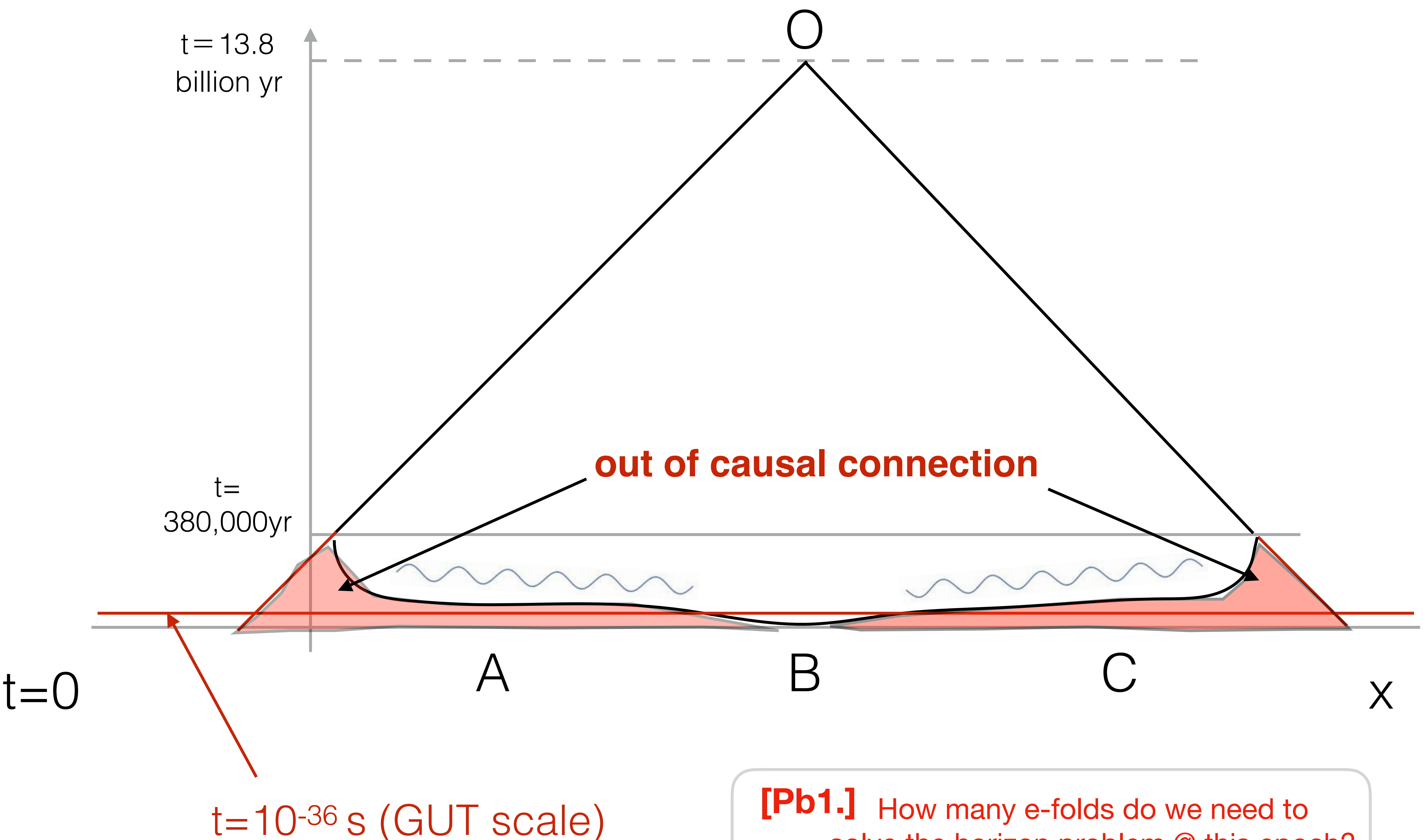
2d sphere，totally 140^2~20,000
causal disconnected region

To solve horizon problem @z=1000, need enlarge the physical size of forward light-cone, by a factor 100.

$e^N \sim 100$, $N \sim 5$ (e-folding number)



continue to push back to GUT scale



[Pb1.] How many e-folds do we need to solve the horizon problem @ this epoch?

flatness problem

$$\boxed{H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$$

$$\Omega = \rho/3M_{pl}^2H^2$$

$$\Omega - 1 = \frac{k}{a^2H^2}$$

10^{60}

$|\Omega_k| < 0.005$

$10^{19}\,GeV$	$10^{-3}\,eV$	$10\ eV$
Planck era	DE era	equality era

$$\frac{\rho_{pl}}{\rho_{de}} = \left(\frac{E_{pl}}{E_{de}}\right)^4 \sim 10^{124}$$

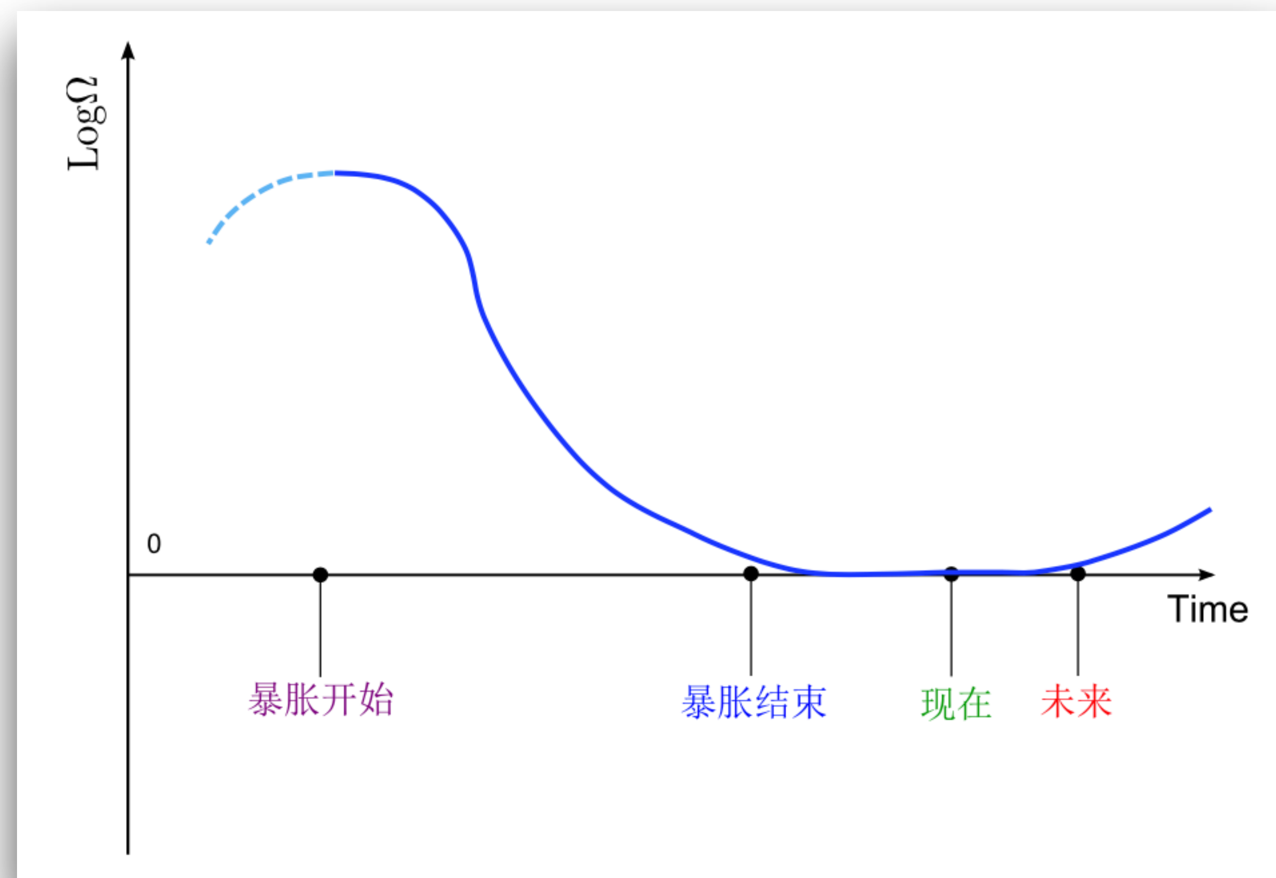
$$\frac{\rho_{pl}}{\rho_{eq}} = \left(\frac{E_{pl}}{E_{eq}}\right)^4 \sim 10^{108}$$

