CMB physics

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4. Polarization

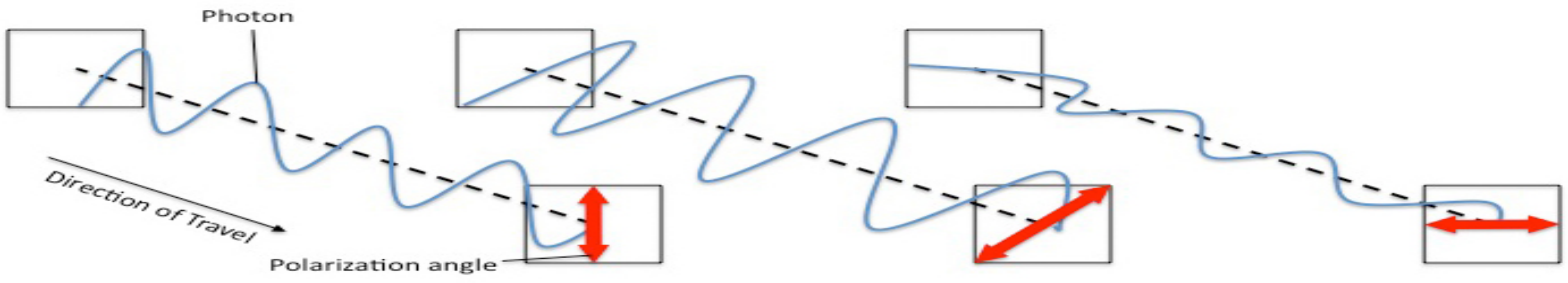
Key concept

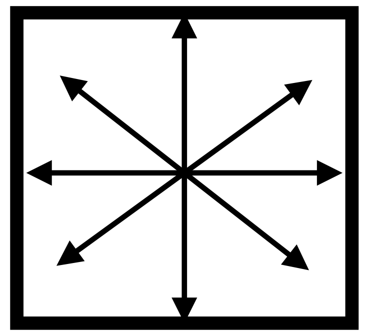
- Stokes parameters
- quadrupole anisotropy generate linear polarization from unpolarised light
- polarization field trace the velocity field at recombination
- Radial/Tangential polarization mode
- Q/U (frame dependent) => E/B (frame independent)
- TS only generate E mode, large scale modulation convert local E mode to B mode.
- LoS projection transfer local quadrupole into higher ell modes
- Powers are distributed according Clebosh-Gordan coef in the high ell modes. Hence, modulation can mix E and B.

Polarization

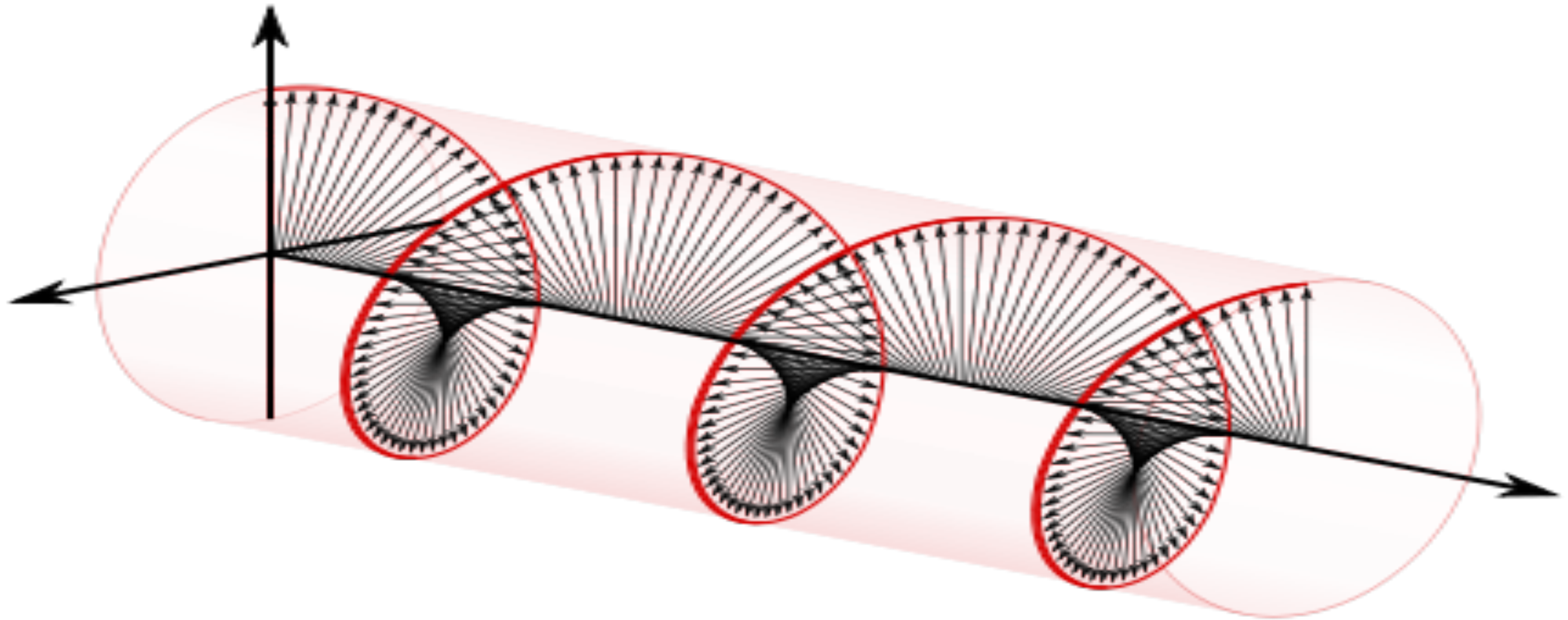
至此，我们讨论的都只是**无偏振**CMB光子的温度扰动

但实际上，大约**10%**的CMB光子存在着**线**偏振 / linear polarization





CMB光子**不**存在**圆**偏振



The Thomson scattering cross section depends on polarization as (see e.g. Chandrasekhar 1960)

$$\frac{d\sigma_T}{d\Omega} \propto |\hat{\epsilon} \cdot \hat{\epsilon}'|^2 \, , \tag{1}$$

where $\hat{\epsilon}$ ($\hat{\epsilon}'$) are the incident (scattered) polarization directions. Heuristically, the incident light sets up oscilla-

