Cosmic Large-scale Structure Formations

Bin HU

bhu@bnu.edu.cn

Astro@BNU

Office: 京师大厦9907

18 weeks

outline

Background (1 w)

- universe geometry and matter components (1 hr)
- Standard candle (SNIa) (0.5 hr)
- Standard ruler (BAO) (0.5 hr)

Linear perturbation (9 w)

- relativistic treatment perturbation (2 hr)
- primordial power spectrum (2 hr)
- linear growth rate (2 hr)
- galaxy 2-pt correlation function (2 hr)
- Bayron Acoustic Osciilation (BAO) (2 hr)
- Redshift Space Distortion (RSD) (2 hr)
- Weak Lensing (2 hr)
- Einstein-Boltzmann codes (2 hr)

Non-linear perturbation (6 w)

- Non-linear power spectrum (2 hr)
- halo model (2 hr)
- N-body simulation algorithms (2 hr)
- Press-Schechter (PS) halo mass function (2 hr)
- Extended-PS (EPS) halo mass function (2 hr)
- halo bias & halo density profile (2 hr)

Statistical analysis (2 w)

- Monte-Carlo Markov Chain sampler (2 hr)
- CosmoMC use (2 hr)

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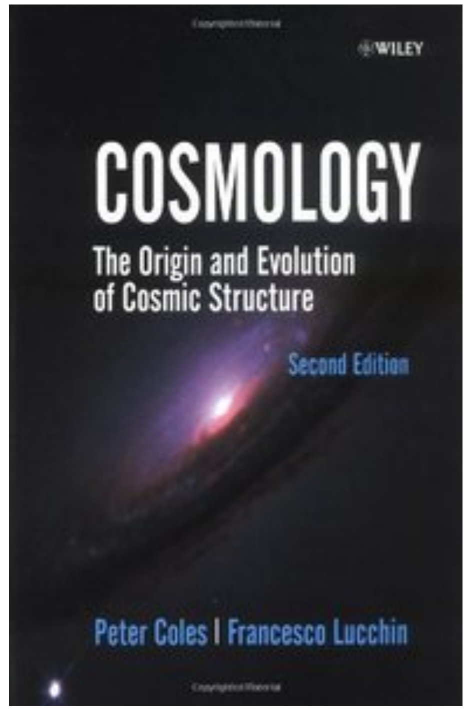
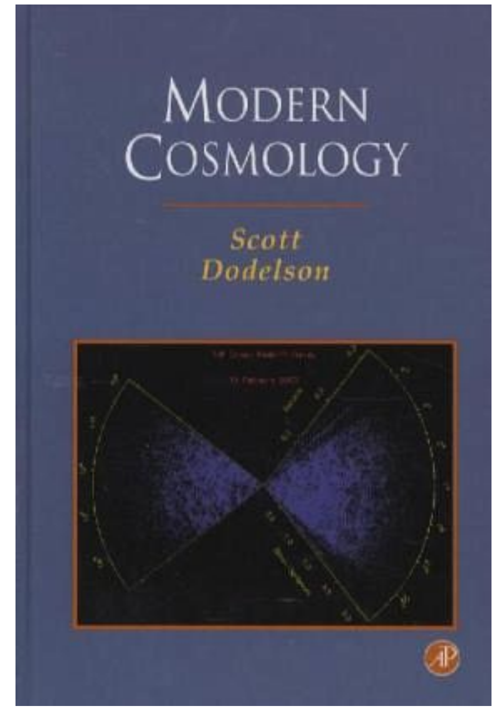
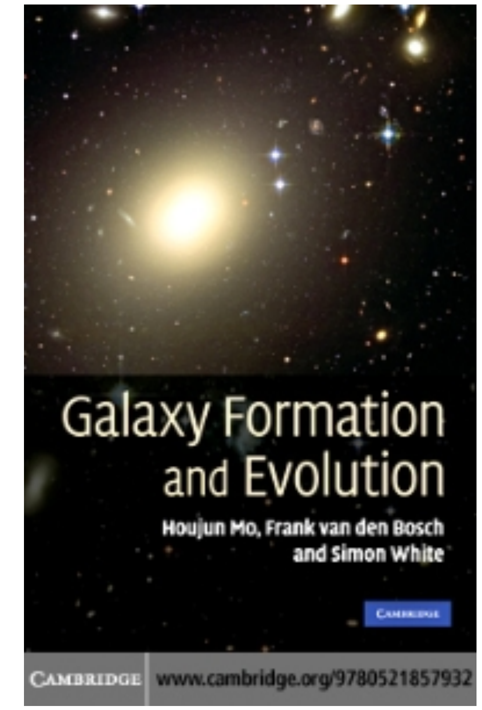
1. Gaussian Random Field/ Power spectrum/Correlation function/ Phase
2. BAO
3. Galaxy Clustering
4. RSD
5. Lensing: WL/ Strong Lensing
6. Linear Growth
7. Nonlinear growth (spherical collapse)
8. Halo model: Press-Schesther formalism, merge tree

references







- 宇宙大尺度结构的形成 向守平、冯珑珑
- Cosmology Peter Coles & Francesco Lucchin
- Modern Cosmology Scott Dodelson
- Galaxy Formation and Evolution Houjun Mo & van den Bosch and Simon White
- Baumann lecture note

http://101.96.8.165/www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf

成绩计算（百分制）

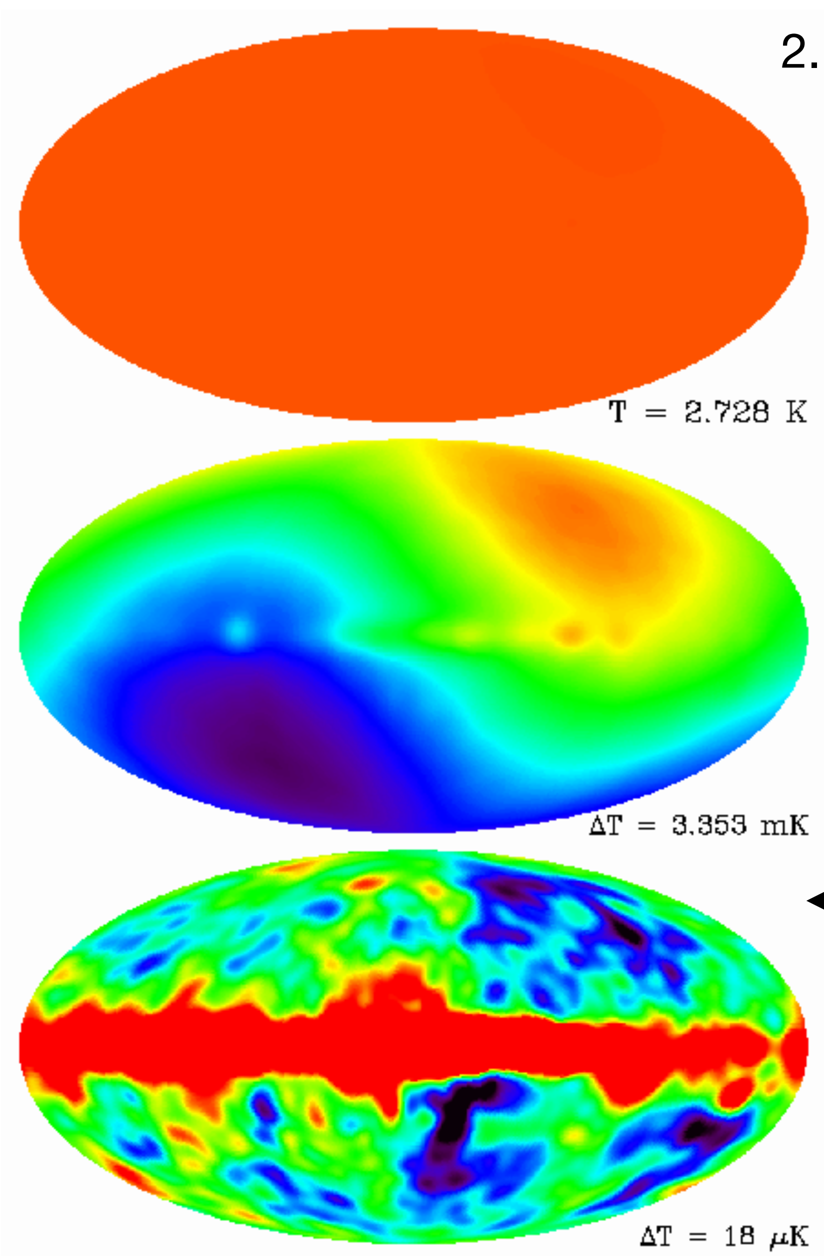
1. 平时作业：40%
2. 期末随堂考试：60%

Lecture 1

1.cosmological principle/CP

For a **co-moving observer**, on the **large** scale, the universe is **homogenous** and **isotropic**.