$$H^2 \propto \rho \propto a^{-4}$$

radiation era

radiation era covers most parts of the energy scale

$$10^{54} \longleftarrow a^2H^2 \propto \sqrt{\rho}$$



monopole problem

GUT $\longrightarrow$ huge mount of stable magnetic monopole

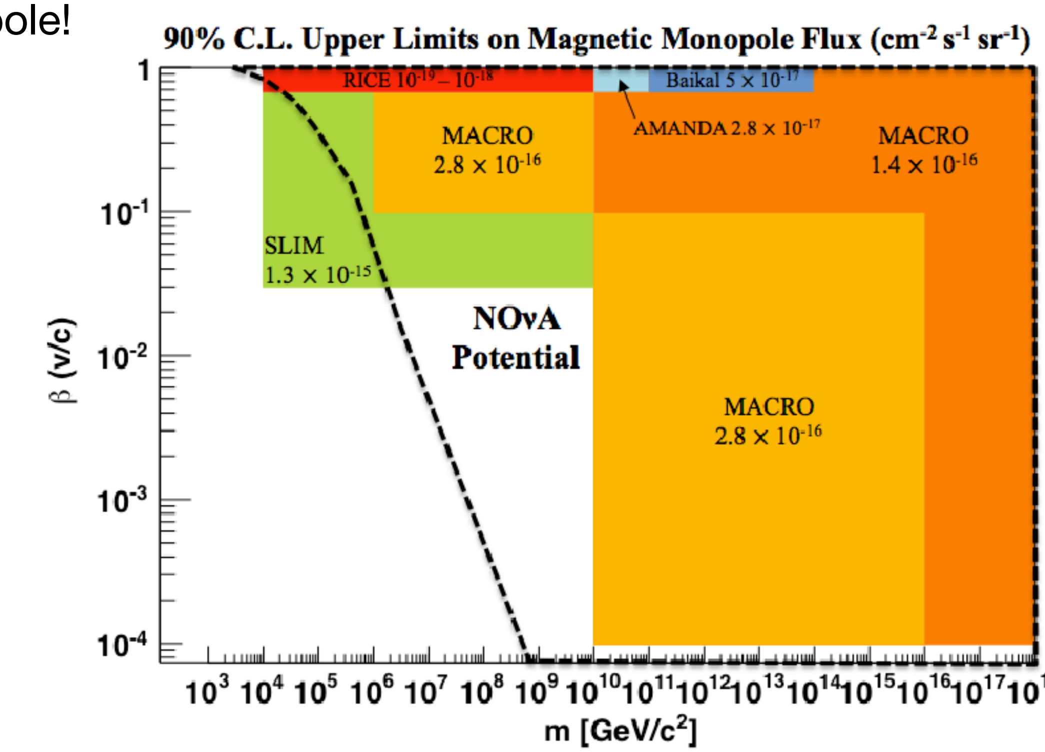
m~10^{16} GeV

$$\rho_c \sim 10^{-29}[gm/cm^3]$$

$$\rho_{mon} > 10^{-18}[gm/cm^3]$$

$$\Omega = \rho_{mon}/\rho_c > 10^{11}$$

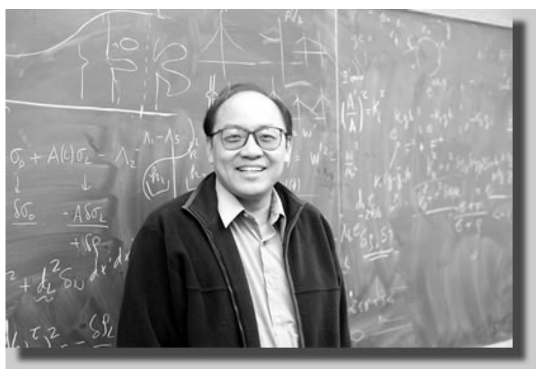
completely dominated by monopole!



The way out?

within 10^{-36} s, stretch the physical scale of the forward light-cone by a factor e^{60}

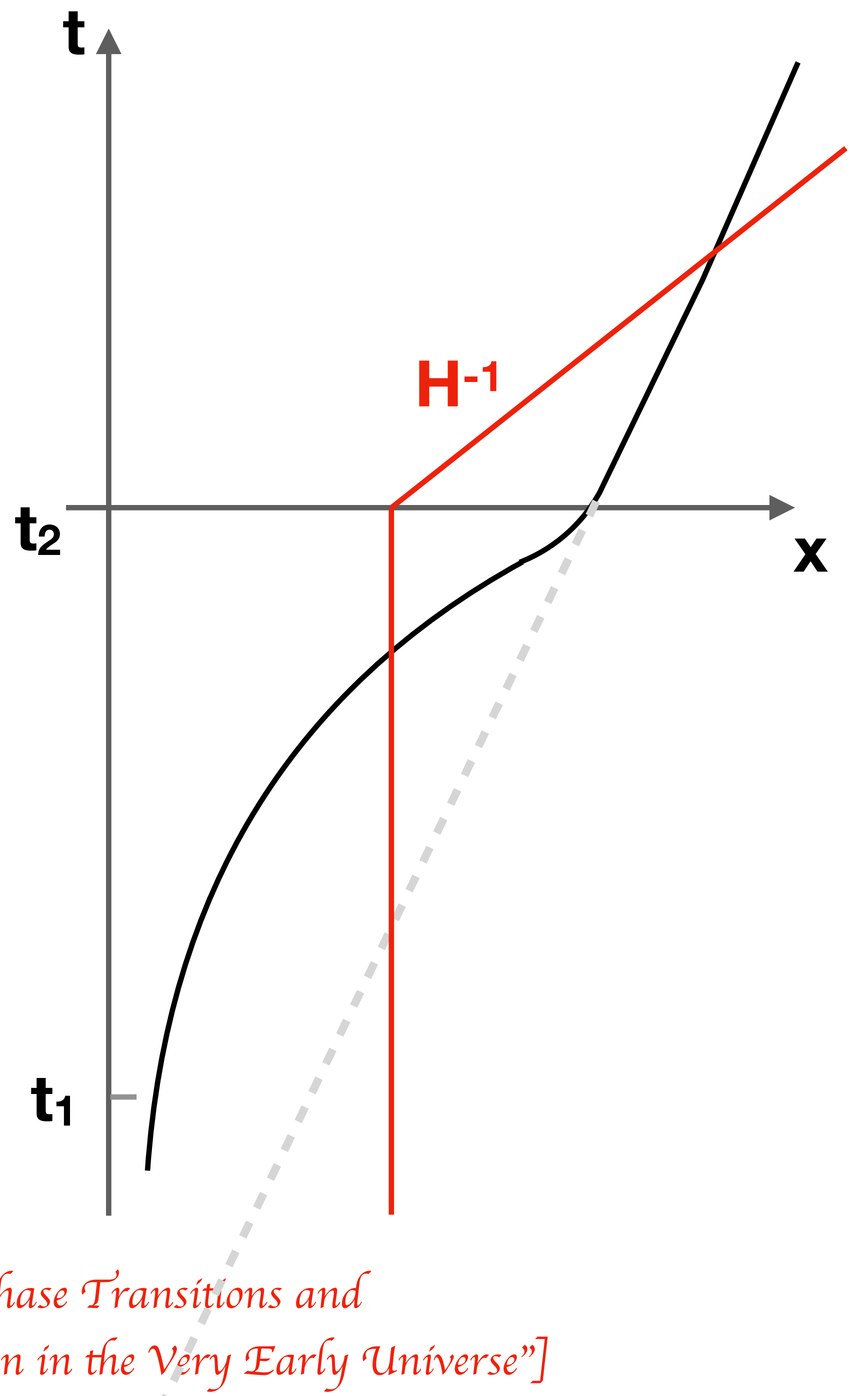
Guth 1980

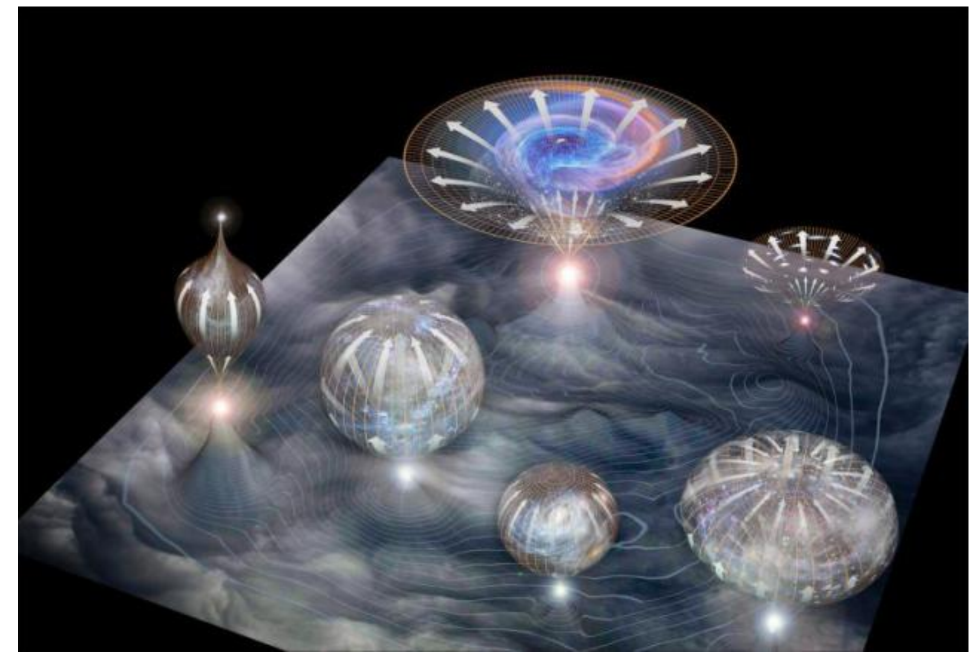


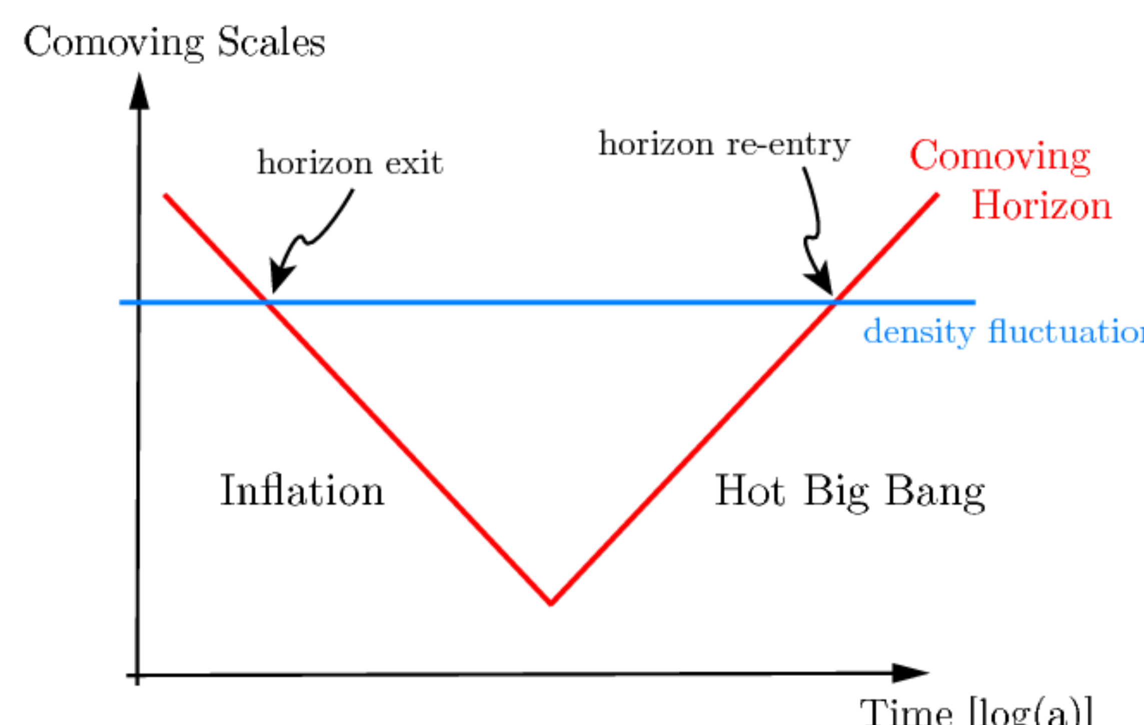
Henry Tye

how to：qausi-de Sitter phase ⟶ exponential expansion

in RD/MD era, a~t# (power law), too slow! $a = e^{H \bullet \Delta t}$ $H \bullet \Delta t = 60$ **H~ const**