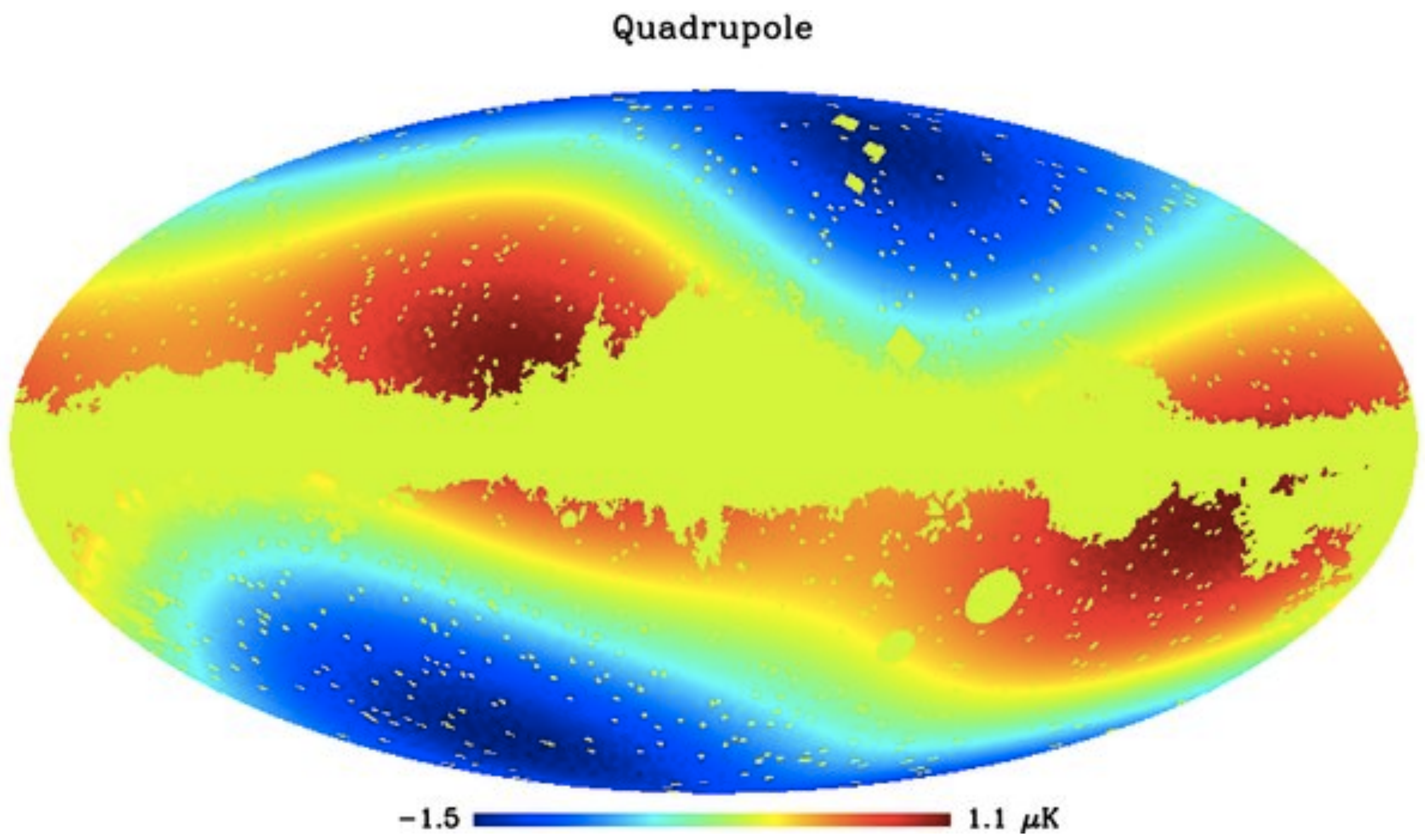
Unpolarized light, Q=0

$$Q = \frac{3\sigma_T}{8\pi} \left(\sum_{j=1}^{2} |\hat{\epsilon}_1 \cdot \hat{\epsilon}_j'(\hat{n}')|^2 - \sum_{j=1}^{2} |\hat{\epsilon}_2 \cdot \hat{\epsilon}_j'(\hat{n}')|^2 \right).$$

[credit:Valeria Pettorino]

How much polarized?

The quadrupole is zero at early times and then grows while approaching decoupling.



Any source of quadrupole anisotropy leaves its imprint in the polarization.

The fraction depends on the duration of last scattering. It is 10% on a characteristic scale of degree scale. Since temperature anisotropies are at the 10^{-5} level, the polarized signal is at (or below) 10^{-6} level representing a significant experimental challenge.

[credit:Valeria Pettorino]

Intensity tensor

$$I_{ab} = \frac{1}{2} I \sigma_0 + P_{ab}$$

Total Intensity

Polarization tensor

I~Electric field^2

[why electric field not magnetic field?]

$$P_{ab} = \frac{1}{2} (Q\sigma_3 + U\sigma_1 - V\sigma_2)$$

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \boldsymbol{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Q, U, V Stokes parameters

Q: linear polarization; U linear polarization rotated of 45°

V circular polarization

Not convenient under rotations

Stokes Vector consists of four parameters **(called Stokes parameters)**:

intensity I,

the degree of polarization Q,

the plane of polarization U,

the ellipticity V.

Notation

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \quad \text{or} \quad \{I, Q, U, V\}$$

- **Stokes parameters** are defined via the intensities which can be measured:

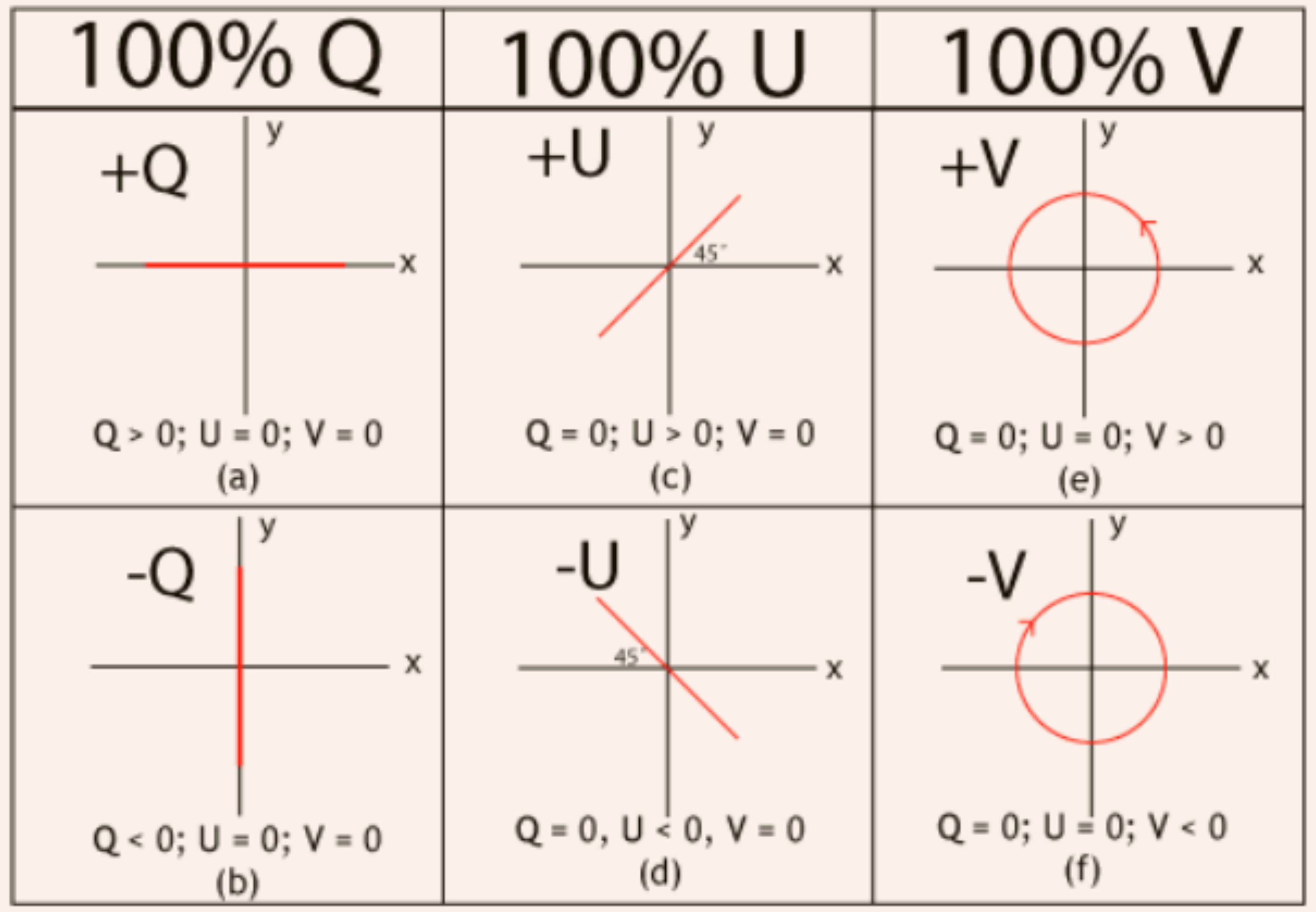
I = total intensity

$Q = I_0 - I_{90}$ = differences in intensities between horizontal and vertical linearly polarized components;

$U = I_{+45} - I_{-45}$ = differences in intensities between linearly polarized components oriented at $+45^0$ and -45^0

$V = I_{rcl} - I_{lcr}$ = differences in intensities between right and left circular