1. Observer: co-move with the background expansion

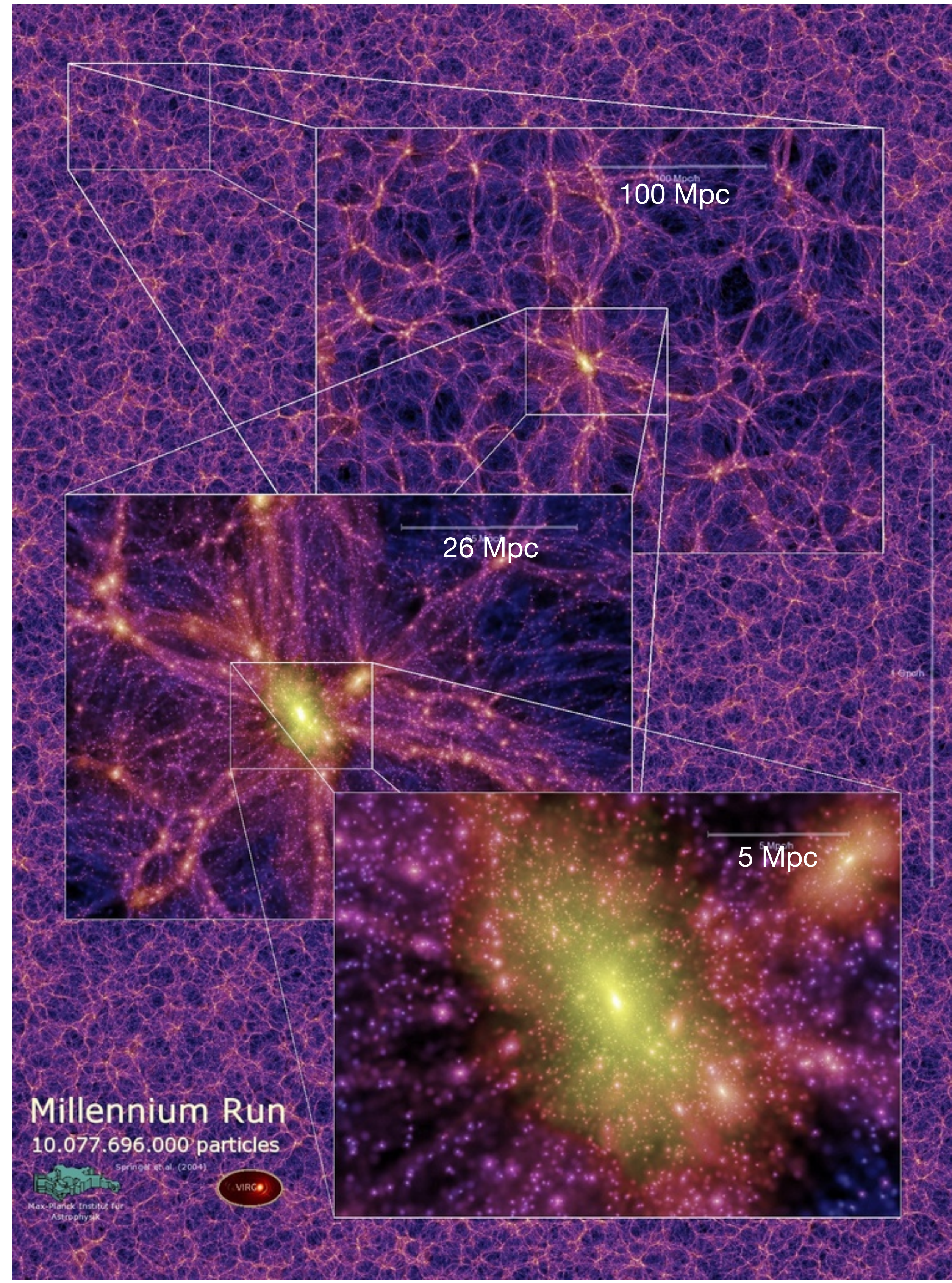


2. On this scale (> 1 Mpc): each galaxy is l ike a test particle

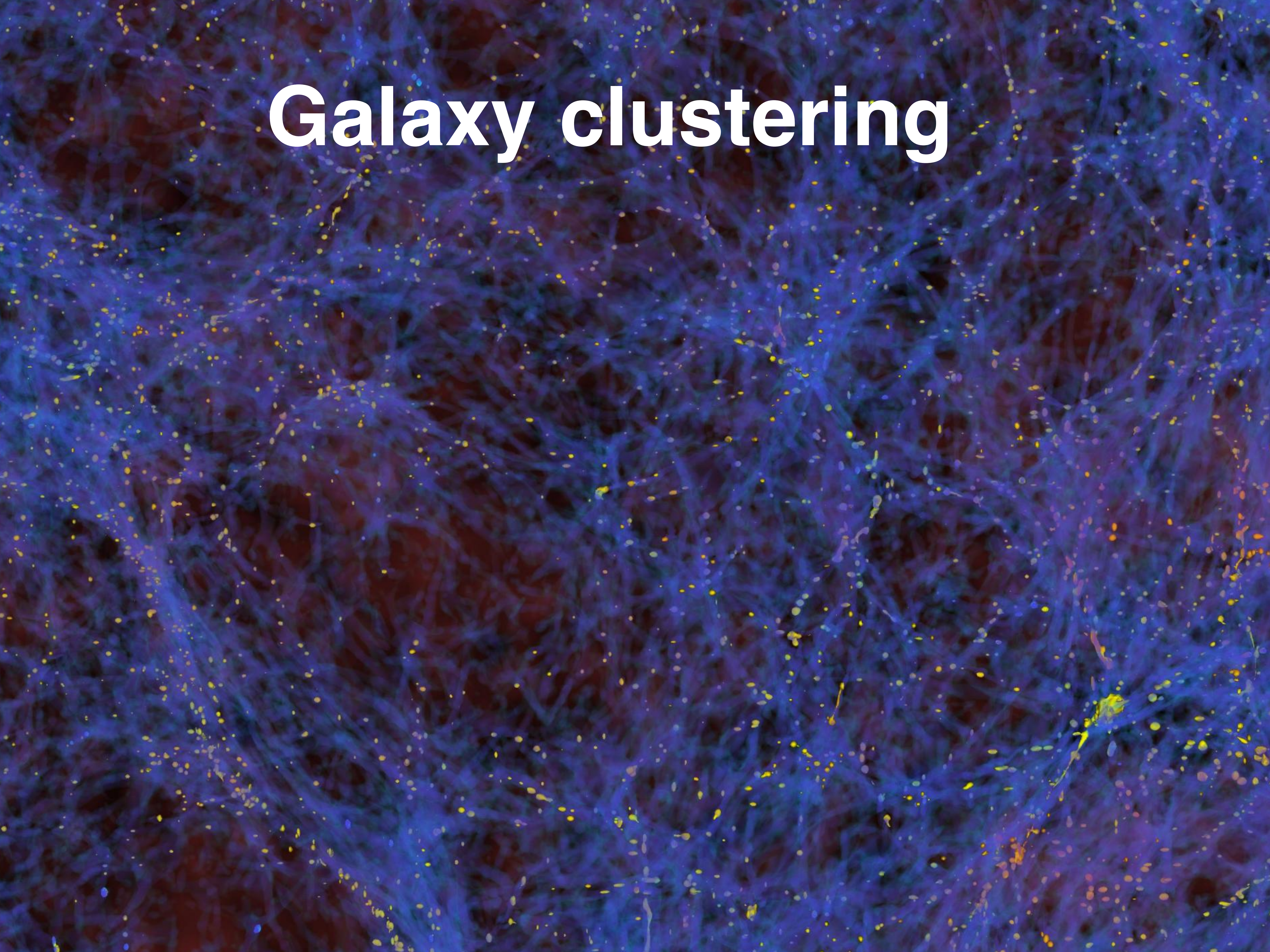
[milky way ~ 15kpc, 1pc ~ 3 ly]

relativistic photon distribution

non-relativistic matter distribution



Galaxy clustering



2. FRWL metric [Friedmann–Robertson–Walker-Lemaître]

$$[ds^2 = g_{\mu\nu} dx^\mu dx^\nu]$$

$g_{\mu\nu} \rightarrow$ describe the d.o.f. gravity sector

similar to: E/B field in Maxwell eq.

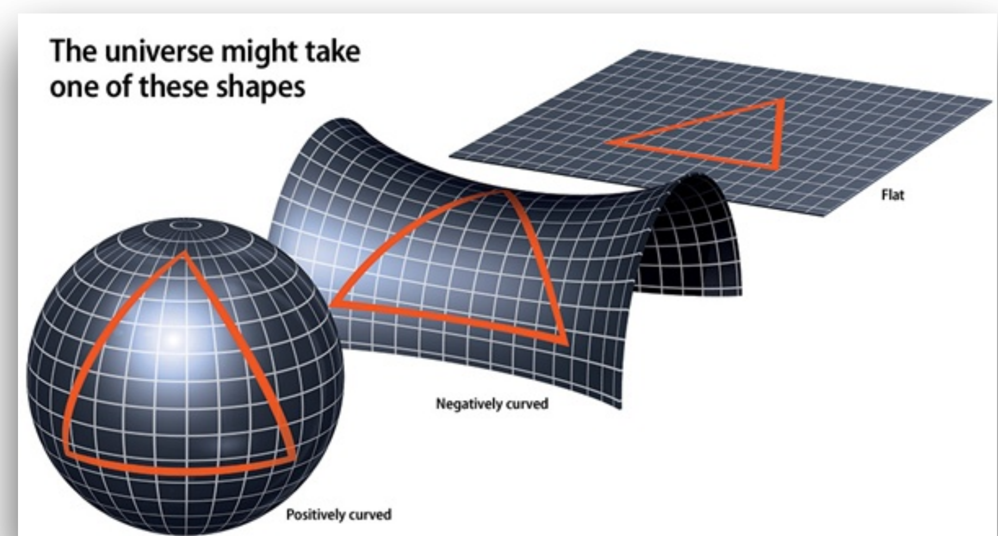
$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$\xrightarrow[\text{format}]{\text{covariant}}$ $F^{\mu\nu}{}_{;\nu} = J^\mu$

[c=1] $$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-K(t)r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right]$$

3 spatial curvature

$$K(t) = \begin{cases} > 0 \rightarrow (close) \\ 0 \rightarrow (flat) \\ < 0 \rightarrow (open) \end{cases}$$

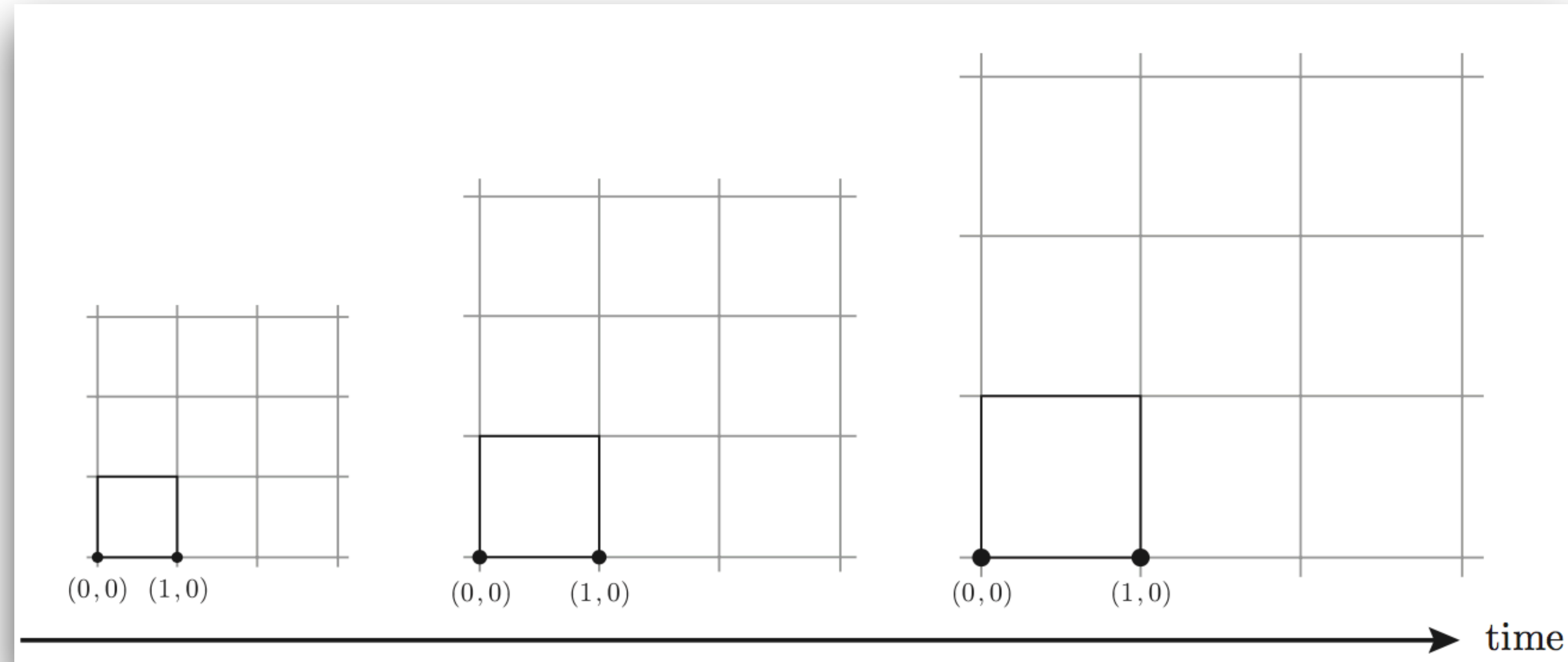


a(t) [scale factor 标度因子]: tells the physical size of the universe.

cosmic redshift: $a = 1/(1+z)$ or $z = 1/a - 1$ with $a_0 = 1, z = 0$

[http://101.96.8.165/www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf]

[Baumann lecture note]



$$ds^2 = -dt^2 + a^2(t)\Big[\underbrace{\frac{dr^2}{1-K(t)r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2}_{\text{(co-moving) } du^2}\Big] \Rightarrow dv^2 \text{ (physical)}$$

dv = a du

here, du = 1, but dv increase w.r.t. time

For a **co-moving observer**, on the **large** scale, the universe is **homogenous** and **isotropic**.

physical meaning of FRWL metric:

the only metric compatible with the cosmological principle!

e.g. flat case $ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2] \longrightarrow$ isotropic (rotation symm.)

coordinate transformation

$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \longrightarrow$ homo (spatial shift symm.)

$$[ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \tilde{g}_{\mu\nu}d\tilde{x}^\mu d\tilde{x}^\nu]$$

iso/hom is purely geometry property of the space-time, so it does NOT depends on the coord. But, some properties are more easily demonstrated in some specific coordinates.

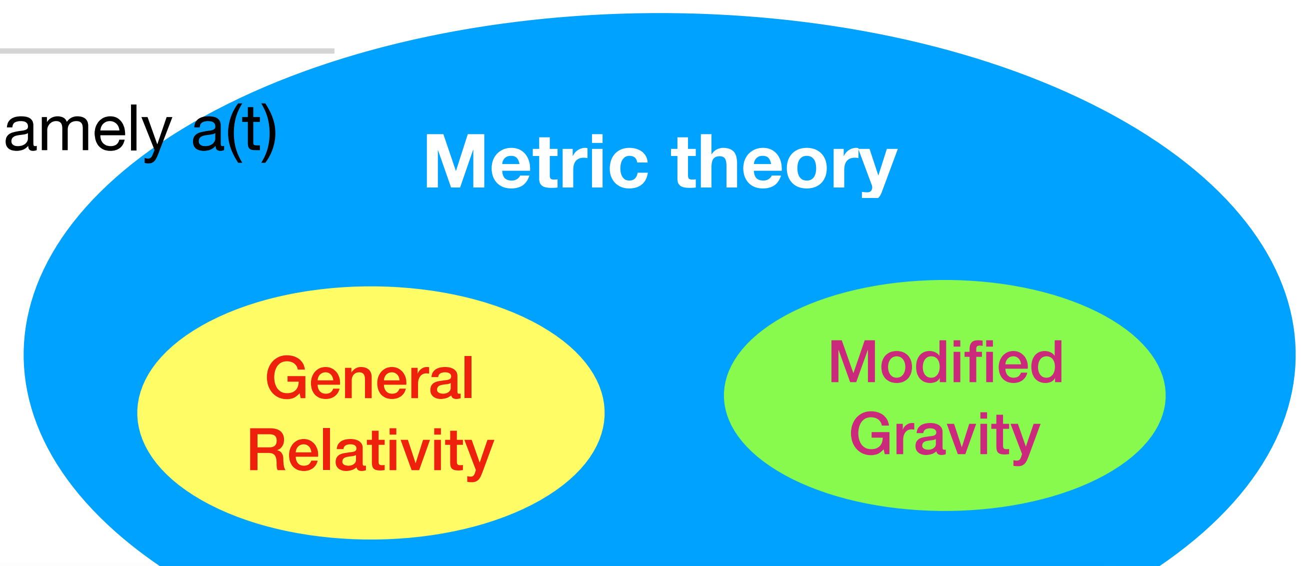
More importantly, FRWL metric defines **a unique clock** with time coordinate ’t’

All the observers, **who satisfy the CP**, have to **co-move with this clock**

Now, the metric is fixed up to a function of ’t’, namely a(t)

This is because, up to now, we only use the geometric/symm. property of the space-time

In order to fix a(t), we need to solve the dynamical equation of gravity sector



e.g. GR: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ f(R) gravity: $F(R)R_{\mu\nu}(g) - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = \kappa^2 T^{(M)}_{\mu\nu}$

3. Friedmann eq.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

(EoM of gravitational d.o.f.) → Einstein eq.