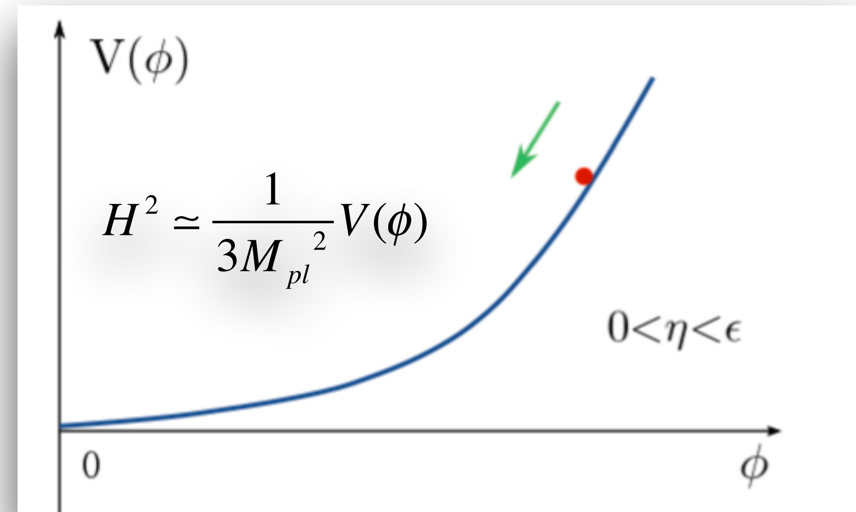
[Guth & Tye, 1979, PRL, "Phase Transitions and Magnetic Monopole Production in the Very Early Universe"]

[Guth, 1980, PRD, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems"]

mechanism: a scalar field slowly roll in its potential

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial^\mu\phi\partial_\mu\phi-V(\phi)\right]$$

$$\rho=\frac{1}{2}\dot{\phi}^2+V(\phi) \qquad P=\frac{1}{2}\dot{\phi}^2-V(\phi) \qquad \dot{\phi}^2\ll V(\phi)\Leftrightarrow P\simeq-\rho \qquad H^2=\frac{1}{3M_{pl}^{\;2}}\left[\frac{1}{2}\dot{\phi}^2+V(\phi)\right]$$

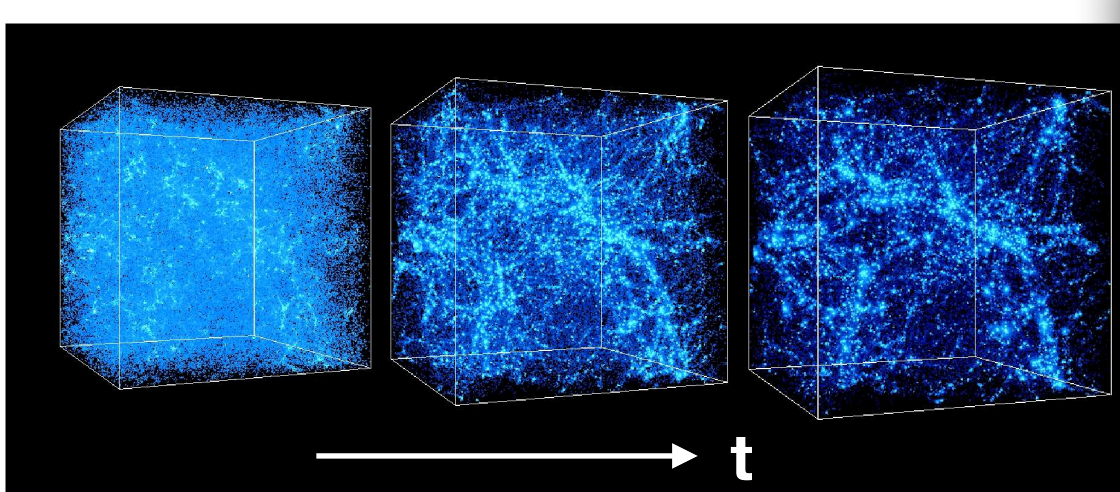


$$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = M_{pl}^2\left(\frac{V''}{V}\right),$$

$$\epsilon\ll 1\,, \qquad |\eta|\ll 1\,.$$

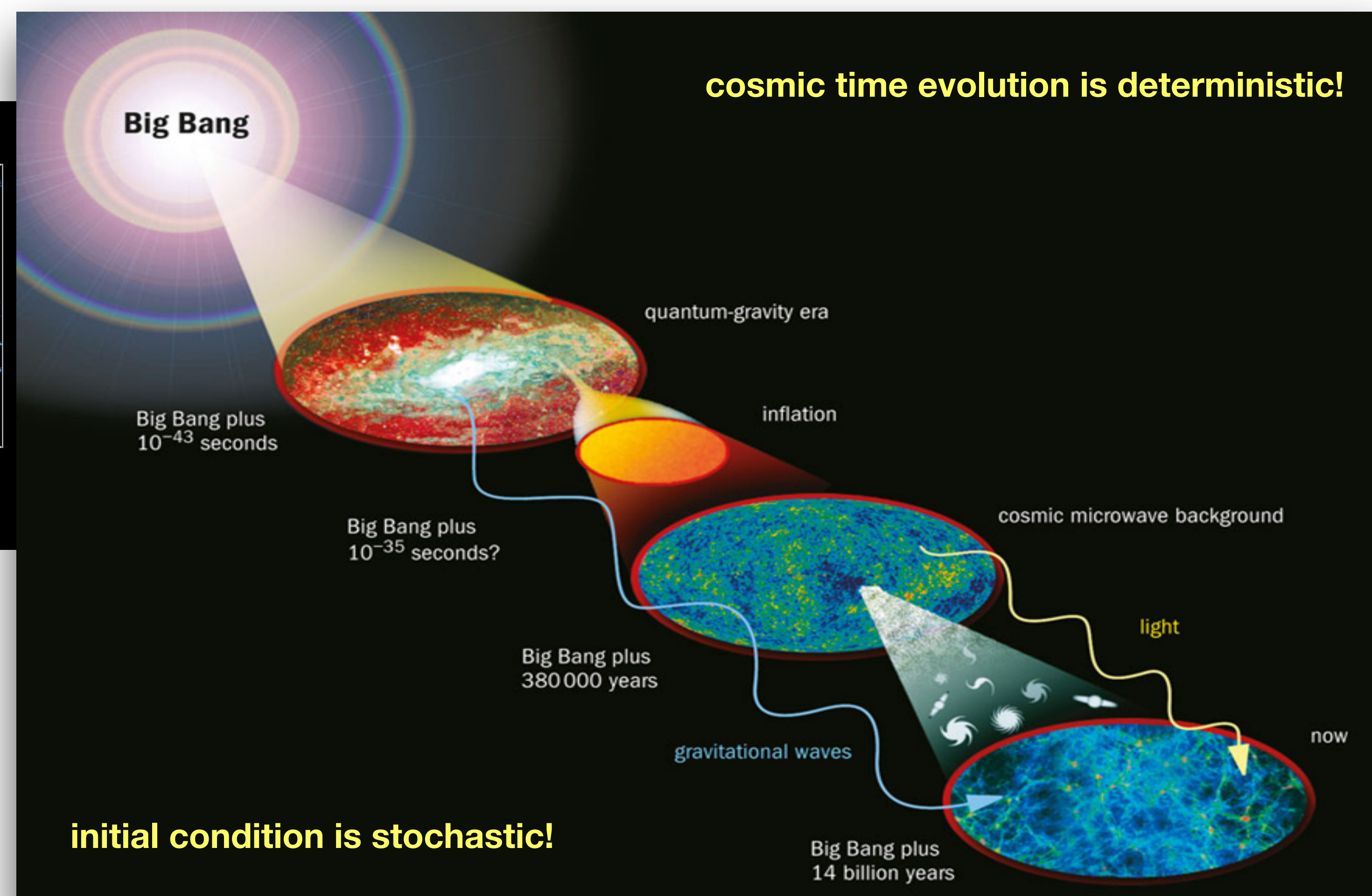
inflationary mechanism does not only solve several problems on the background level,

but also, naturally gives the initial conditions needed by the CMB and LSS formation! (we force on this)



$$P(k,z_0)=D^2(z_i,z_0)P_i(k)$$

obs evolution IC



inflaton action

$$S=\int d\tau d^3x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)\right] \xrightarrow{\text{plug unperturbed FRWL metric}} S=\int d\tau d^3x\left[\frac{1}{2}a^2\left((\phi')^2-(\nabla\phi)^2\right)-a^4V(\phi)\right]$$

$$\phi(\tau,\boldsymbol{x})=\bar\phi(\tau)+\frac{f(\tau,\boldsymbol{x})}{a(\tau)}$$

linear order action

$$S^{(1)}=\int d\tau d^3x\underbrace{\left[a\bar\phi' f'-a'\bar\phi' f-a^3V_{,\phi}f\right]}_{\text{(deriv)}}=-\int d\tau d^3x\, a\underbrace{\left[\bar\phi''+2\mathcal{H}\bar\phi'+a^2V_{,\phi}\right]}_{\text{(deriv)}}f$$

background field e.o.m

$$\bar\phi''+2\mathcal{H}\bar\phi'+a^2V_{,\phi}=0$$

quadratic action

$$S^{(2)}=\frac{1}{2}\int d\tau d^3x\left[(f')^2-(\nabla f)^2-2\mathcal{H}ff'+\underbrace{\left(\mathcal{H}^2-a^2V_{,\phi\phi}\right)}_{\text{(deriv)}}f^2\right]=\frac{1}{2}\int d\tau d^3x\left[(f')^2-(\nabla f)^2+\underbrace{\left(\frac{a''}{a}-a^2V_{,\phi\phi}\right)}_{\text{(deriv)}}f^2\right]$$

$$\boxed{S^{(2)}\approx\int d\tau d^3x\,\frac{1}{2}\left[(f')^2-(\nabla f)^2+\frac{a''}{a}f^2\right]}$$

$$\frac{V_{,\phi\phi}}{H^2}\approx\frac{3M_{\rm pl}^2V_{,\phi\phi}}{V}=3\eta_{\rm v}\ll 1 \qquad \frac{a''}{a}\approx 2a'H=2a^2H^2\gg a^2V_{,\phi\phi}$$