STOKES PARAMETERS FORMALISM



$$\left\{ \begin{matrix} I \\ Q \\ U \\ V \end{matrix} \right\}$$

★ ***I***, *intensity*

★ ***Q***, ***U***, *linear polarization*

★ ***V***, *circular polarization*

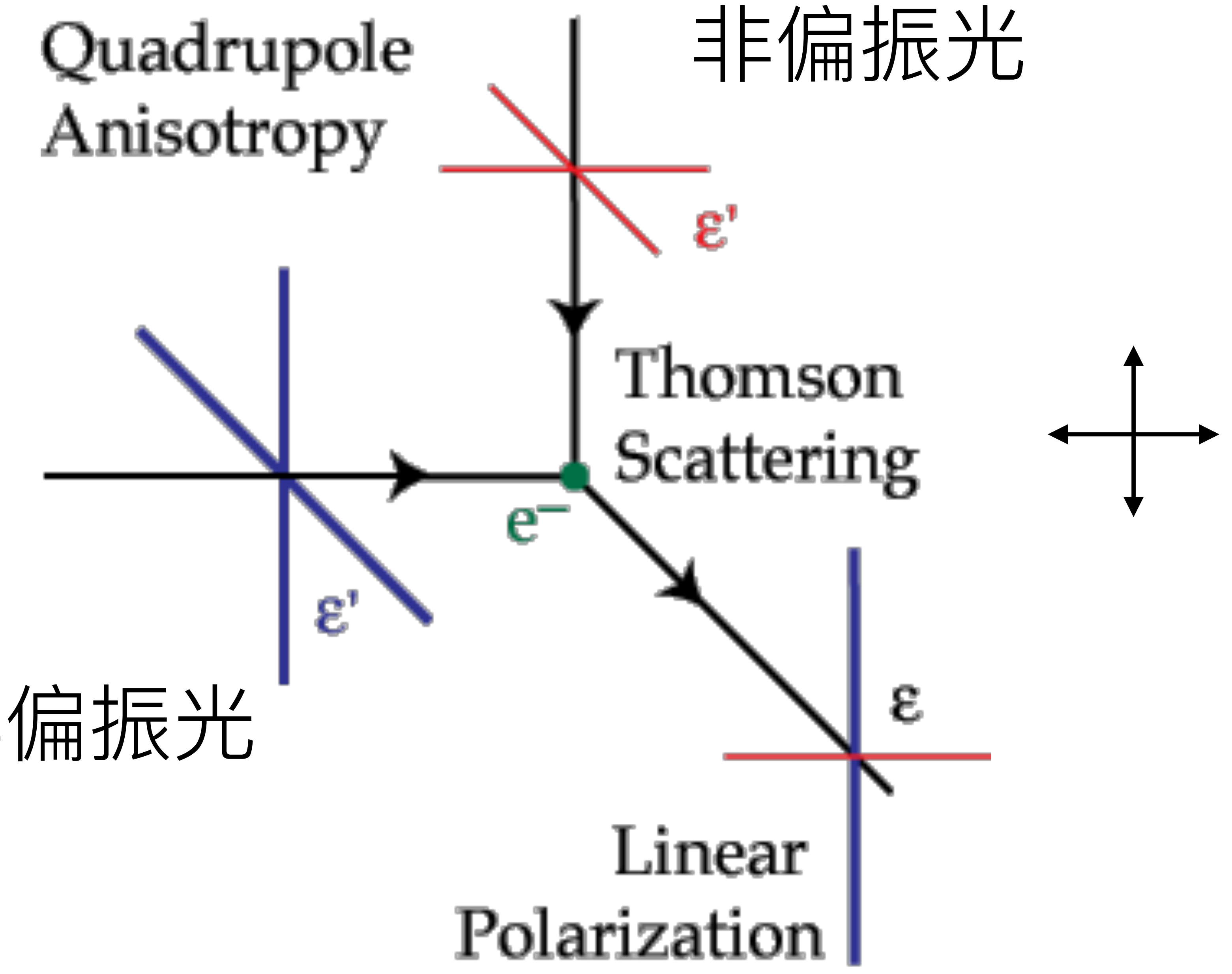
★*in the case of the CMB,* $V = 0$

原初引力波扰动，会产生一定比例的B模计划

这是用来了解，宇宙极早期暴胀机制 / inflation，
研究量子引力的**最主要**窗口

物质密度扰动（标量 / 自旋0/单极矩）、速度扰动（矢量 / 自旋1/偶极矩），
在最后散射面上，不会产生四极距（张量 / 自旋2）

前两者(360°/180°)，
不会在90°方向上，
产生不均匀性，
只有引力波才有可能

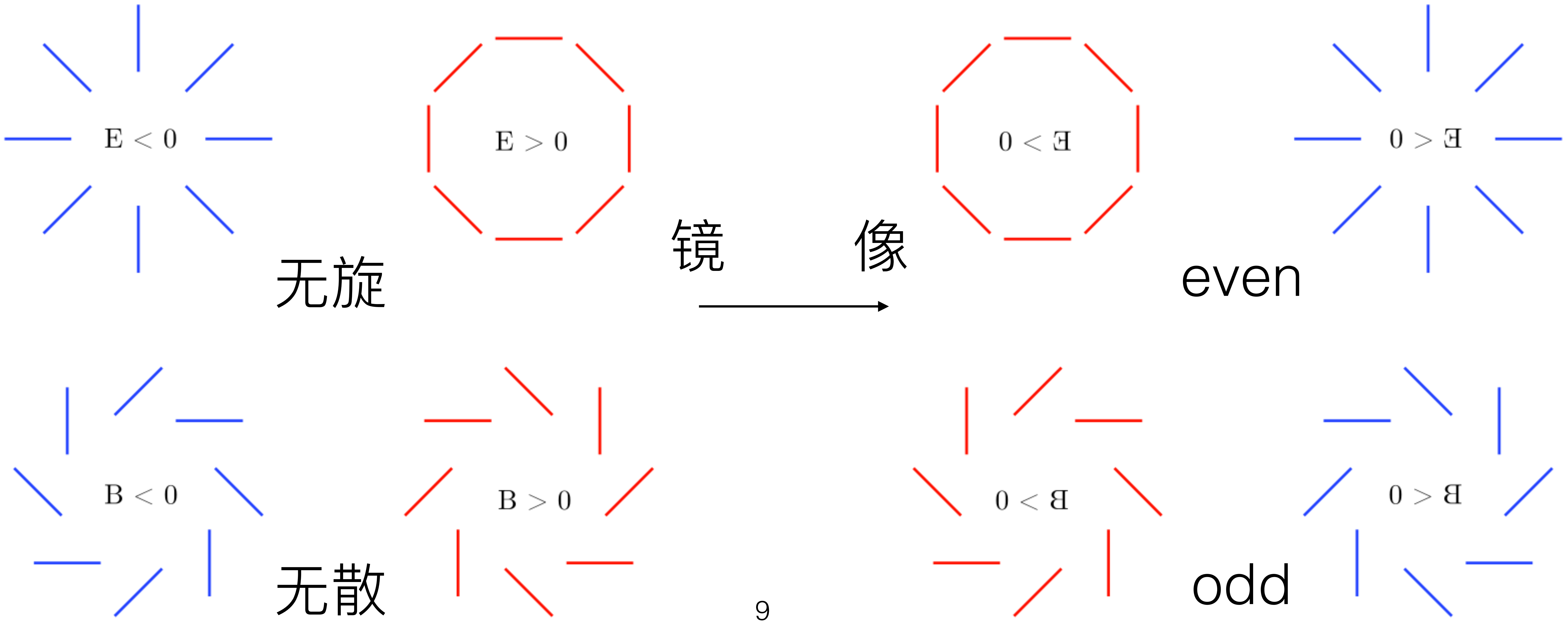


光学中，描述光子强度张量 I_{ij}, 有4个Stokes参量

$$T = (I_{11} + I_{22})/4 \qquad Q = (I_{11} - I_{22})/4 \qquad U = I_{12}/2 \qquad V = 0$$

非极化光　　　　线性极化光　　　　圆性极化光

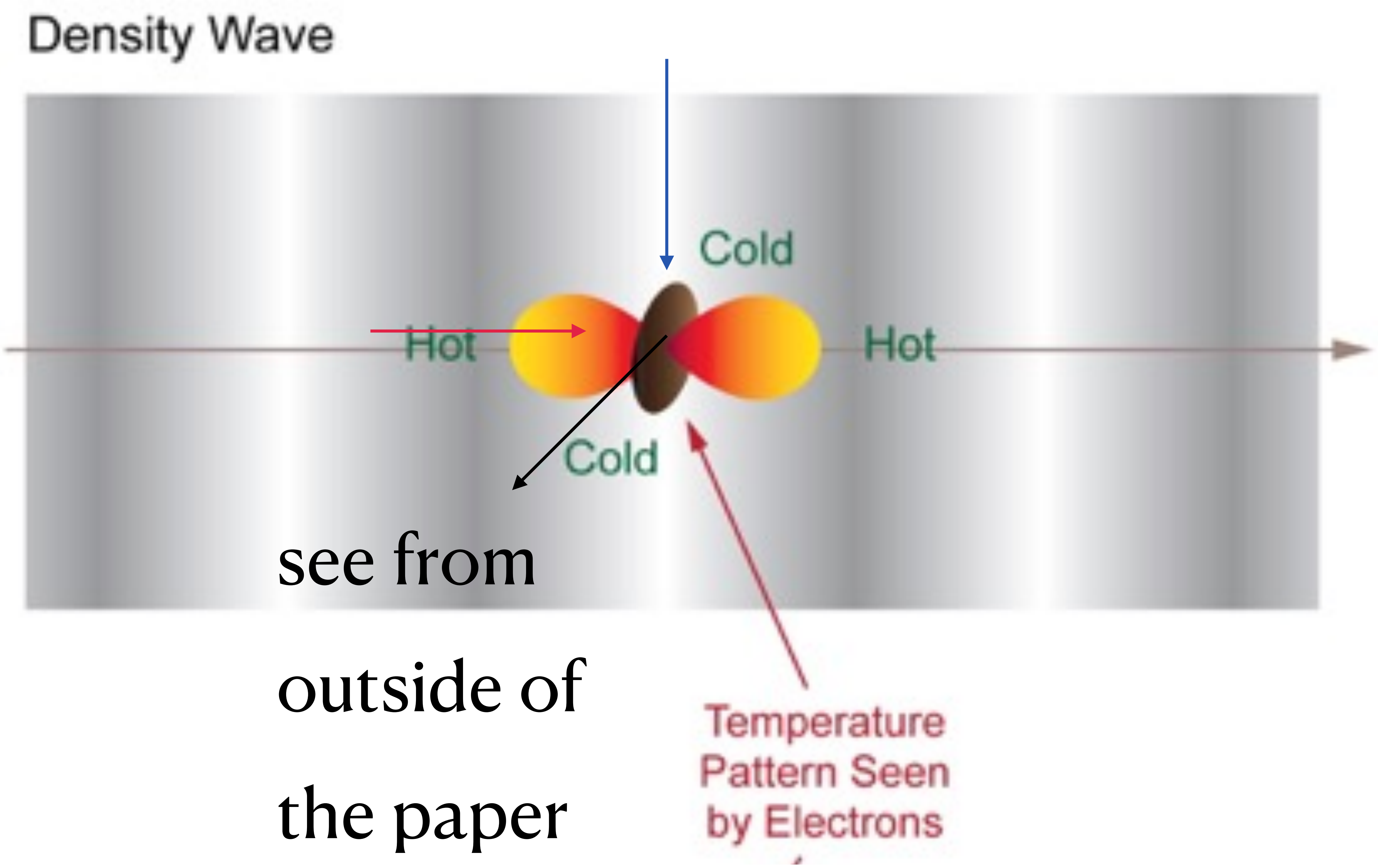
进一步，将（Q，U）组合成我们更为熟悉的，
电磁场的电部分 / E模，和磁部分 / B模