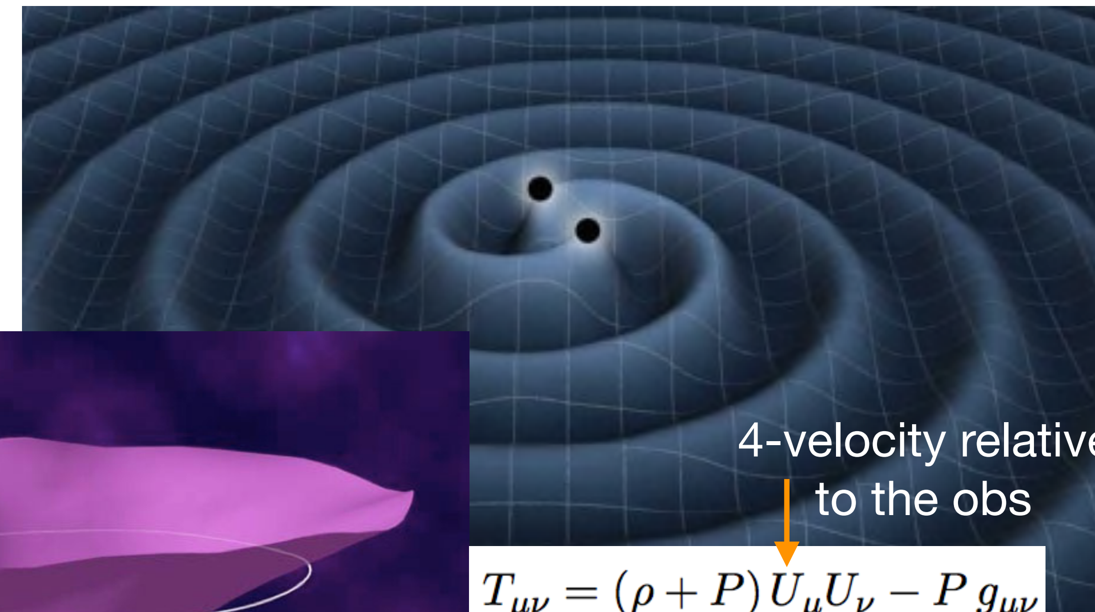
gravity sector (Geometry of space-time)

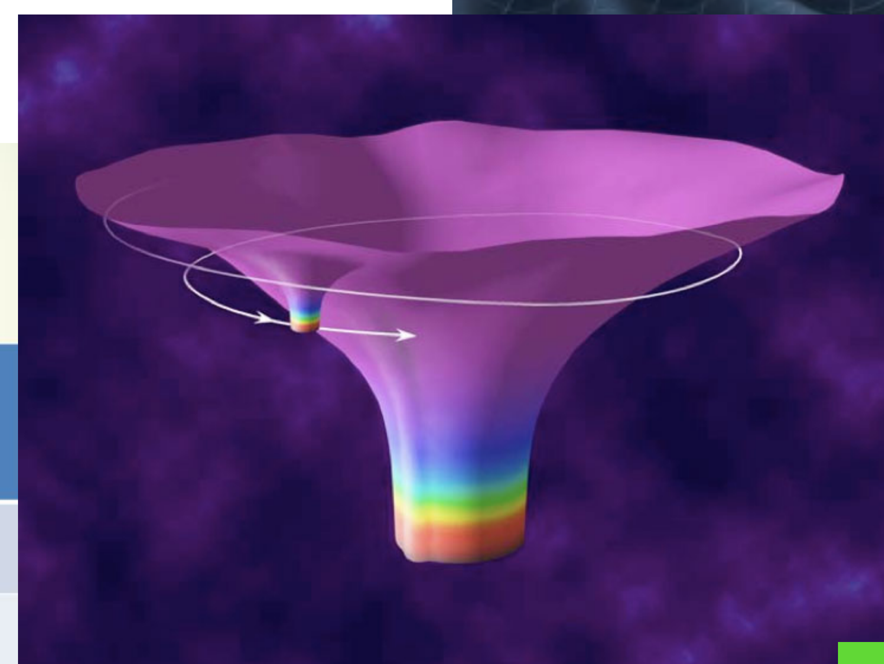
gravitational coupling constant

matter sector (stress-energy tensor)

John Wheeler

Spacetime tells matter how to move; matter tells spacetime how to curve





4-velocity relative to the obs

$$T_{\mu\nu} = (\rho + P)\,U_\mu U_\nu - P\,g_{\mu\nu}$$

$$\begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & +P & 0 & 0 \\ 0 & 0 & +P & 0 \\ 0 & 0 & 0 & +P \end{pmatrix}$$

model cosmic matter distribution by the fluid approach!

Gravitation and the other fundamental interactions

Fundamental Interaction	Crucial years	Fundamental constant	Normalized Intensity
Gravity	1687	$Gm_p^2/\hbar c$	5.1×10^{-39}
Weak nuclear force	1934	$G_{Fermi}\,(m_p c^2)^2$	1.03×10^{-5}
Electromagnetism	1864	$e^2/(4\pi\varepsilon_0\hbar c)$	$7.3\times10^{-3} \simeq 1/137$
Strong nuclear force	1935/1947	α_s	0.119

need a lots of energy to bend the space-time!

$$\nabla_\mu T^{\mu\nu} = 0$$

→ energy-momentum conservation eq. (EoM of matter)

$$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ Einstein tensor

$R_{\mu\nu} \equiv \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\lambda\rho}\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\rho}$ Ricci tensor

$R = R^\mu{}_\mu = g^{\mu\nu} R_{\mu\nu}$ Ricci scalar

$\boxed{\Gamma^\mu_{\alpha\beta} \equiv \frac{1}{2} g^{\mu\lambda}(\partial_\alpha g_{\beta\lambda} + \partial_\beta g_{\alpha\lambda} - \partial_\lambda g_{\alpha\beta})}$ connection/Christoffel symbol

$G_{\mu\nu}$ contains $(g_{\mu\nu}, \dot{g}_{\mu\nu}, \ddot{g}_{\mu\nu})$ $\longrightarrow$ good! does not need acceleration of acceleration

Classical dynamics tell us: a canonical dynamical system,
shall at most contain the 2nd order time derivative of its dynamical variables.

However, $G_{\mu\nu}(g_{\mu\nu})$ is a **non-linear** functional. $\longrightarrow$ bad! very hard to solve

e.g. for merger stage of binary black hole system, EE is very very hard to solve!

EE is written @1915, but the first bbh solution is got @2005

For FRWL metric: $ds^2 = -dt^2 + a^2(t)\delta_{ij} dx^i dx^j$

[Pb1.] $R_{00} = -3\frac{\ddot{a}}{a}$

$R_{ij} = [a\ddot{a} + 2\dot{a}^2]\delta_{ij}$

$\longrightarrow$

$$G^0{}_0 = 3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right],$$

$$G^i{}_j = \left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]\delta^i_j$$

$\longrightarrow$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \quad \text{1st Friedmann eq.}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad \text{2nd Friedmann eq.}$$

We need:

For a co-moving obs: $U^{\mu} = (-1,0,0,0)$

For a perfect fluid: $T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & +P & 0 & 0 \\ 0 & 0 & +P & 0 \\ 0 & 0 & 0 & +P \end{pmatrix}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}\,,$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

Not independent with each other

[Pb2.] Check the relationship between 1st & 2nd Friedmann eq.

$$\nabla_{\mu} T^{\mu\nu} = 0 \longrightarrow \boxed{\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0}$$

Besides the conservation eq. we also need the thermal dynamical info of the fluid

$$\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda$$

(CAN SNeIa measure H_0?)

e.g. Equation of State $w = P/\rho$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$V \sim a^{-3}$$

$$\rho \propto a^{-3(1+w)}\,,$$

$$\rho \propto \begin{cases} a^{-3} & \text{matter} \\ a^{-4} & \text{radiation} \\ a^{0} & \text{vacuum} \end{cases}$$

- for non-relativistic particle, E is conserved
- for relativistic photon, E is **NOT** conserved!

$$E(\nu_a) > E(\nu_b)$$

- for vacuum energy, E is **NOT** conserved!

$$dU = -PdV$$

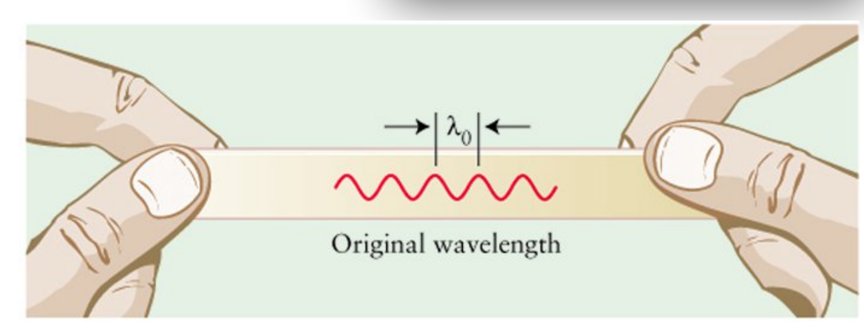
vacuum energy:
where there is space, there it is

$$dU = \rho dV$$

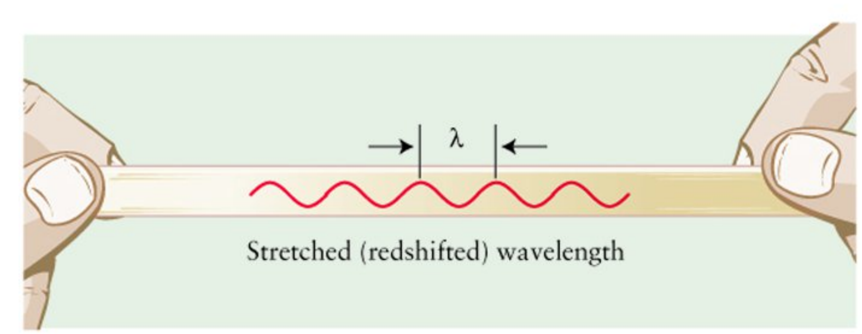
$$\rho_{\mathrm{crit},0} = \frac{3H_0^2}{8\pi G} = 1.9 \times 10^{-29}\, h^2 \,\mathrm{grams\,cm^{-3}}$$
$$= 2.8 \times 10^{11}\, h^2\, M_\odot\, \mathrm{Mpc}^{-3}$$
$$= 1.1 \times 10^{-5}\, h^2 \,\mathrm{protons\,cm^{-3}}$$

$$\Omega_{I,0} \equiv \frac{\rho_{I,0}}{\rho_{\mathrm{crit},0}}$$

A negative EoS means, after a system work to the environment, its internal energy is **increased** instead of decreased.



(a) A wave drawn on a rubber band ...

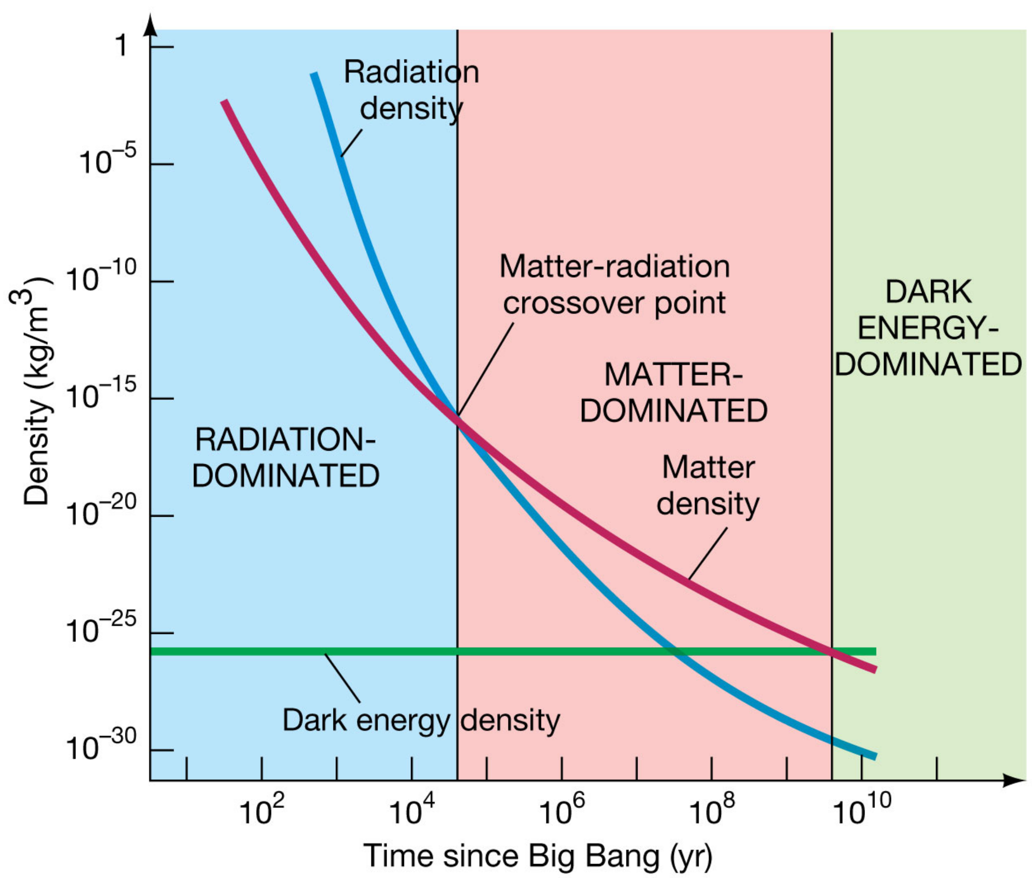


(b) ... increases in wavelength as the rubber band is stretched.