Mukhanov-Sasaki eq.

$$\boxed{f_{\boldsymbol{k}}''+\left(k^2\boxed{-\frac{a''}{a}}\right)f_{\boldsymbol{k}}=0}$$

'm_f^2'(negative mass sq)

sub-horizon limit

$$k^2\gg a''/a\approx 2\mathcal{H}^2 \qquad f_{\boldsymbol{k}}''+k^2f_{\boldsymbol{k}}\approx 0 \longrightarrow$$

Simple Harmonic oscillator with 0-mass in Minkowski space (no feel of curvature)

$V_{,\phi\phi}\propto m_f^{\,2}; m_f\sim$ H

in this energy level (M_{pl}>>H), inflaton behaves as massless particle

e.g.

$$V(\phi)=\frac{1}{2}m_f^{\,2}\phi^2 \qquad H^2\simeq\frac{1}{3M_{pl}^{\,2}}V(\phi)$$

$$\boxed{\bar\phi\sim M_{pl};\delta\phi\sim H}$$

validation of our calculation!

$$H\ll M_{pl}$$

we quantise $\delta\phi$ NOT $\bar\phi$

up to now,
no quantum gravity
theory available (@M_{pl} scale)

classical field

$$\boxed{f_k'' + \left(k^2 - \frac{a''}{a}\right) f_k = 0}$$

$$a(t) = e^{Ht} \qquad a(\tau) = \frac{\tau_0}{\tau} \quad \text{(deriv)} \qquad f_k'' + \left(k^2 - \frac{2}{\tau^2}\right) f_k = 0$$

general solution

$$f_k(\tau) = \alpha \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) + \beta \frac{e^{ik\tau}}{\sqrt{2k}}\left(1 + \frac{i}{k\tau}\right)$$

For a classical vacuum, no reason to excite any state, so it is natural to choose $\alpha = \beta = 0$

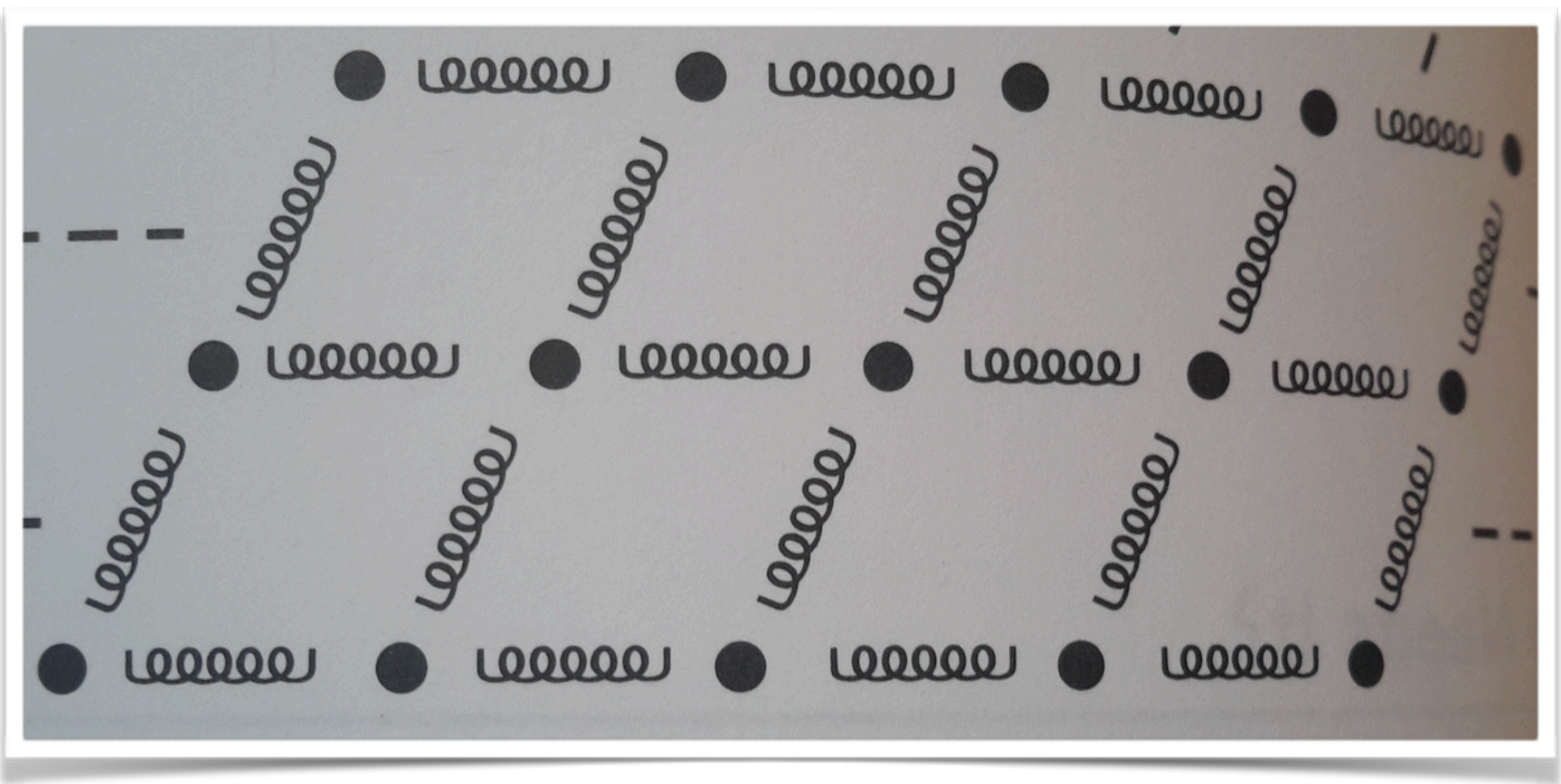
However, the **quantum fluct.** in the curved space-time, will naturally gives

$$\boxed{f_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)}$$

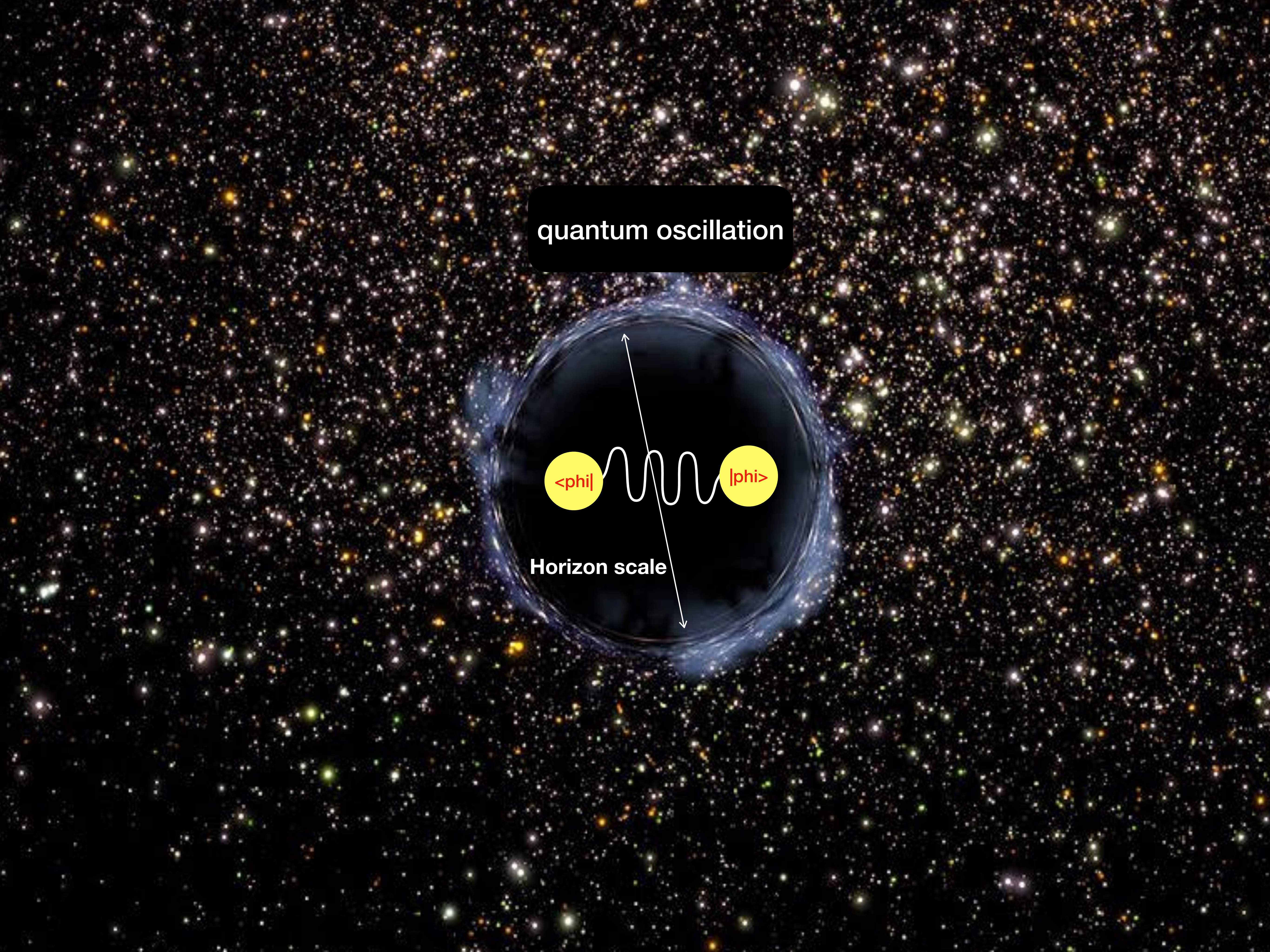
(Bunch-Davis vacuum)
(adiabatic state)
(no particle creation)