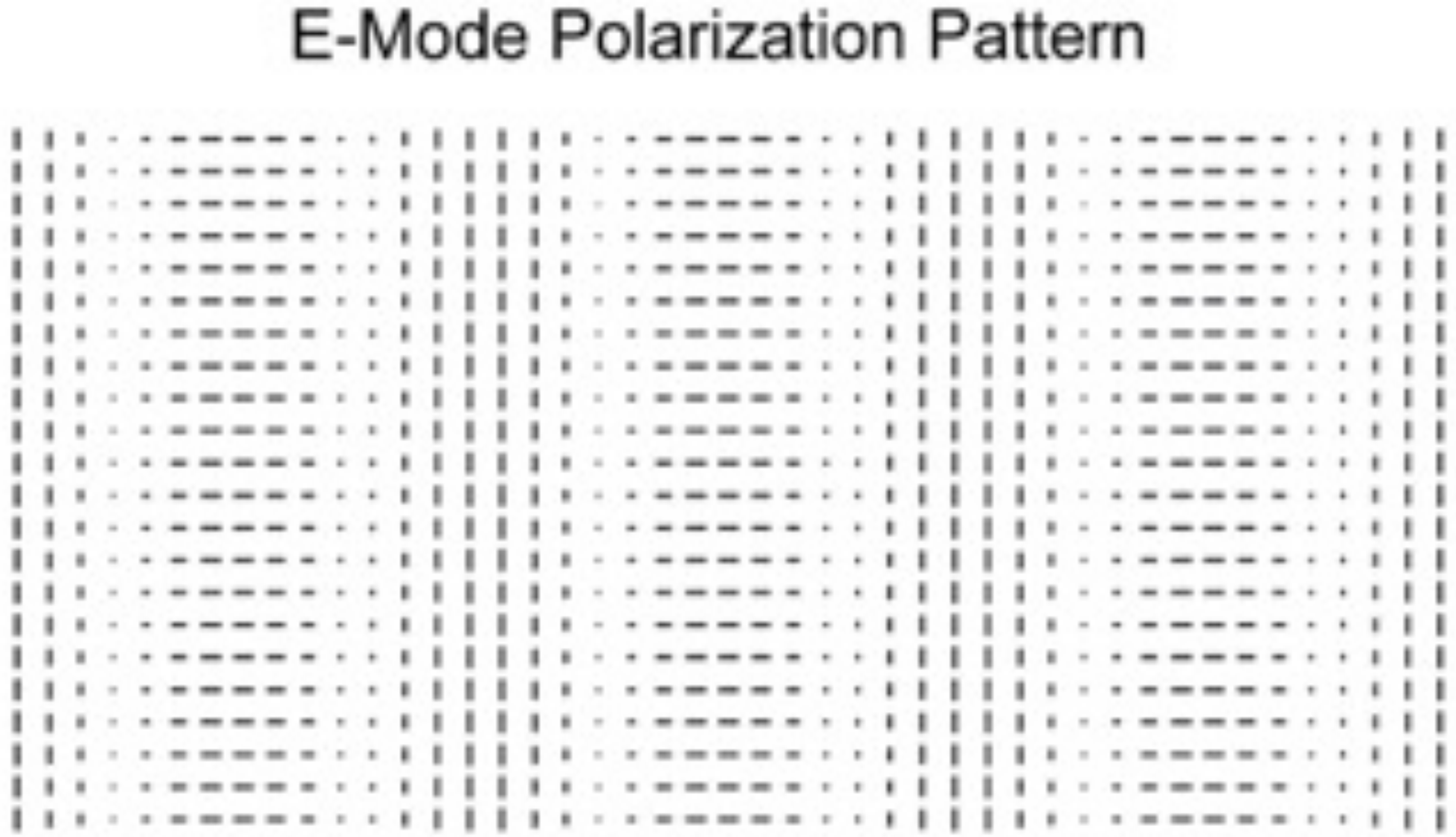


[credit: Peiris]

plane wave case

Temperature quadrupole at surface of last scattering creates polarisation…





Radial (tangential) pattern around hot (cold) spots.

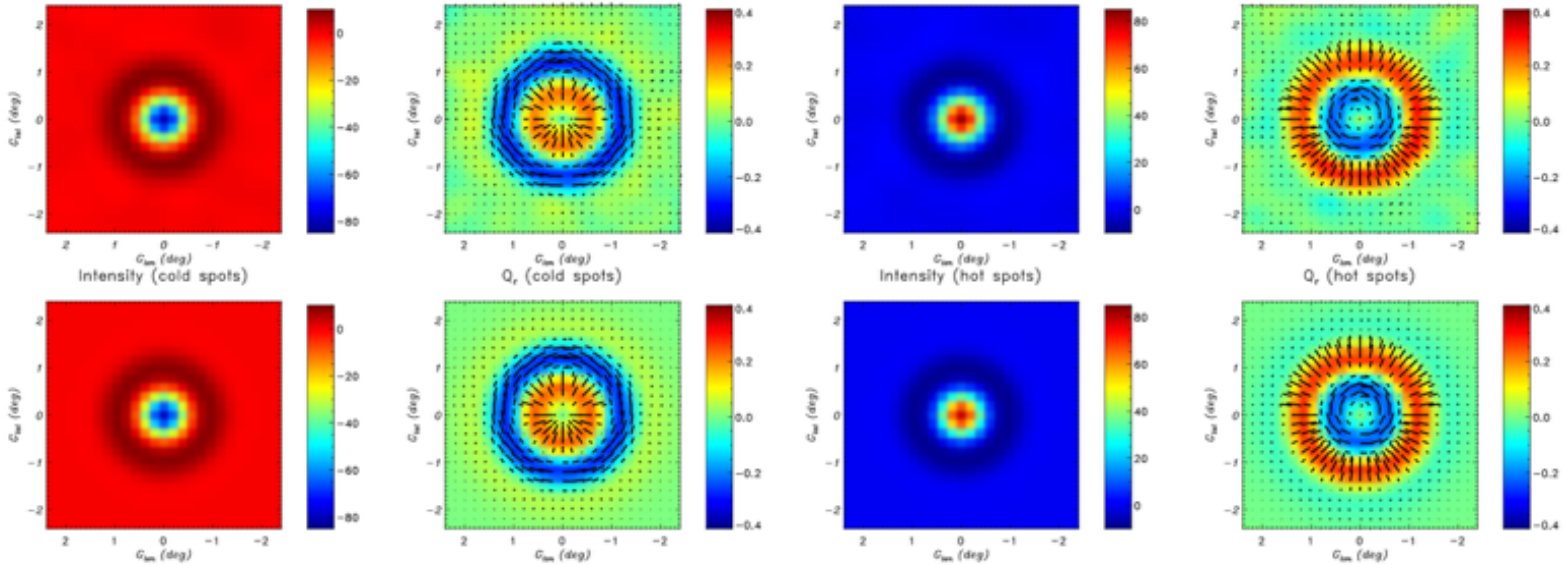
[credit: Peiris]

Spherical density case

Measurement

Cold spot | Hot spot

I | *Q* | *I* | *Q*



Cold spot | Hot spot

Theory prediction

consider:

velocity gradient

from gravity &

neglect photon

pressure

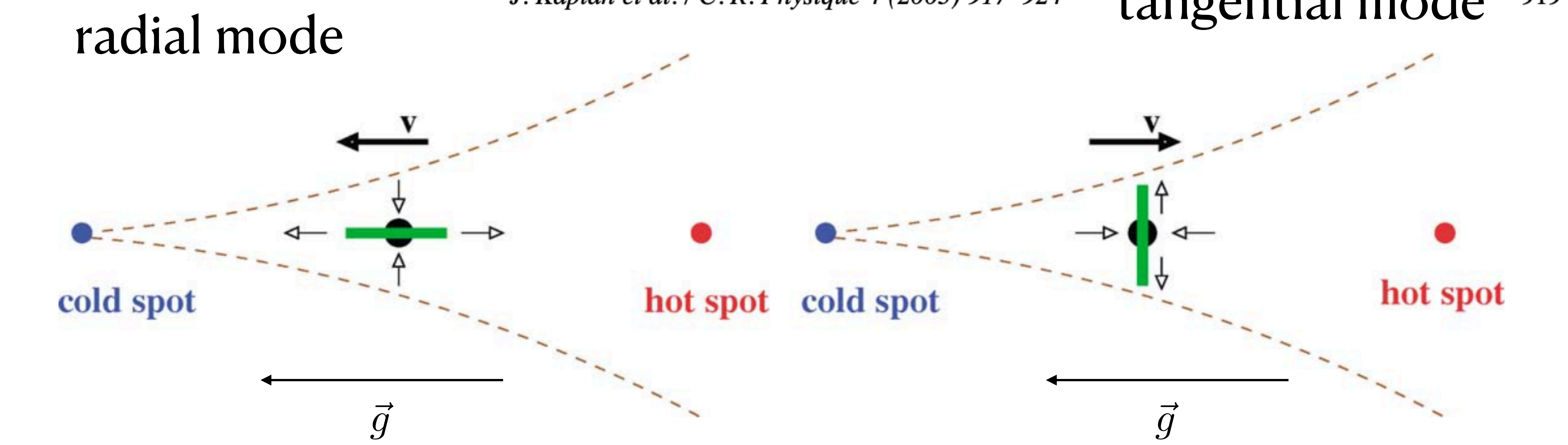
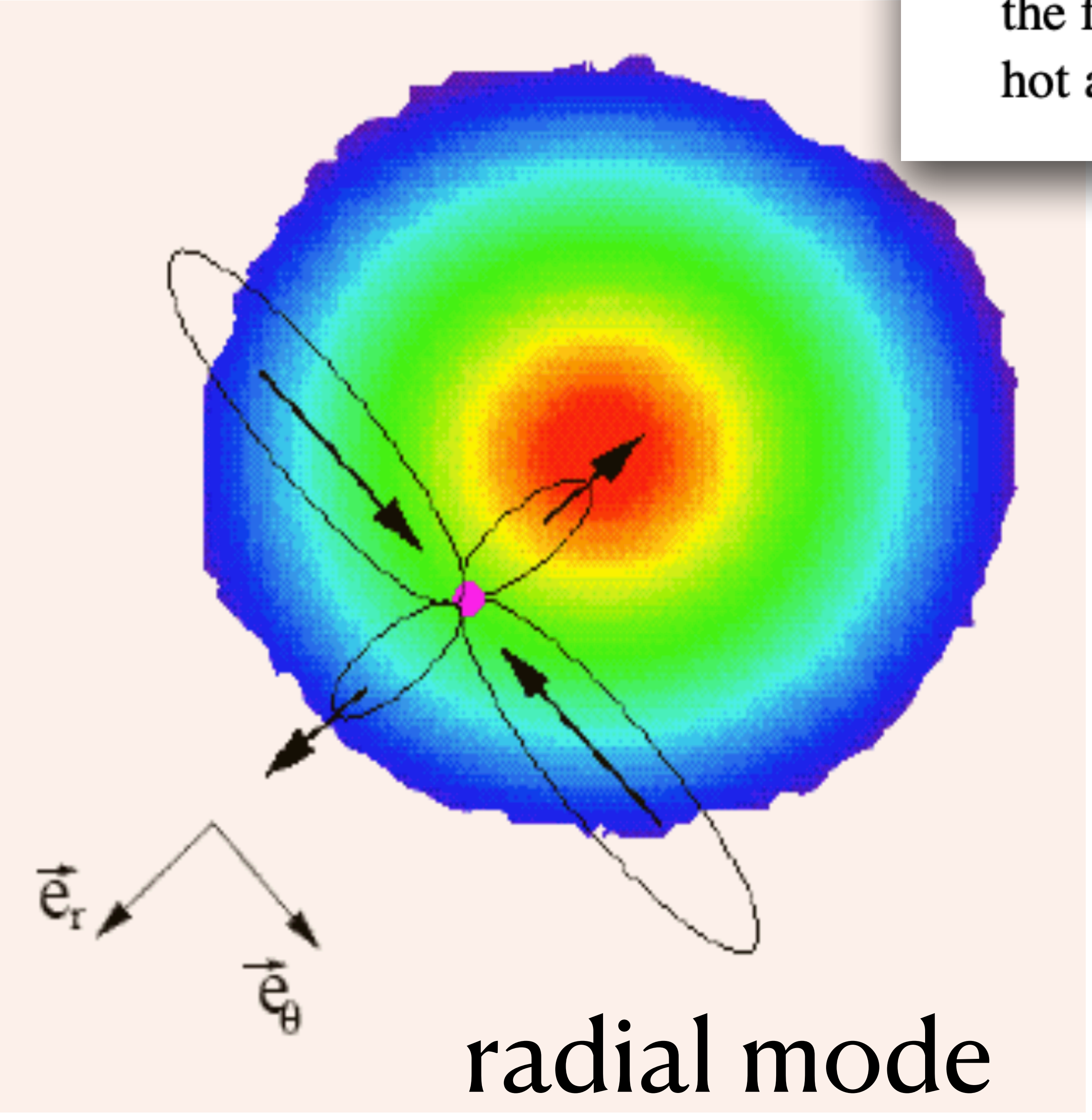


Fig. 2. Polarization directions from velocity gradients. Polarization directions when the fluid is accelerated towards the cold spot (left panel) or decelerated towards a hot spot (right panel). The brown dashed curves are fluid stream lines. The small thin arrows are the fluid velocities in the fluid rest frame near the scattering point. The large thick arrows are the directions of the fluid motion near the scattering point relative to the hot and cold spots.



flow in => more photons => intensity high => hot temperature

Actually, photon pressure repulse the inner flow, it gives tangential mode around the hot spot

Actually, photon pressure drag the outer flow, it gives radial mode around the cold spot