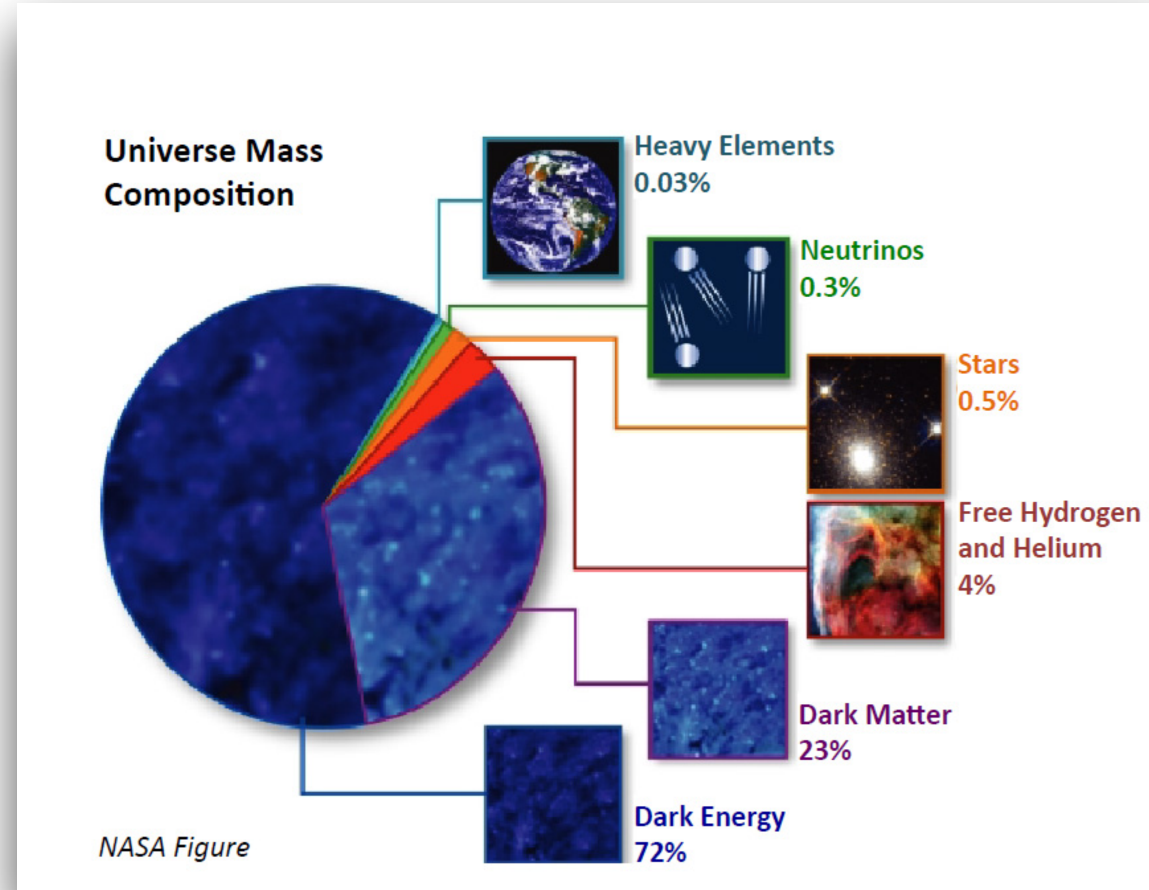
single component universe solution

$$a(t) \propto \begin{cases} t^{2/3(1+w_I)} & w_I \neq -1 & \begin{matrix} t^{2/3} & \text{MD} \\ t^{1/2} & \text{RD} \end{matrix} \\ e^{Ht} & w_I = -1 & \Lambda\text{D} \end{cases}$$

$$a(\tau) \propto \begin{cases} \tau^{2/(1+3w_I)} & w_I \neq -1 & \begin{matrix} \tau^{2} & \text{MD} \\ \tau & \text{RD} \end{matrix} \\ (-\tau)^{-1} & w_I = -1 & \Lambda\text{D} \end{cases}$$



matter ingredient

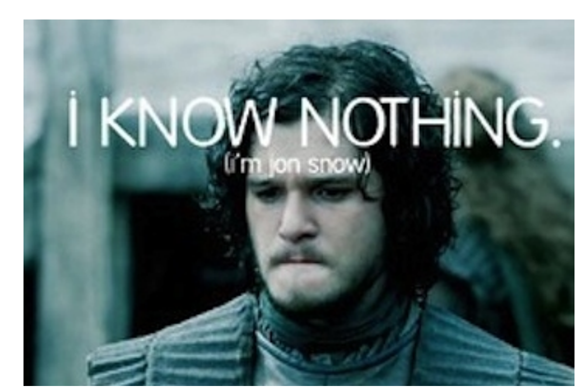


baryon & DM is **indistinguishable** on the large scale

on the small scale, baryon stop collapsing once below its jeans radius
DM will keep collapsing until r~0

photon & neutrino is **indistinguishable** in the early stage (z>200)
once z<200, neutrino will becomes non-relativistic, behaves more like DM

As of DE:



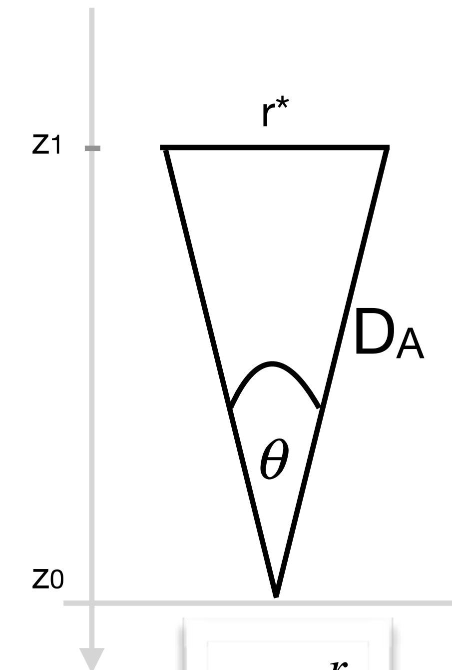
4. Distance

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2]$$ Null-like geodesic $\longrightarrow$ $ds = 0$

a light co-movingly propagate with background expansion along the radial direction from z_1 to z_0 $\longrightarrow$ $dr = dt/a$

- **co-moving distance [along line-of-sight]:**

$$\chi = \int_{z1}^{z0} dr = \int_{z1}^{z0} \frac{dt}{a} = \int_{z1}^{z0} \frac{dz}{H(z)}$$



- **diameter distance [transverse]:** $r_* \rightarrow$ known by prior (physical scale)

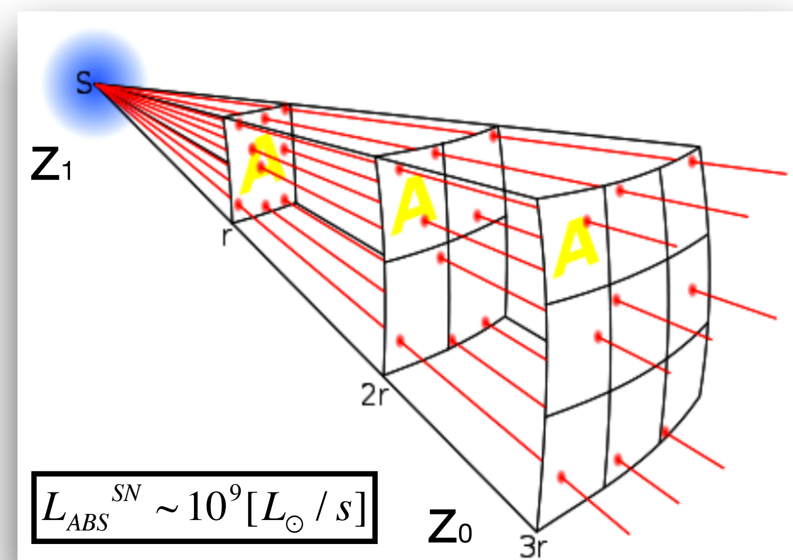
measure the angular separation θ

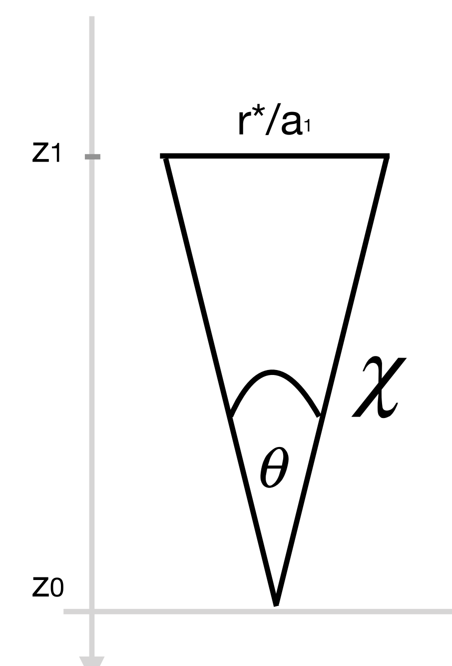
assuming Euclidean geometry, we can define

$$D_A = \frac{r_*}{\theta}$$

But, this is WRONG! Physical geometry is NOT Euclidean,
the co-moving one does! We need re-scale r_* to the co-moving one, namely r_*/a_1

$$D_A = \frac{\chi}{1+z}$$





$$\chi = \frac{(1+z_1) r_*}{\theta}$$

- **luminosity distance:**

$$F_{obs}(z_0) = \frac{L_{ABS}(z_1)}{4\pi * D_L^2}$$

In Euclidean geometry, we shall have

$$F_{obs}(z_0) = \frac{L_{ABS}(z_0)}{4\pi * \chi^2}$$

$$\boxed{L_{ABS}(z_0) = \frac{E_0}{\delta t_0} = \frac{E_1/(1+z)}{(1+z)\delta t_1} = L_{ABS}(z_1)/(1+z)^2}$$

$$D_L = (1+z)\chi$$

●●●○○ 中国联通 17:22 48 %

返回 超级数学建模

为什么宇宙的年龄是 130 亿年，而我们却能看到 470 亿光年远的东西?

2017-09-04 土豆泥 超级数学建模

超级数学建模

数学思维的聚集地

麻烦来个通俗解释

> 宇宙的年龄约130亿年，可观宇宙半径约为470亿光年。资料在维基百科等很多地方都可以查到。我只是不理解如果光速不可超越的话，怎么会在130亿年时间里产生了470亿

[Pb3.]

calculate this number now!