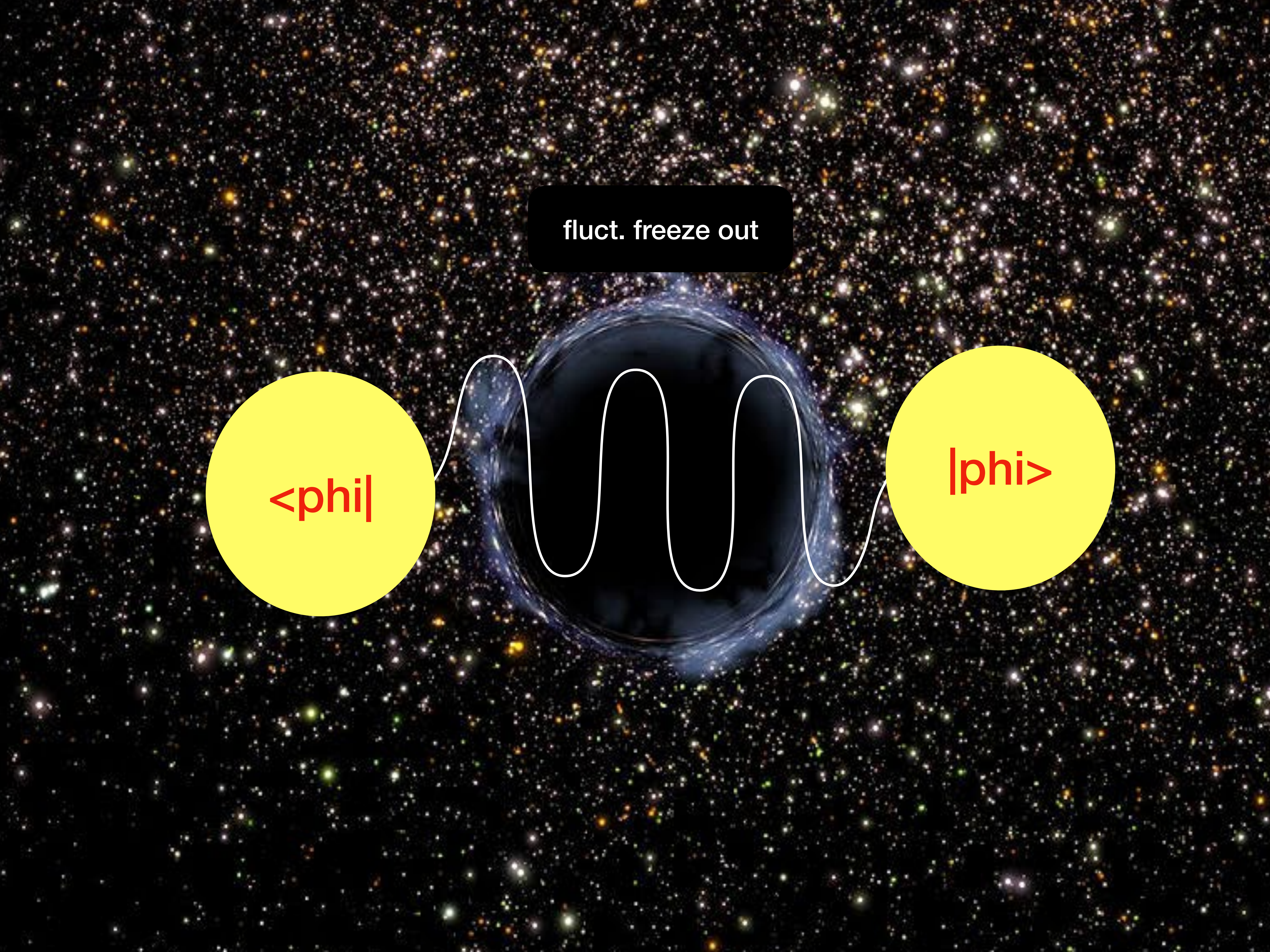
If we zoom in (time & space), a classical vacuum, is full of instantaneous particle creations and annihilations.

(off-set of the equilibrium position denotes for the particle creation/annihilation)

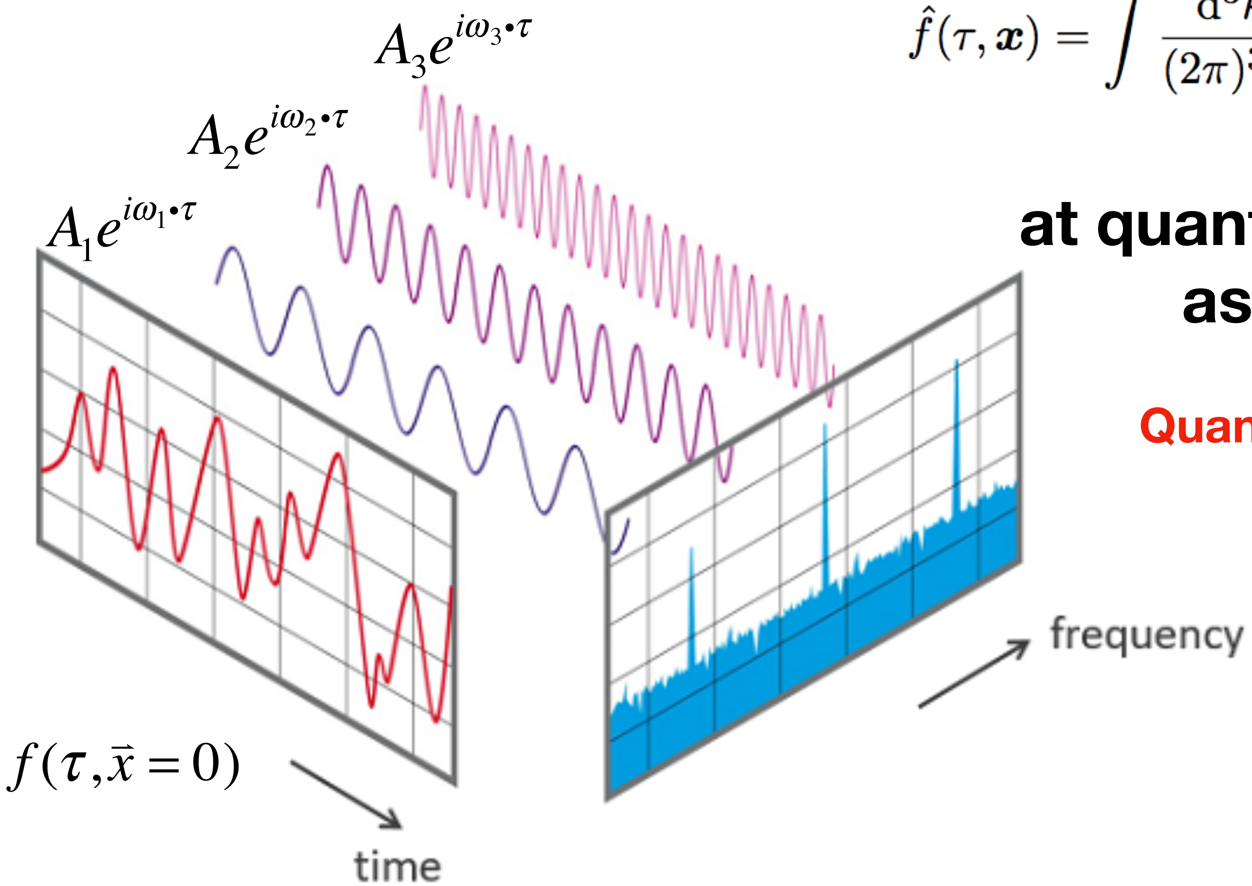


the quantum field view of space-time: string matrix


quantum oscillation
<phi|
|phi>
Horizon scale


fluct. freeze out
<phi|
|phi>

Let us fix a space point $\vec{x}=0$ **, record scalar field amplitude** $f(\tau,\vec{x}=0)$



$$\hat{f}(\tau,\boldsymbol{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \left[f_{\boldsymbol{k}}(\tau)\hat{a}_{\boldsymbol{k}} + f^*_{\boldsymbol{k}}(\tau) a^\dagger_{\boldsymbol{k}} \right] e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

at quantum level, the scalar field can be treated as an assembly of simple harmonics!

Quantum Field is a collection of Quantum mechanics

$$\langle \hat{f} \rangle = 0 \qquad f(\tau,\vec{x}) = \sqrt{<\hat{f}\cdot\hat{f}>}$$

classical solution

quantum operator

$$\hat{f} = f \cdot \hat{\delta} \qquad <\hat{\delta}>=0, <\hat{\delta}\cdot\hat{\delta}>=1$$

Gaussian random variables

quantization of the pert.

$$\hat{f}(\tau, \boldsymbol{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} \left[f_k(\tau)\hat{a}_{\boldsymbol{k}} + f_k^*(\tau) a_{\boldsymbol{k}}^\dagger \right] e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\boxed{[\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{k}'}^\dagger] = \delta(\boldsymbol{k} + \boldsymbol{k}')}$$

$$\langle |\hat{f}|^2 \rangle \equiv \langle 0 | \hat{f}^\dagger(\tau, \boldsymbol{0}) \hat{f}(\tau, \boldsymbol{0}) | 0 \rangle$$

$$= \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3k'}{(2\pi)^{3/2}} \langle 0 | \left(f_k^*(\tau)\hat{a}_{\boldsymbol{k}}^\dagger + f_k(\tau)\hat{a}_{\boldsymbol{k}} \right) \left(f_{k'}(\tau)\hat{a}_{\boldsymbol{k}'} + f_{k'}^*(\tau)\hat{a}_{\boldsymbol{k}'}^\dagger \right) | 0 \rangle$$

$$= \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3k'}{(2\pi)^{3/2}} f_k(\tau) f_{k'}^*(\tau) \langle 0 | [\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{k}'}^\dagger] | 0 \rangle$$

$$= \int \frac{\mathrm{d}^3k}{(2\pi)^3} |f_k(\tau)|^2 \hbar$$

$$= \int \mathrm{d}\ln k \, \frac{k^3}{2\pi^2} |f_k(\tau)|^2 \hbar$$

(deriv)

$$\langle \hat{f} \rangle = 0$$

mode function $f_k(\tau)$: is chosen to be the classical field solution

$$\boxed{f_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau} \right)} \sqrt{\hbar}$$

conjugate momentum

$$[\hat{f}_{\vec{k}}(\tau), \hat{\pi}_{\vec{k}'}(\tau)] = i\delta(\vec{k} + \vec{k}')$$

$$\pi \equiv \frac{\partial \mathcal{L}}{\partial f'} = f'$$

quantum effect

$$\boxed{f_k(\tau) = \alpha \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau} \right) + \beta \frac{e^{ik\tau}}{\sqrt{2k}} \left(1 + \frac{i}{k\tau} \right)}$$