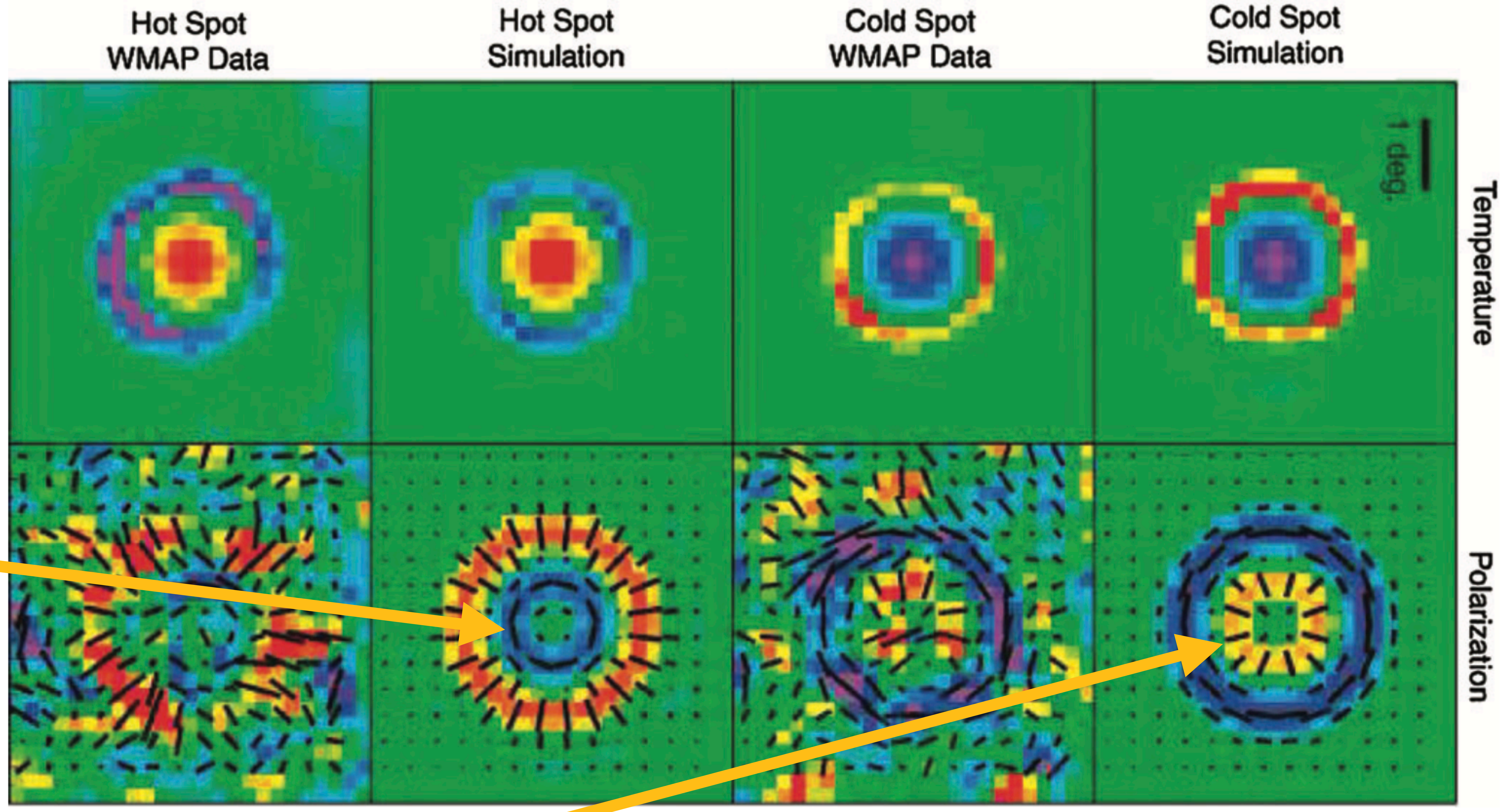
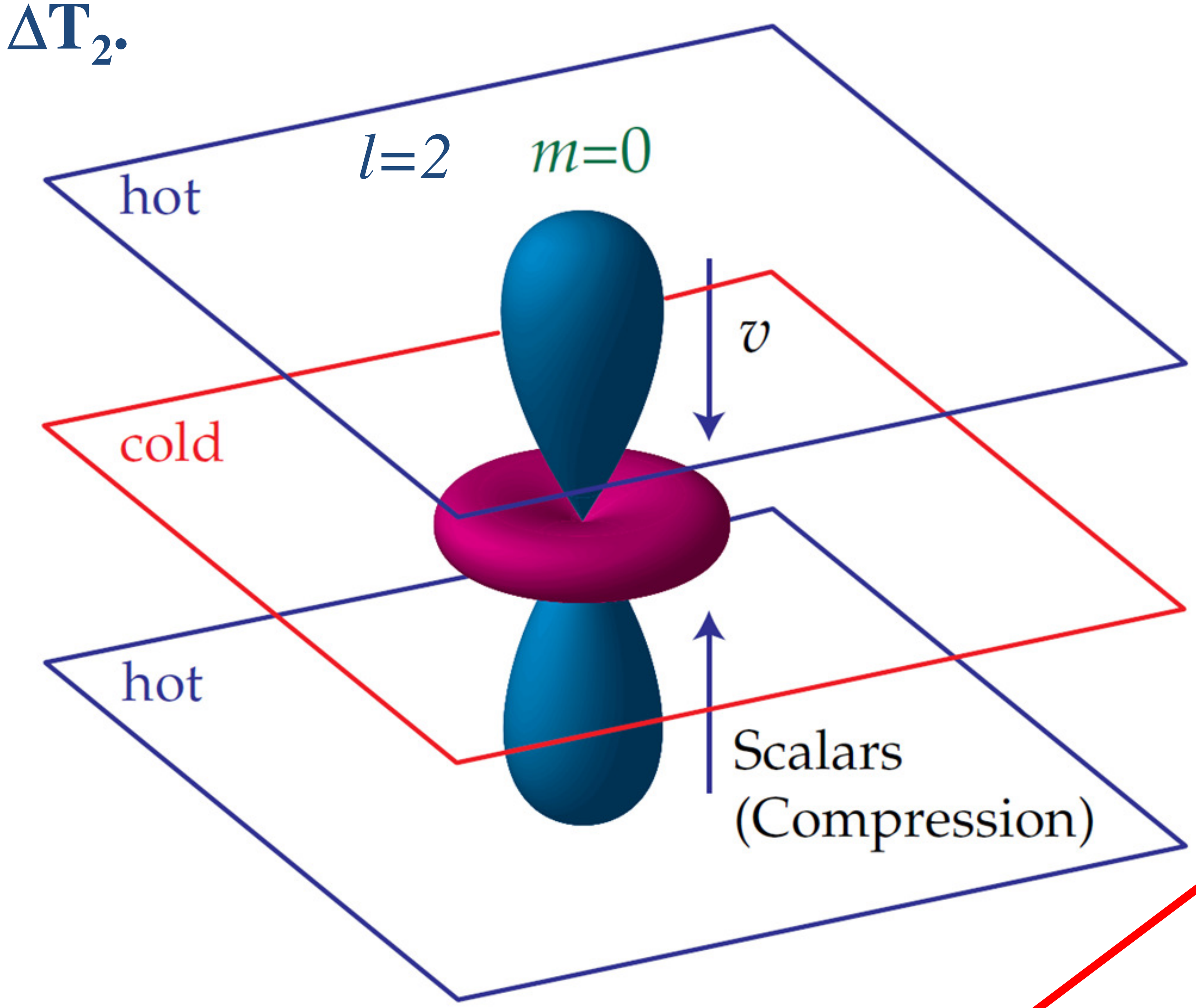
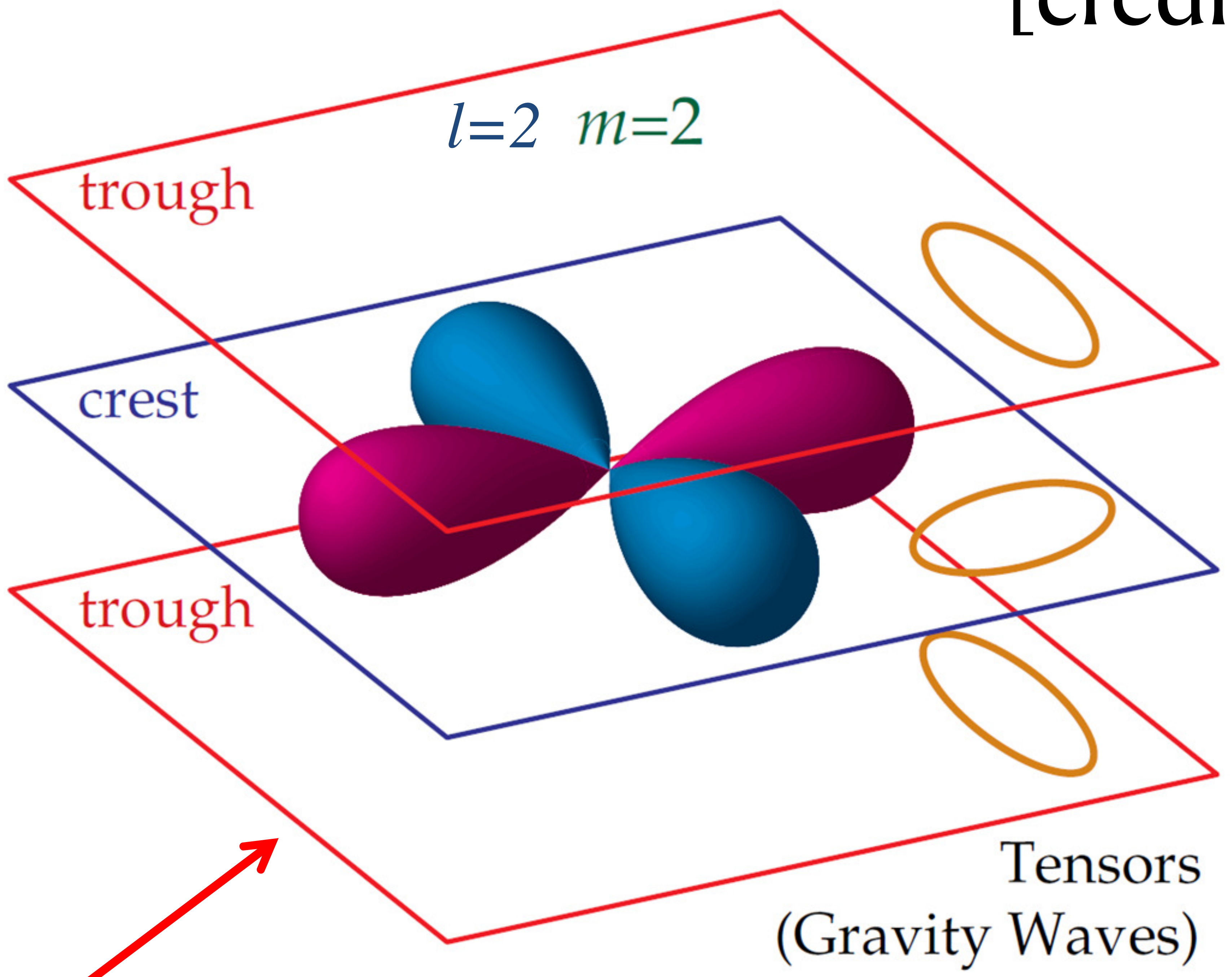