●●●○○ 中国联通 17:22 48 %
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麻烦来个通俗解释

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$$\Omega_{m,0} = 0.3$$

$$\Omega_{\Lambda,0} = 0.7$$

$$\Omega_{r,0} = 10^{-5}$$

$$H_0 = 68[km/s/Mpc]$$

$$c = 30 * 10^4 km/s$$

$$z_0 = 0; z_1 = 1100$$

$$\chi = \frac{c}{H_0}\int_{z1}^{z0}\frac{dz}{E(z)}; E^2(z) = \Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0}$$

~14 Gpc

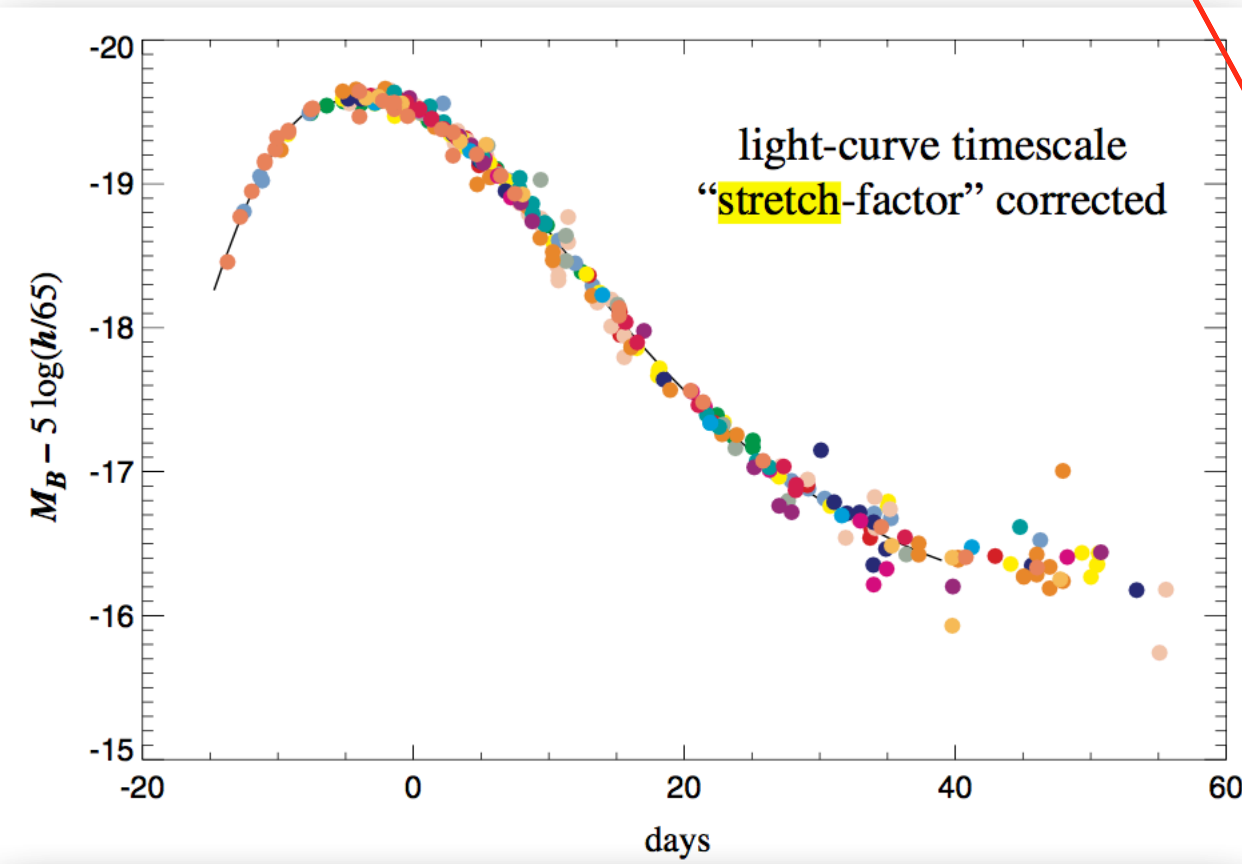
$$t = \frac{c}{H_0}\int_{z1}^{z0}\frac{dz}{E(z)(1+z)}$$

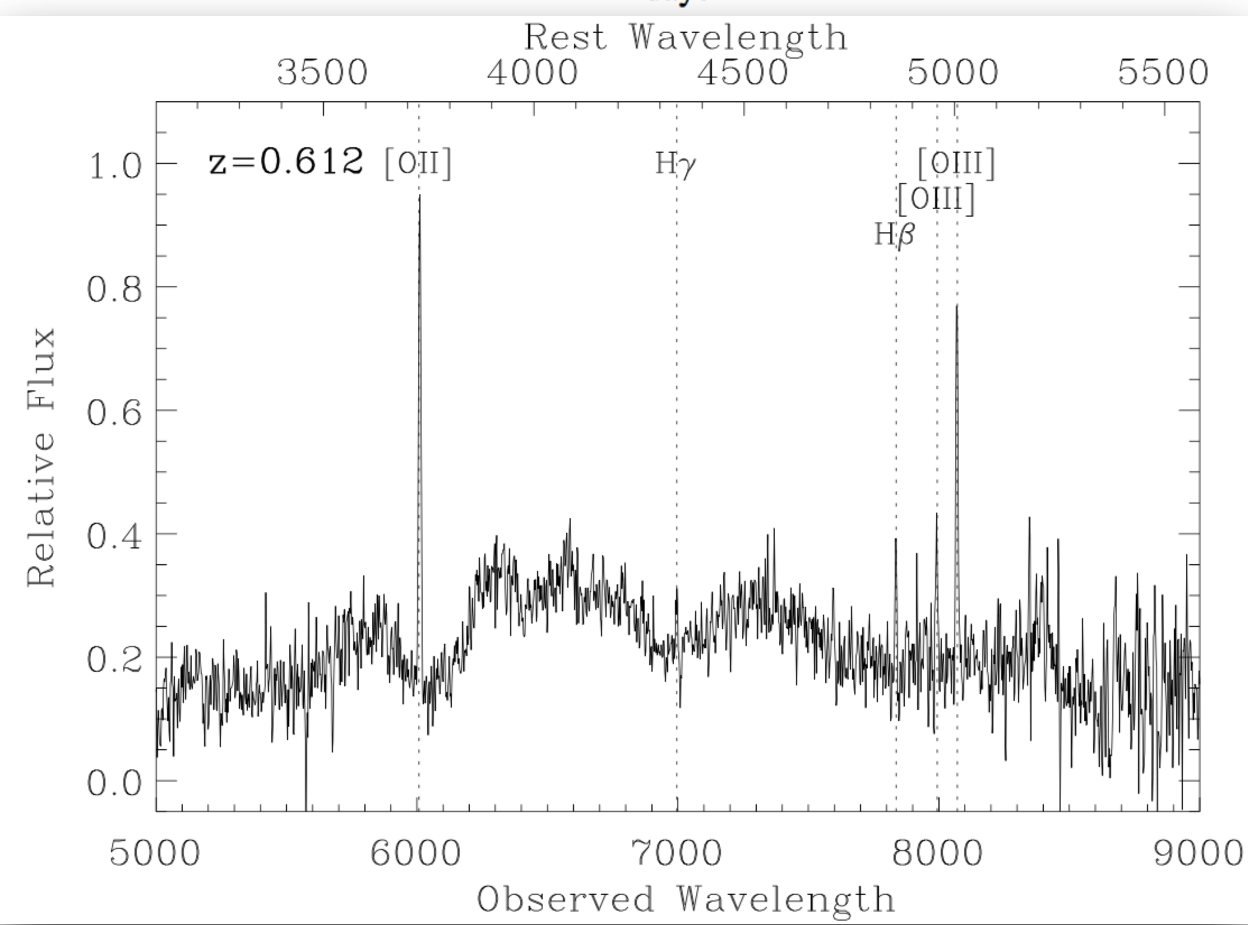
~138 billion yr

5. Standard candle

$$F_{obs}(z_0) = \frac{L_{ABS}(z_1)}{4\pi * D_L{}^2}$$

$$\boxed{L_{ABS}{}^{SN} \sim 10^9 [L_\odot / s]}$$

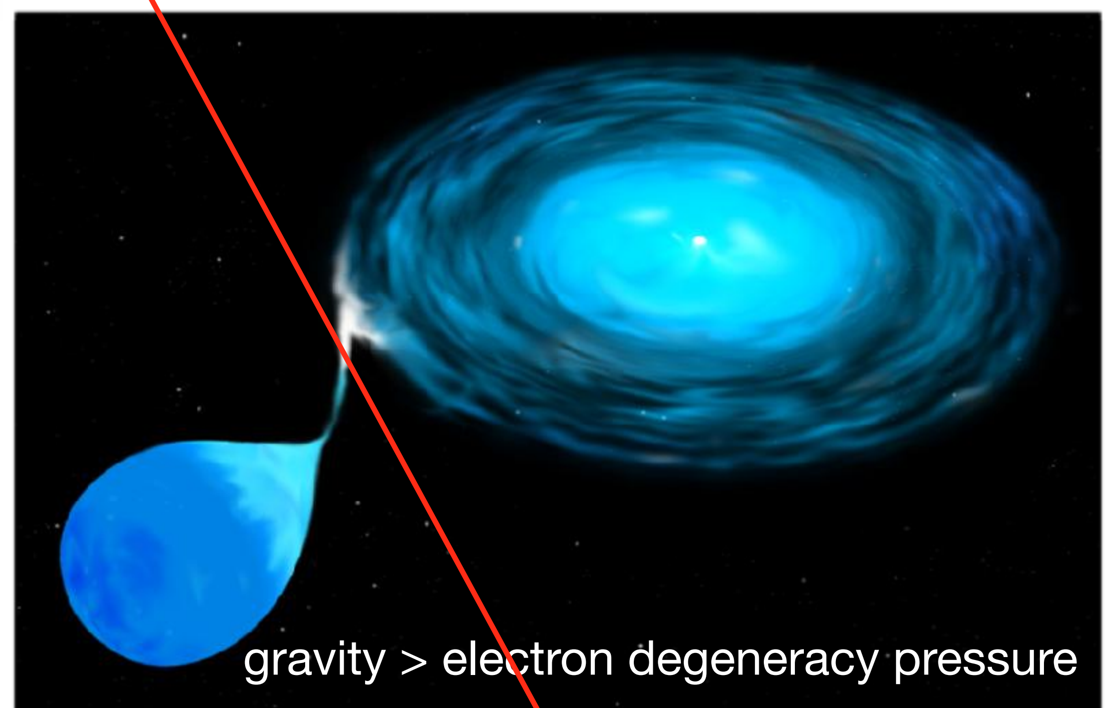




$$M_{WD} > 1.44 M_\odot$$

(Chandrasekhar limit)

SN Ia explode from a binary star system, typically one white dwarf, one giant star



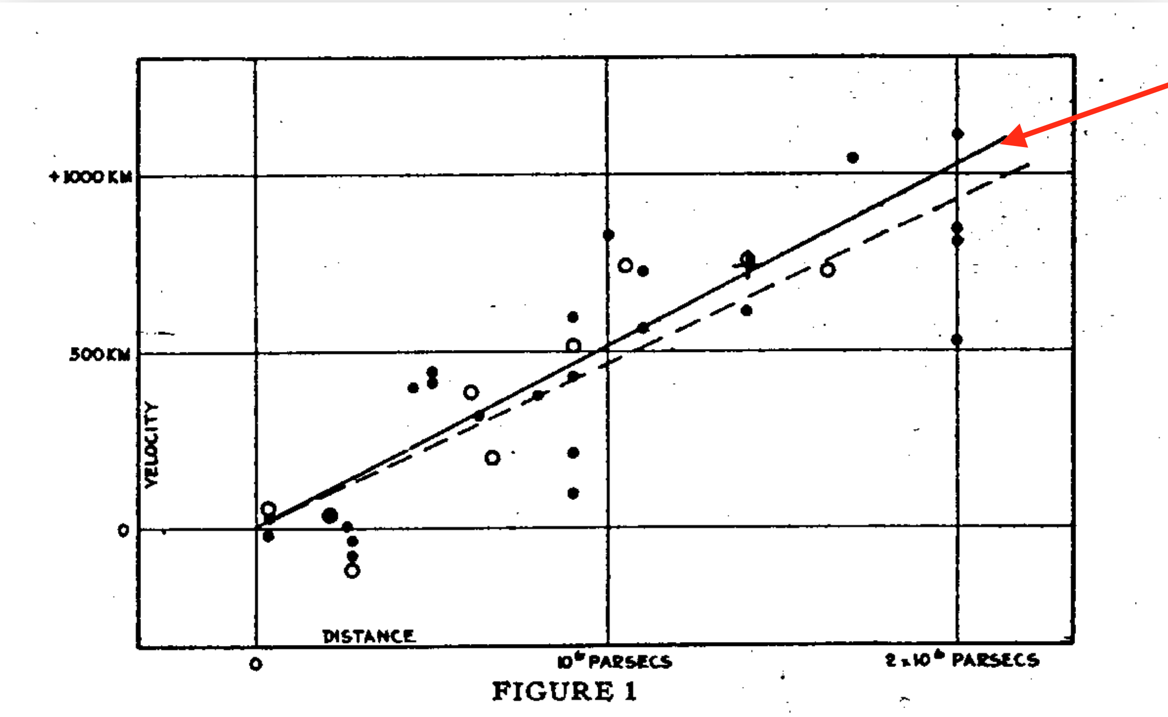
$$D_L = (1+z)\chi$$

$$\lambda_0 \sim 3700\,\mathring{A}$$

$$\chi = \int_{z1}^{z0} dr = \int_{z1}^{z0} \frac{dt}{a} = \int_{z1}^{z0} \frac{dz}{H(z)}$$