The difference between classical & quantum pert.

for classical pert. (α, β) could be **arbitrary** large

for quantum pert. the wave function must be **unitary** (probability normalised to unity) $\alpha^2 + \beta^2 = 1$

---

decoherence

two quantum states separated by a scale k^{-1}, are in coherence! (correlated amplitude and phase)

However, the afterward cosmic evolution is classical process, e.g. galaxy formation

quantum $\xrightarrow{\text{decoherence}}$ **classical**

sub-horizon

$$f_k \sim \frac{e^{-ik\tau}}{\sqrt{2k}} \qquad \pi_k \sim -\frac{ike^{-ik\tau}}{\sqrt{2k}}$$

$$< 0 | [\hat{f}_k, \hat{\pi}_{k'}] | 0 >= i\delta(k + k')$$

(deriv)

non-commute ⟶ quantum state

super-horizon

$$f_k \sim -\frac{i}{\sqrt{2}k^{3/2}\tau} \qquad \pi_k \sim \frac{i}{\sqrt{2}k^{3/2}\tau^2}$$

$$< 0 | [\hat{f}_k, \hat{\pi}_k] | 0 >= 0$$

(deriv)

commute ⟶ classical state

primordial scalar power spectrum

$$a(\tau)=\frac{\tau_0}{\tau} \qquad aH=\mathcal{H} \qquad a=-1/H\tau$$ (deriv)

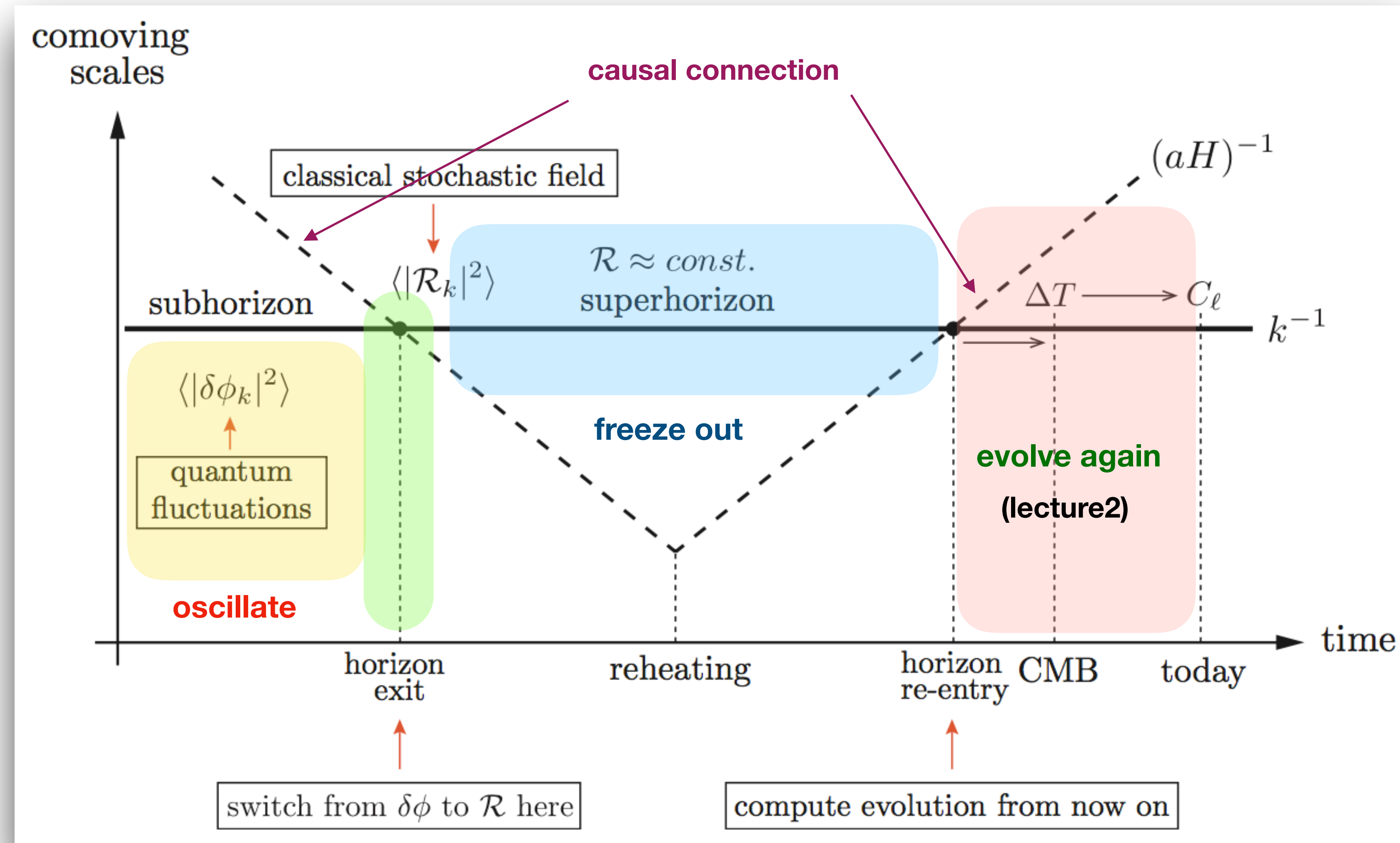
$$\langle|\hat{f}|^2\rangle = \int \mathrm{d}\ln k \, \frac{k^3}{2\pi^2}|f_k(\tau)|^2$$

dimensionless power spectrum

$$\boxed{\Delta_f^2(k,\tau) \equiv \frac{k^3}{2\pi^2}|f_k(\tau)|^2}$$

super-horizon mode

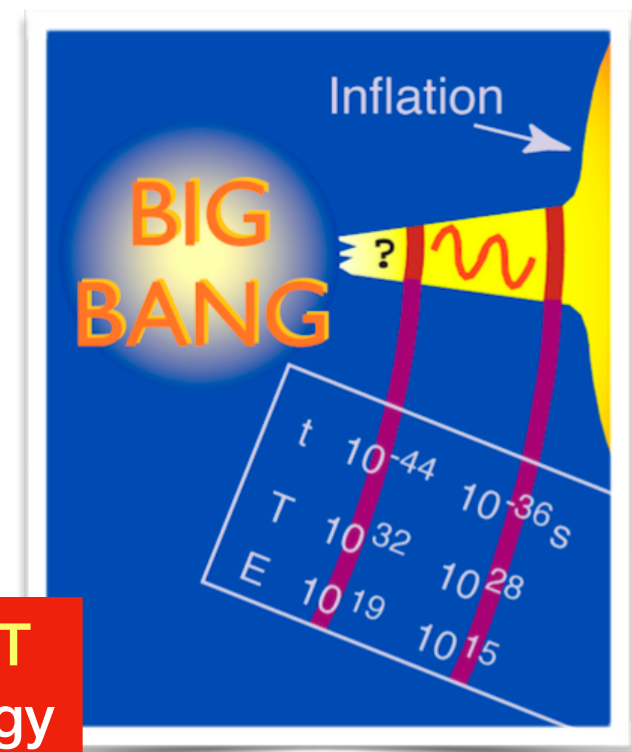
$$f_k \sim -\frac{i}{\sqrt{2}k^{3/2}\tau}$$



$$\Delta_{\delta\phi}^2(k,\tau) = a^{-2}\Delta_f^2(k,\tau) = \left(\frac{H}{2\pi}\right)^2$$ (deriv)

the amplitude of the pert. is proportional to inflationary energy scale!

(by measuring the amp we can 'know' the inflation energy scale)



[Pb2.]

$$\Delta_{\mathcal{R}}^2 = \frac{1}{2\varepsilon}\frac{\Delta_{\delta\phi}^2}{M_{\rm pl}^2}\,, \qquad \text{where} \quad \varepsilon = \frac{\frac{1}{2}\dot{\phi}^2}{M_{\rm pl}^2 H^2}$$

gauge-inv curvature pert.

$$\boxed{\Delta_{\mathcal{R}}^2(k) = \frac{1}{8\pi^2}\frac{1}{\varepsilon}\frac{H^2}{M_{\rm pl}^2}\bigg|_{k=aH}} \quad \textbf{or} \quad \boxed{\Delta_{\mathcal{R}}^2 = \frac{1}{12\pi^2}\frac{V^3}{M_{\rm pl}^6(V')^2}}$$

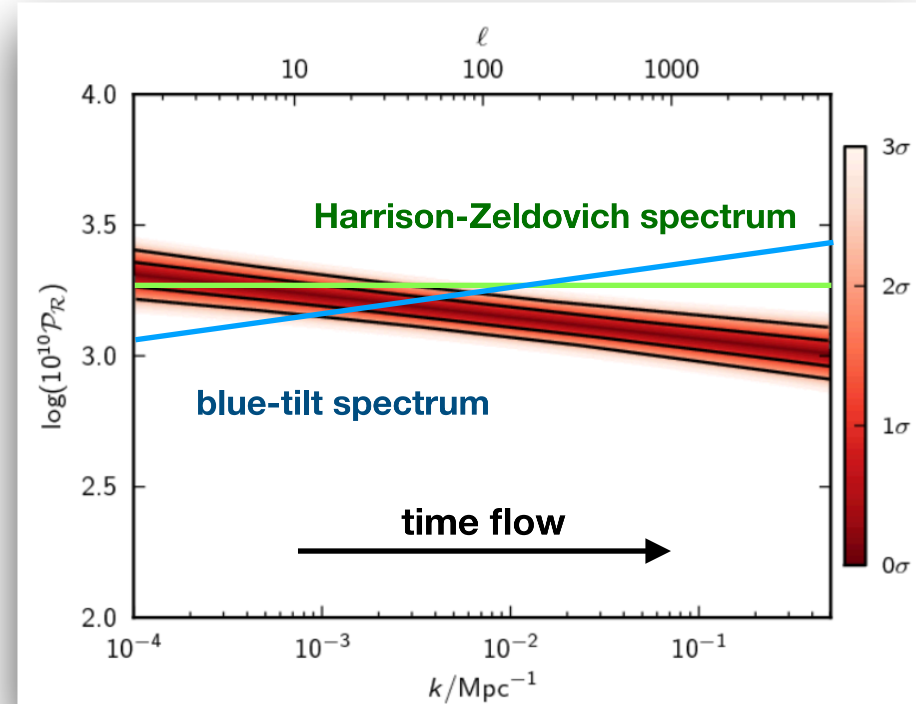
$$H^2 \propto V \qquad \Delta_R \sim (V, V')$$

scalar pert. per. se. **could NOT determine the inflation energy scale!** (its amp also depends **on the potential slop)**

nearly scale-inv power spectrum

$$\boxed{\Delta^2_{\mathcal{R}}(k) = \frac{1}{8\pi^2}\frac{1}{\varepsilon}\frac{H^2}{M^2_{\rm pl}}\bigg|_{k=aH}}$$

if ε, H purely constant $\longrightarrow$ exact scale-inv



$$\varepsilon \equiv -\frac{\dot H}{H^2} \qquad \eta \equiv \frac{d\log\varepsilon}{dN} \qquad a = e^N = e^{\int H\,dt}$$

1st time derivative

2nd time derivative

$$\Delta^2_{\mathcal{R}}(k) \equiv A_s\left(\frac{k}{k_\star}\right)^{n_s-1}$$

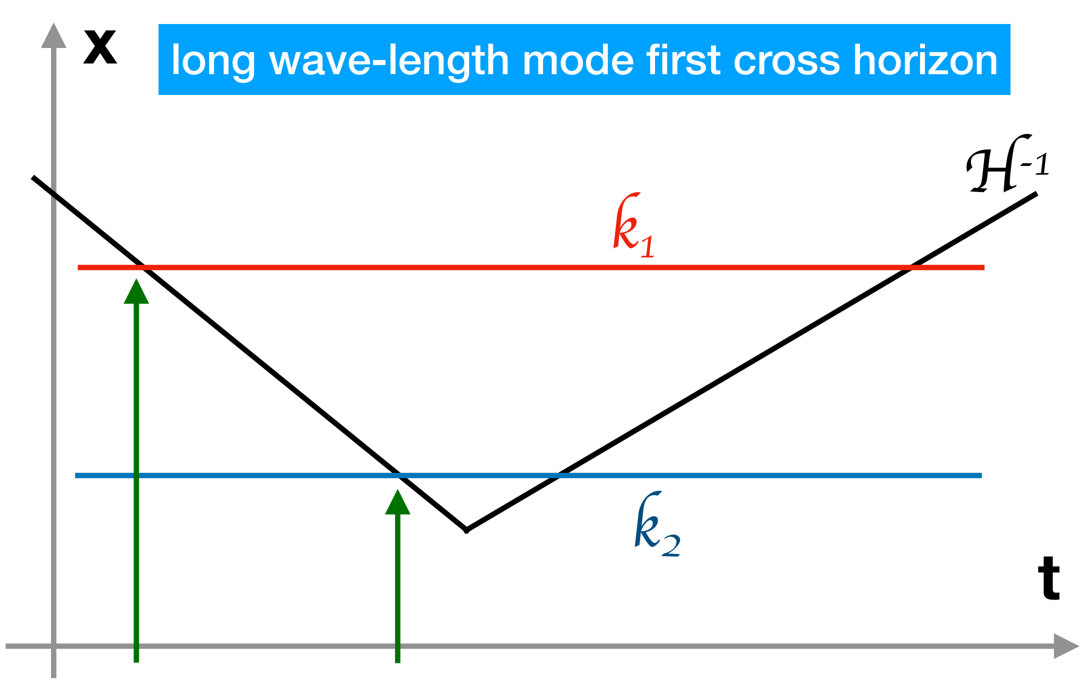
$$n_s - 1 = \frac{d\log\Delta_R^{\;2}}{d\log k} \sim -2\varepsilon - \eta$$

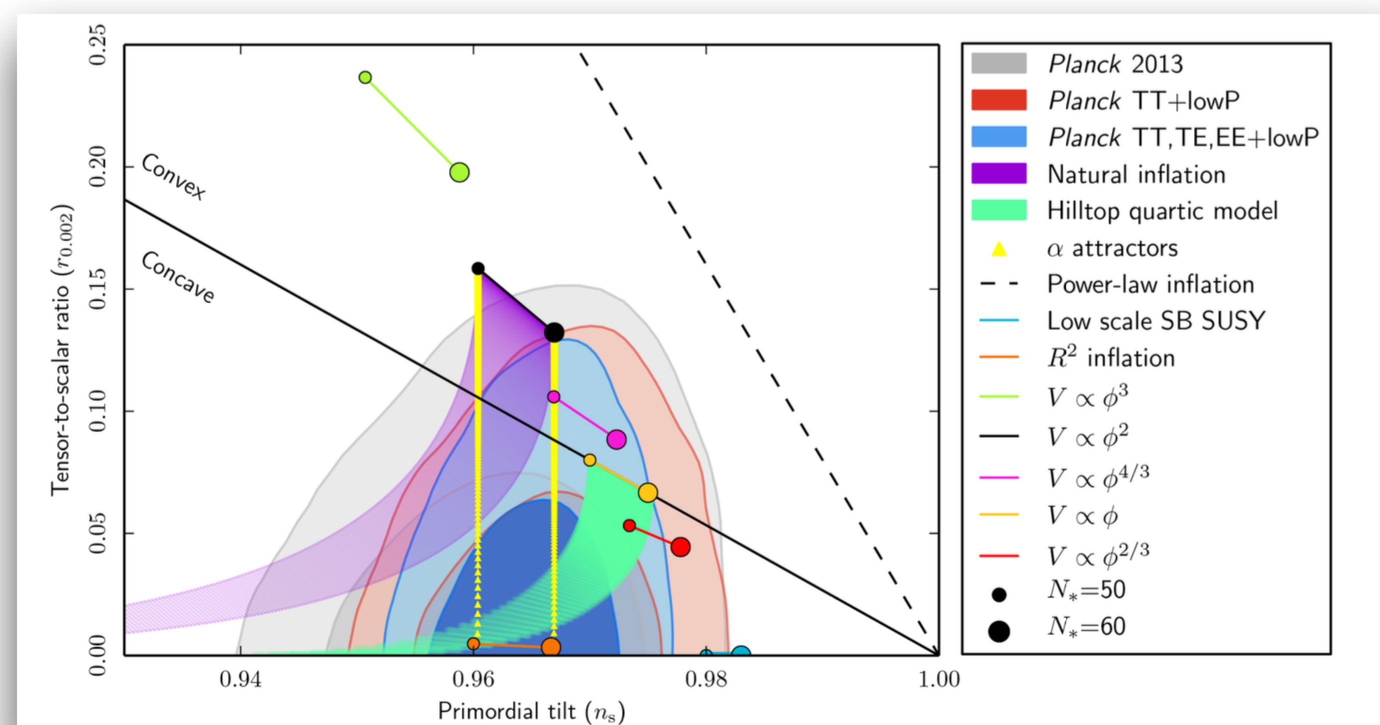
(deriv)

$$\boxed{A_s = (2.196 \pm 0.060)\times 10^{-9}}$$

$$\boxed{n_s = 0.9603 \pm 0.0073}$$

- **red-tilt:** $n_s - 1 < 0$ amp is large on the large scale
- **blue-tilt:** $n_s - 1 > 0$ amp is large on the small scale





tensor pert. (primordial gravitational waves)

@ such high energy scale, if inflaton could have instantaneous particle creation/annihilation, why not the graviton?

no symmetry prevent this!

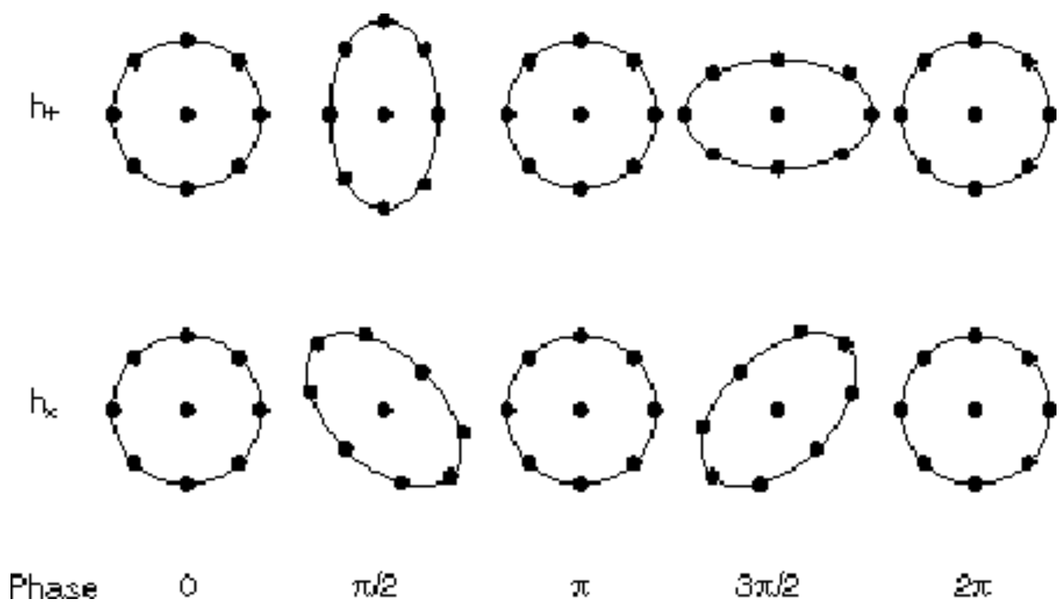
$$ds^2 = a^2(\tau)\left[d\tau^2 - (\delta_{ij} + 2\hat{E}_{ij})dx^i dx^j\right]$$

$$\frac{M_{\rm pl}}{2}\, a\hat{E}_{ij} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} f_+ & f_\times & 0 \\ f_\times & -f_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}\,R \quad \Rightarrow \quad S^{(2)} = \frac{M_{\rm pl}^2}{8}\int d\tau\, d^3x\, a^2\left[(\hat{E}'_{ij})^2 - (\nabla\hat{E}_{ij})^2\right]$$

[Pb3.]

$$S^{(2)} = \frac{1}{2}\sum_{I=+,\times}\int d\tau d^3x\left[(f'_I)^2 - (\nabla f_I)^2 + \frac{a''}{a}f_I^2\right]$$



exactly the same as scalar pert.