z~6000/3700-1~0.6

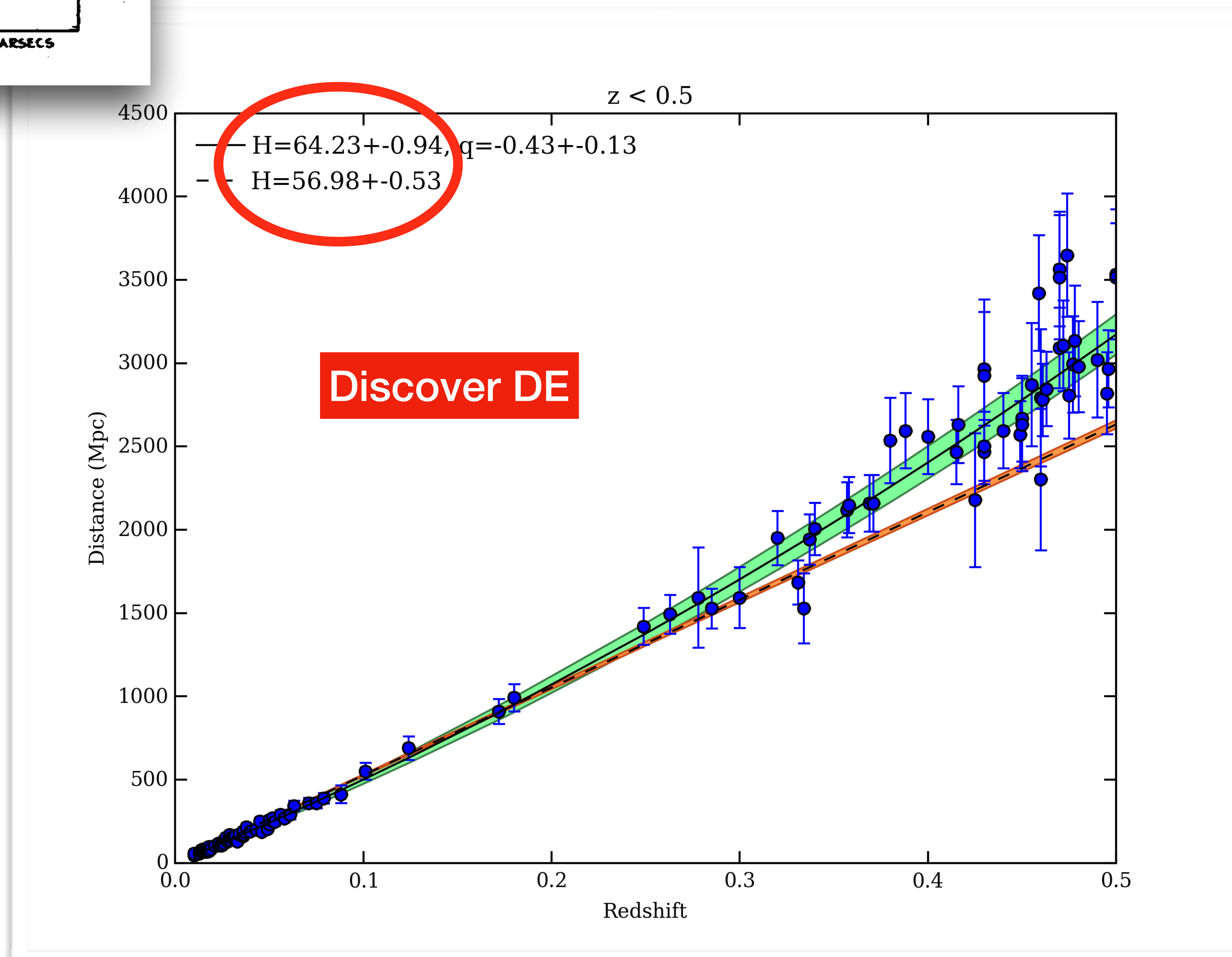
test cosmology



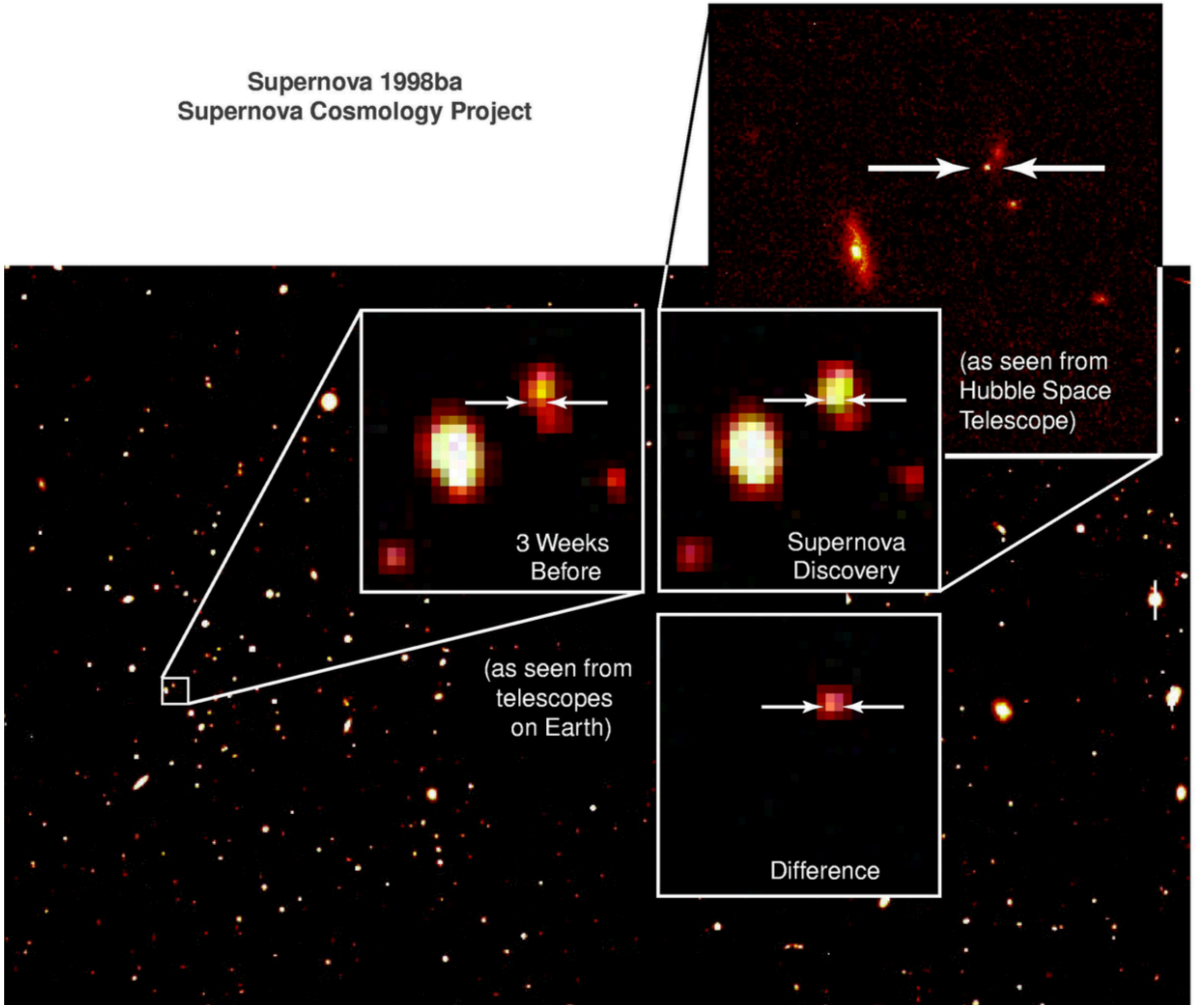
H_0=500

Now

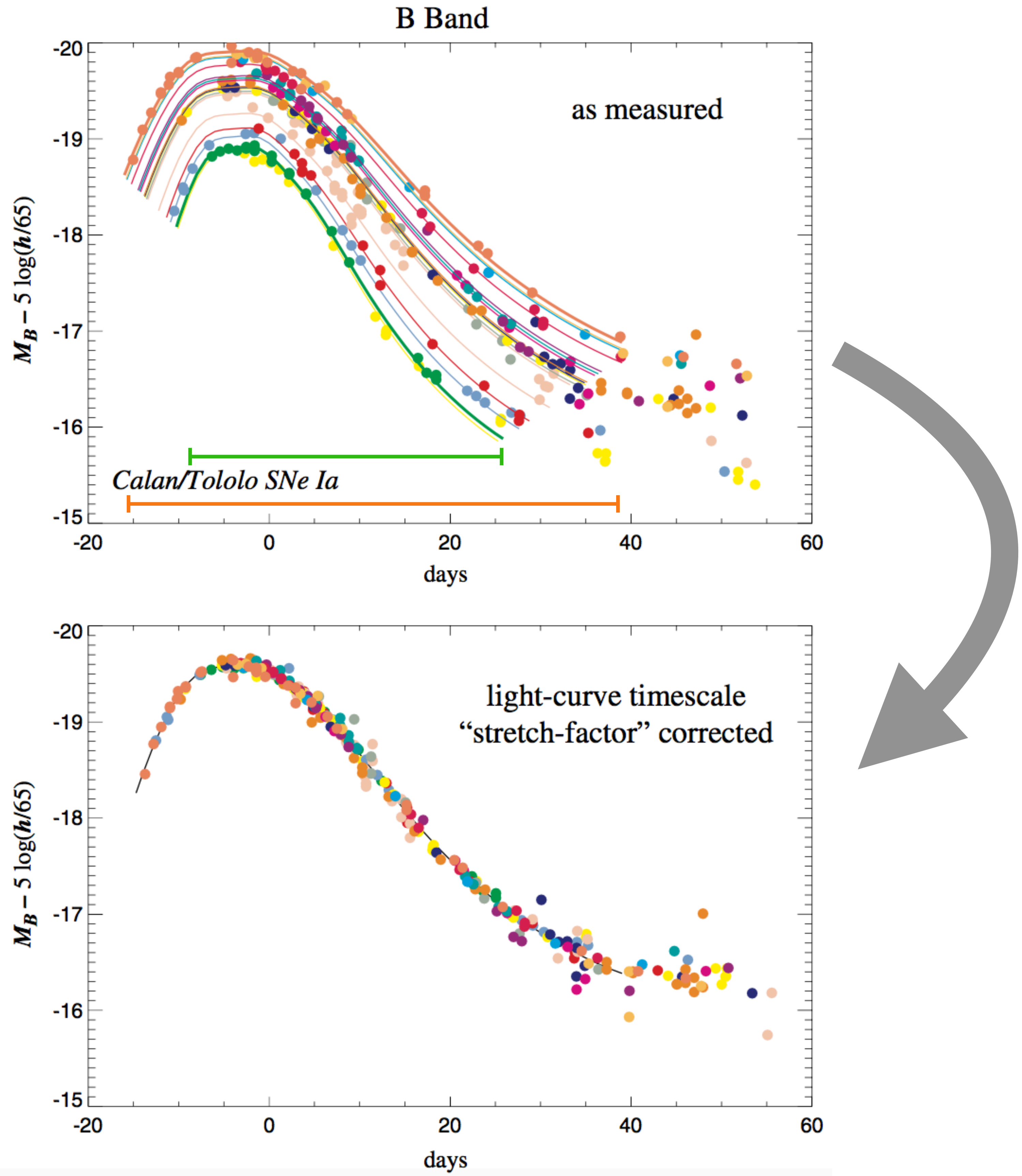
Hubble 1929



how to discover a SN



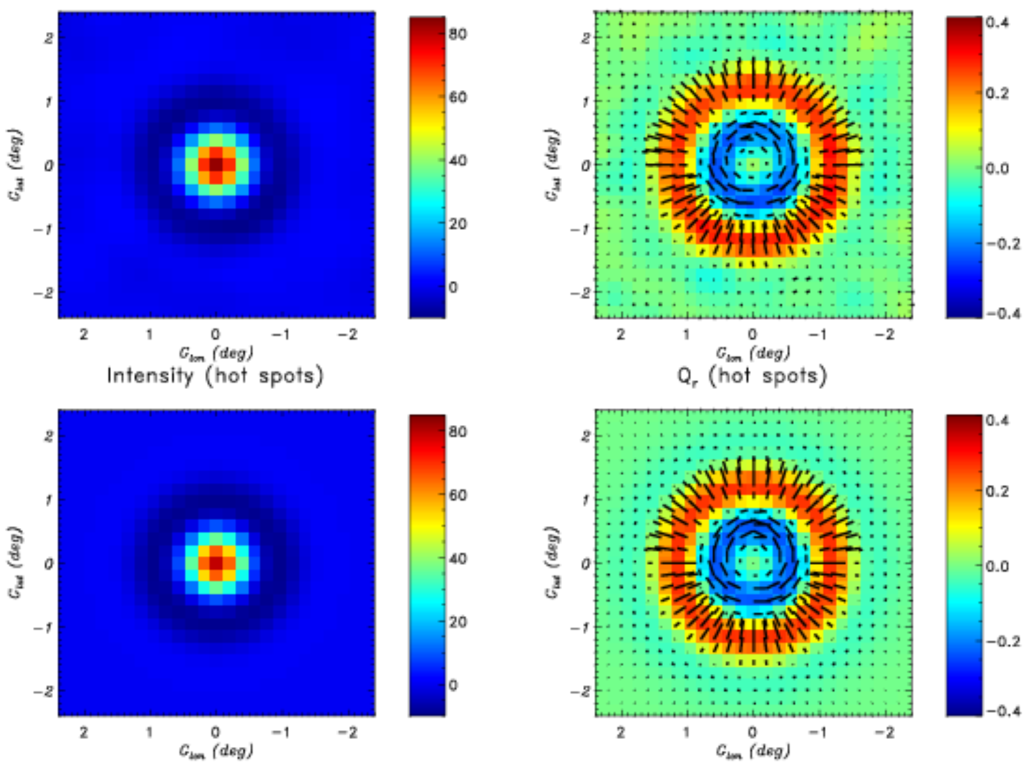
SNIa: standard candle —> standardised candle

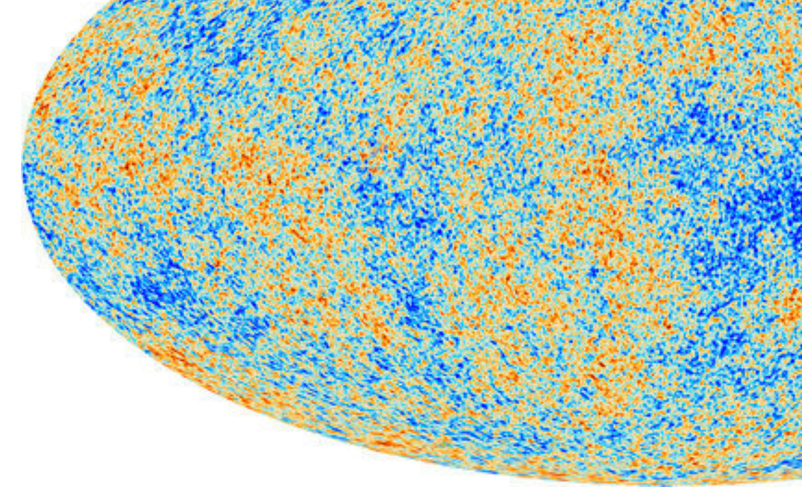


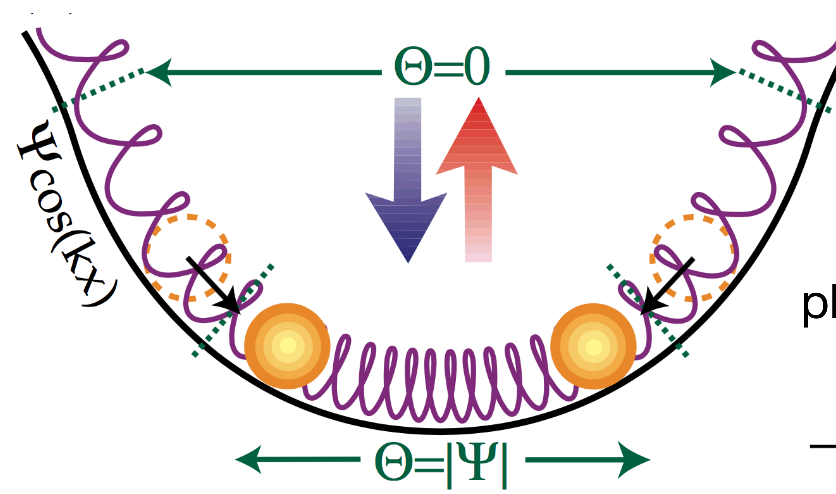
time-scale stretch

Kim, *et al.* (1997)

6. standard ruler → Baryon Acoustic Oscillation

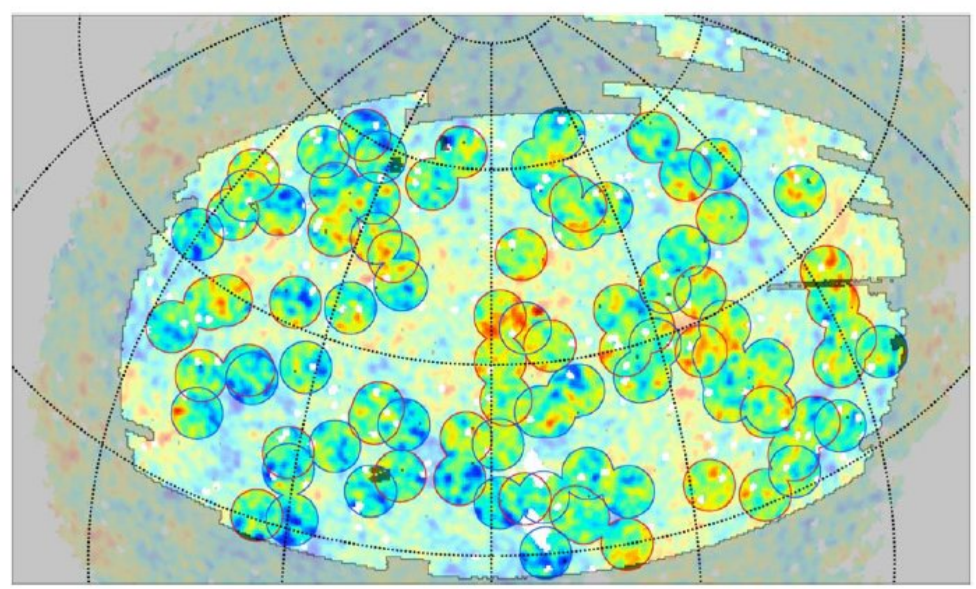






photon pressure balance gravity
@ 150Mpc (co-moving scale)
_ physical scale ~ 150 kpc

$$\chi \sim 150\,Mpc \,/\, D_A \sim 150\,kpc$$



$$D_A = \frac{\chi}{1+z}$$

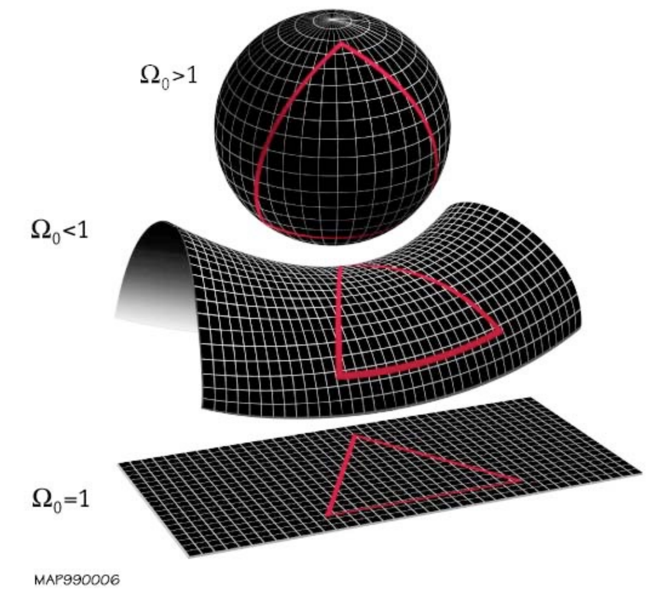
+

z~1100

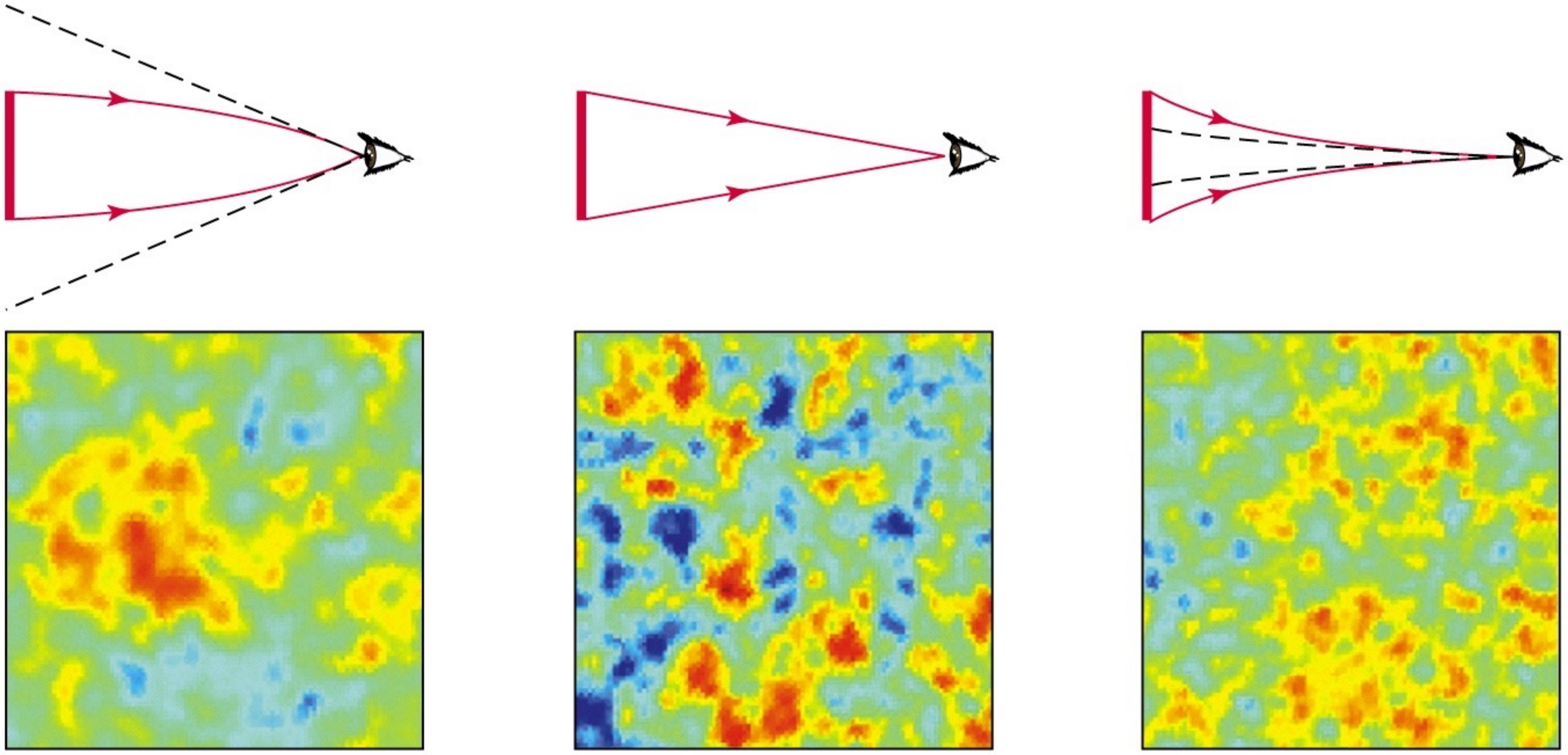
$$\chi = \int_{z1}^{z0} dr = \int_{z1}^{z0} \frac{dt}{a} = \int_{z1}^{z0} \frac{dz}{H(z)}$$

→ **test cosmology**

e.g. measure the 3d spatial curvature



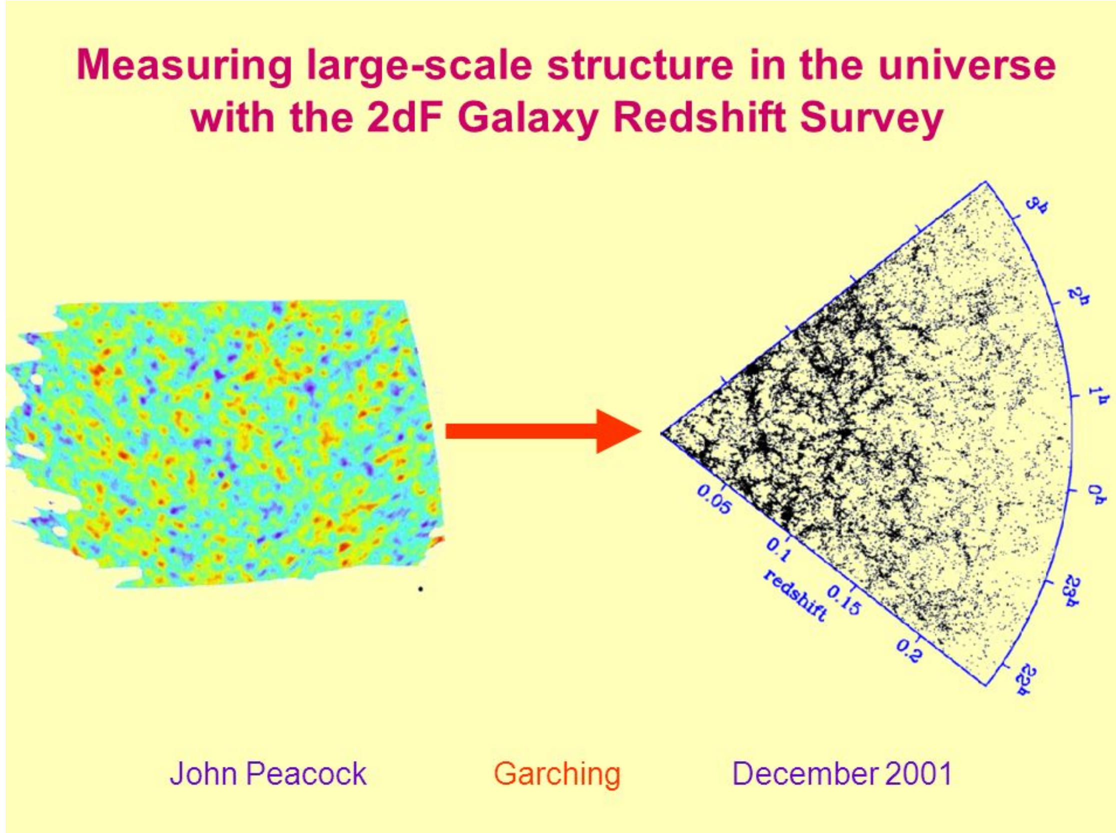
$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-K(t)r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right]$$