[Pb4.]

$$\boxed{\Delta_t^2(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\rm pl}^2}\bigg|_{k=aH}} \quad \textbf{V.S.} \quad \boxed{\Delta_{\mathcal{R}}^2(k) = \frac{1}{8\pi^2}\frac{1}{\varepsilon}\frac{H^2}{M_{\rm pl}^2}\bigg|_{k=aH}}$$

only depends on H!

direct probe of inflation scale! that is why we need measure PGW! **fundamental physics**

(see pic in prev)

$$\Delta_t^2(k) \equiv A_t\left(\frac{k}{k_\star}\right)^{n_t} \qquad r \equiv \frac{A_t}{A_s}$$

Exercise.—Show that

[Pb5.]

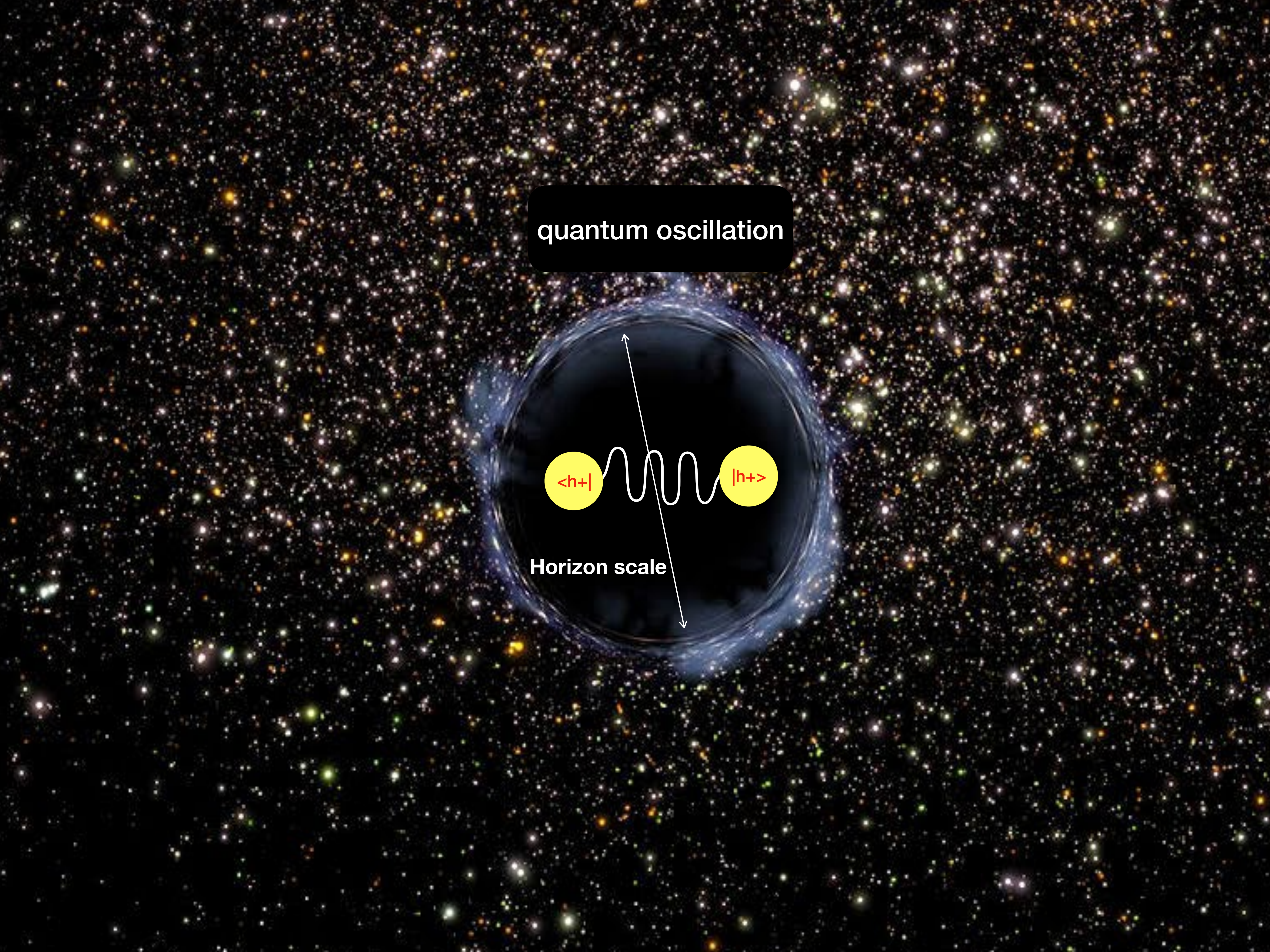
$$r = 16\varepsilon$$
$$n_t = -2\varepsilon\ .$$

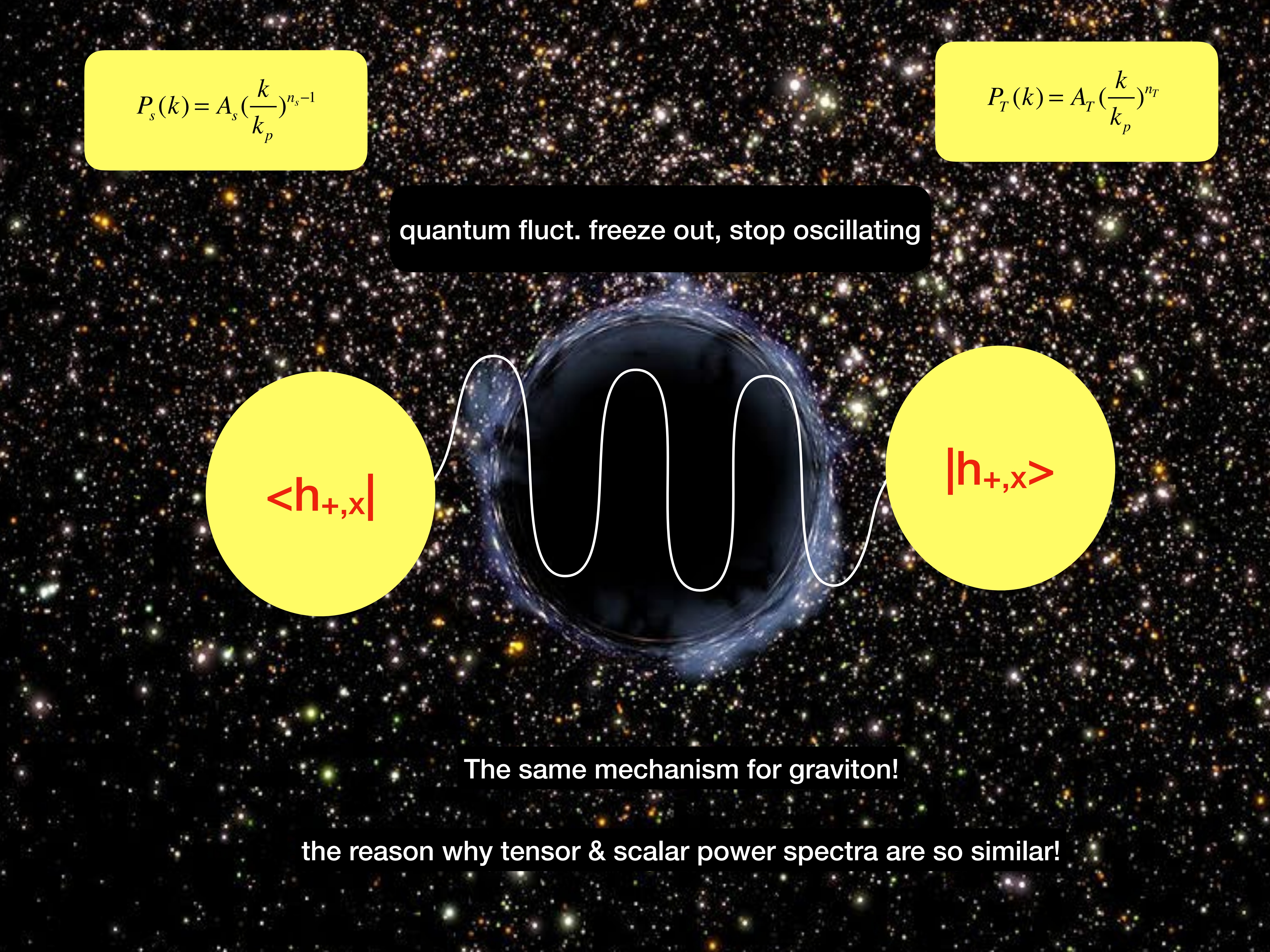
Notice that this implies the consistency relation $n_t = -r/8$.

scalar spec can be both red & blue

tensor spec must be both blue!

(otherwise, violate null energy condition)


quantum oscillation
<h+|
|h+>
Horizon scale


$P_s(k) = A_s(\frac{k}{k_p})^{n_s-1}$
$P_T(k) = A_T(\frac{k}{k_p})^{n_T}$
quantum fluct. freeze out, stop oscillating
$\langle h_{+,x}|$
$|h_{+,x}\rangle$
The same mechanism for graviton!
the reason why tensor & scalar power spectra are so similar!

Further reading

- **Baumann lecture note/**Chapter 6

- **Physical Foundations of Cosmology/**Mukhanov