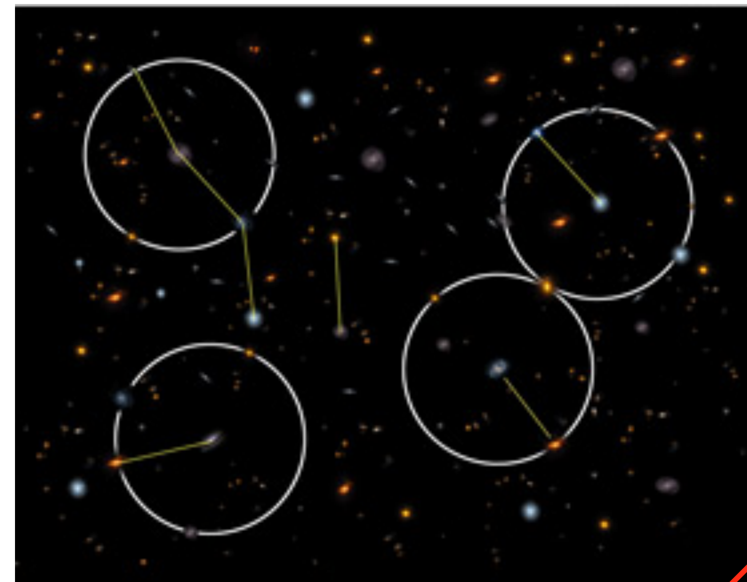
a If universe is closed, "hot spots" appear larger than actual size

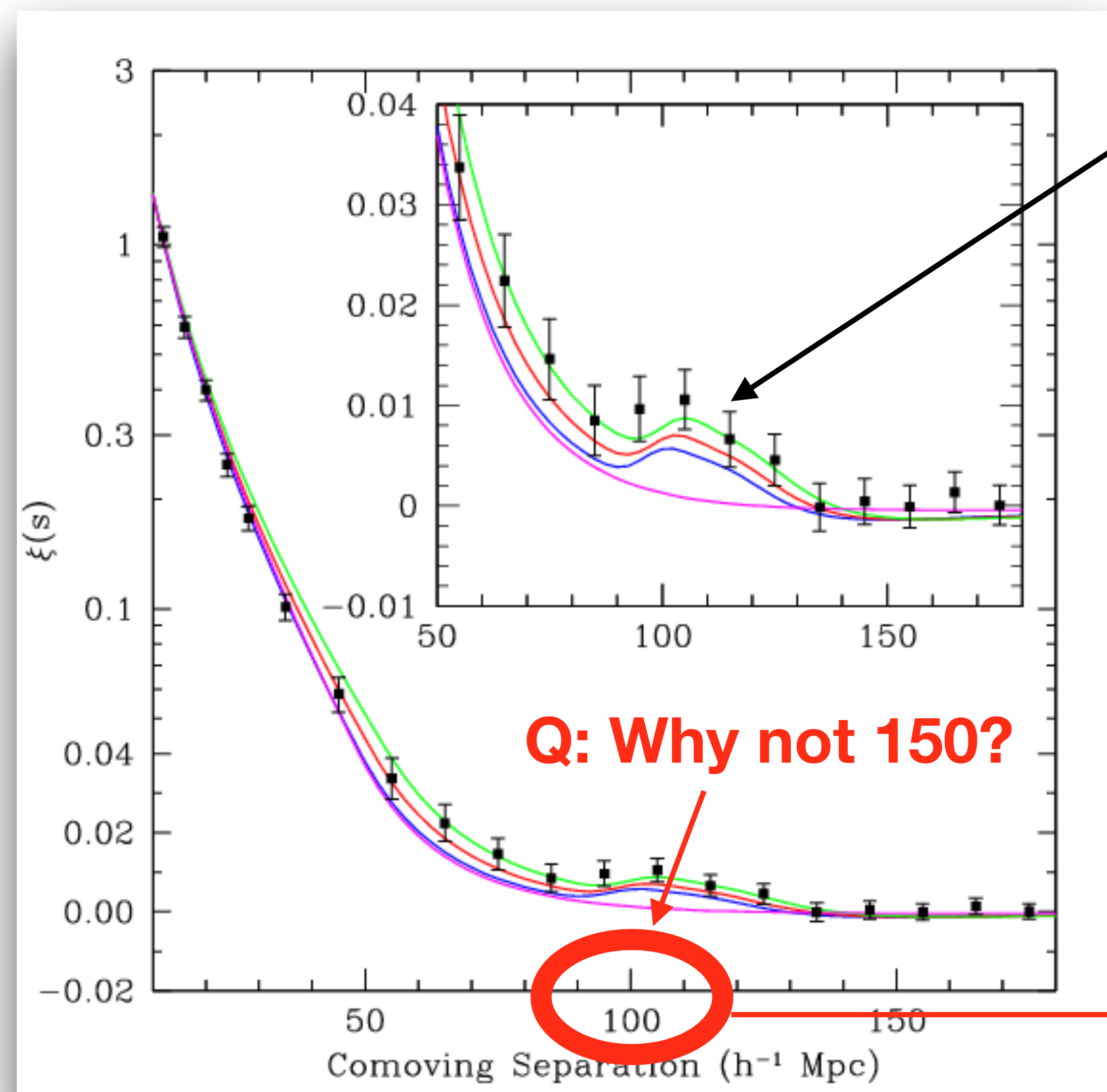
b If universe is flat, "hot spots" appear actual size

c If universe is open, "hot spots" appear smaller than actual size



BAO signal can also imprint on the matter distribution, e.g. galaxies





exceed

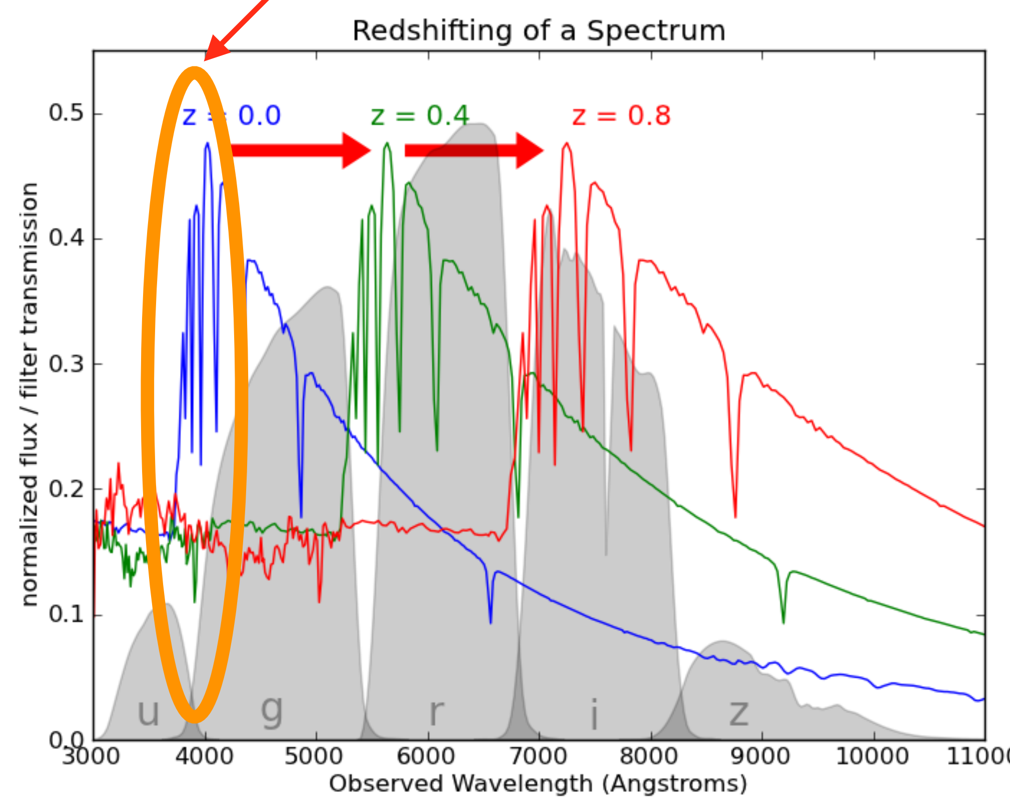
Q: Why not 150?

$$\xi(r)=\frac{DD(r)}{RR(r)}-1$$

photo-z

(photometric redshift)

characteristic break

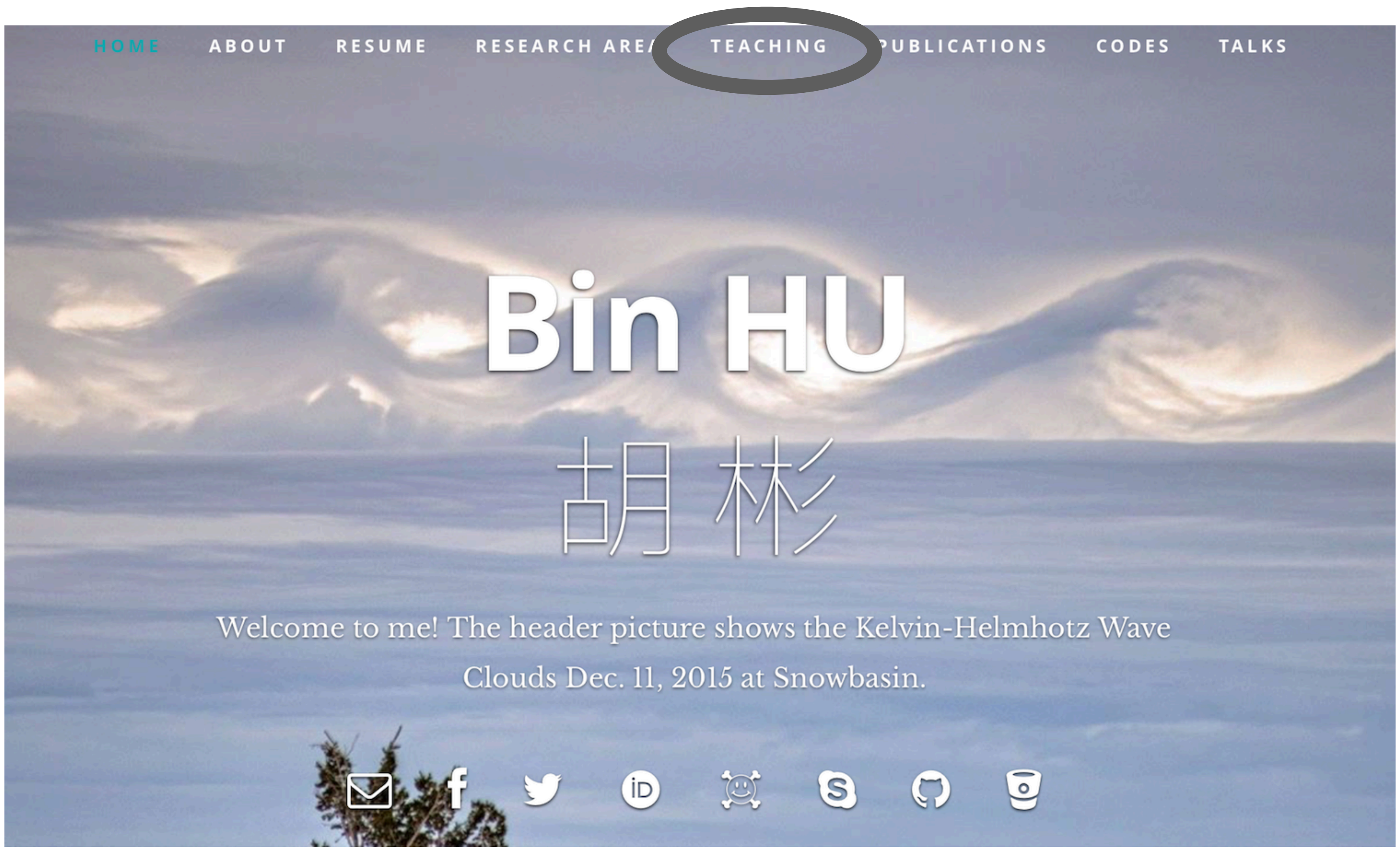


$$\chi=\int_{z1}^{z0}dr=\int_{z1}^{z0}\frac{dt}{a}=\int_{z1}^{z0}\frac{dz}{H(z)}$$

test cosmology

Further reading:

- Baumann Lecture note/Chapter 1
- 宇宙大尺度结构的形成 向守平、冯珑珑/Chapter 1,2,3



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