Fig. 8. Average images of temperature and polarization data. 12387 hot spots and 12628 cold spots are found in the *WMAP* seven-year temperature maps, and the average images of hot and cold spots are shown in the top panels along with the corresponding simulated images. The bottom panels show the average images of the polarization maps around the locations of hot and cold temperature spots, as well as the corresponding simulated images. The size of each image is 5° by 5°. The lines show the polarization directions, and their lengths are proportional to the magnitude of polarization. The colors of the polarization images are chosen such that blue and red show the tangential and radial polarization patterns, respectively. The data show the predicted tangential and radial polarization patterns (E-mode polarization), in excellent agreement with the predictions. The maximum of radial polarization around hot spots occurs at 1.2° from the center, whereas the maximum of tangential polarization around hot spots occurs at 0.6° from the center. Figure adapted from http://wmap.gsfc.nasa.gov/media/101079/index.html (Credit: NASA/WMAP Science Team).

[credit: Carbone]

The scalar (density) quadrupole moment . Flows from hot (blue) regions into cold (red), v||k in Fourier space (irrotational fluid), produce an azimuthally symmetric pattern for ΔT_2.

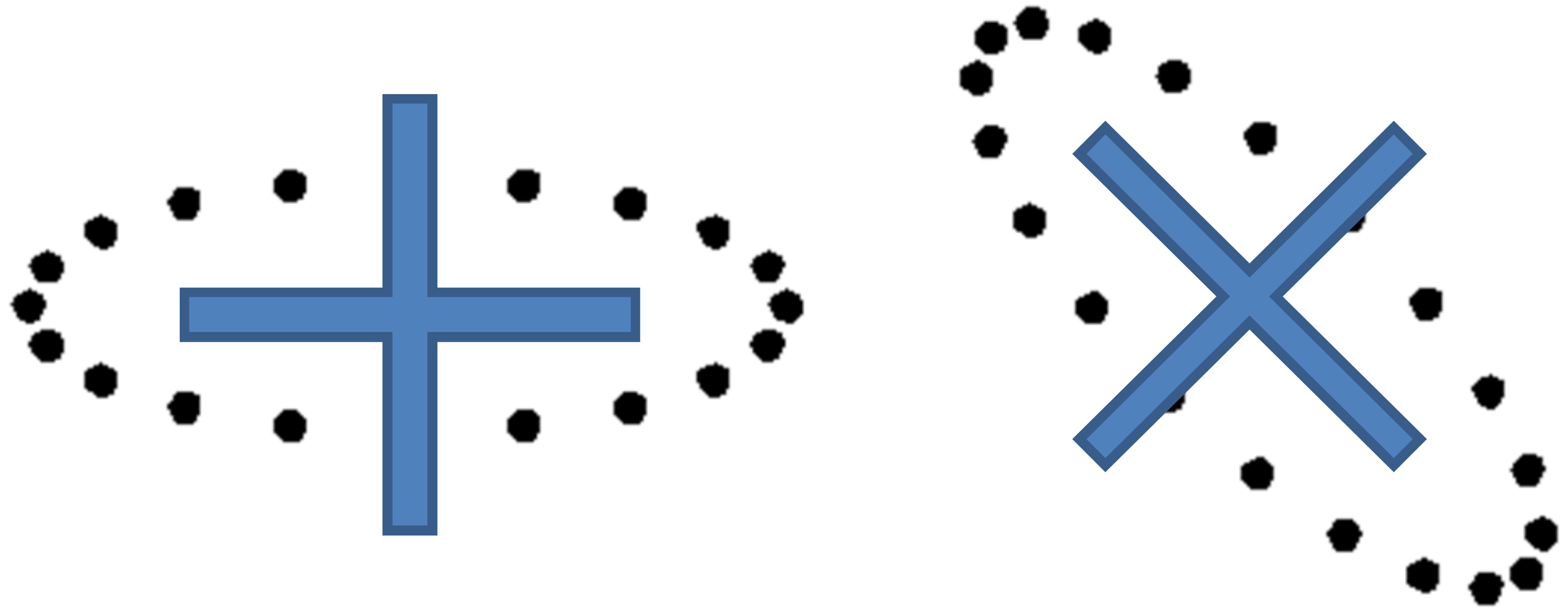




The tensor quadrupole moment (m = 2). Since gravity waves distort space in the plane of the perturbation, changing a circle of test particles into an ellipse, the radiation acquires an m = 2 quadrupole moment. No azimuthal simmetry for ΔT_2

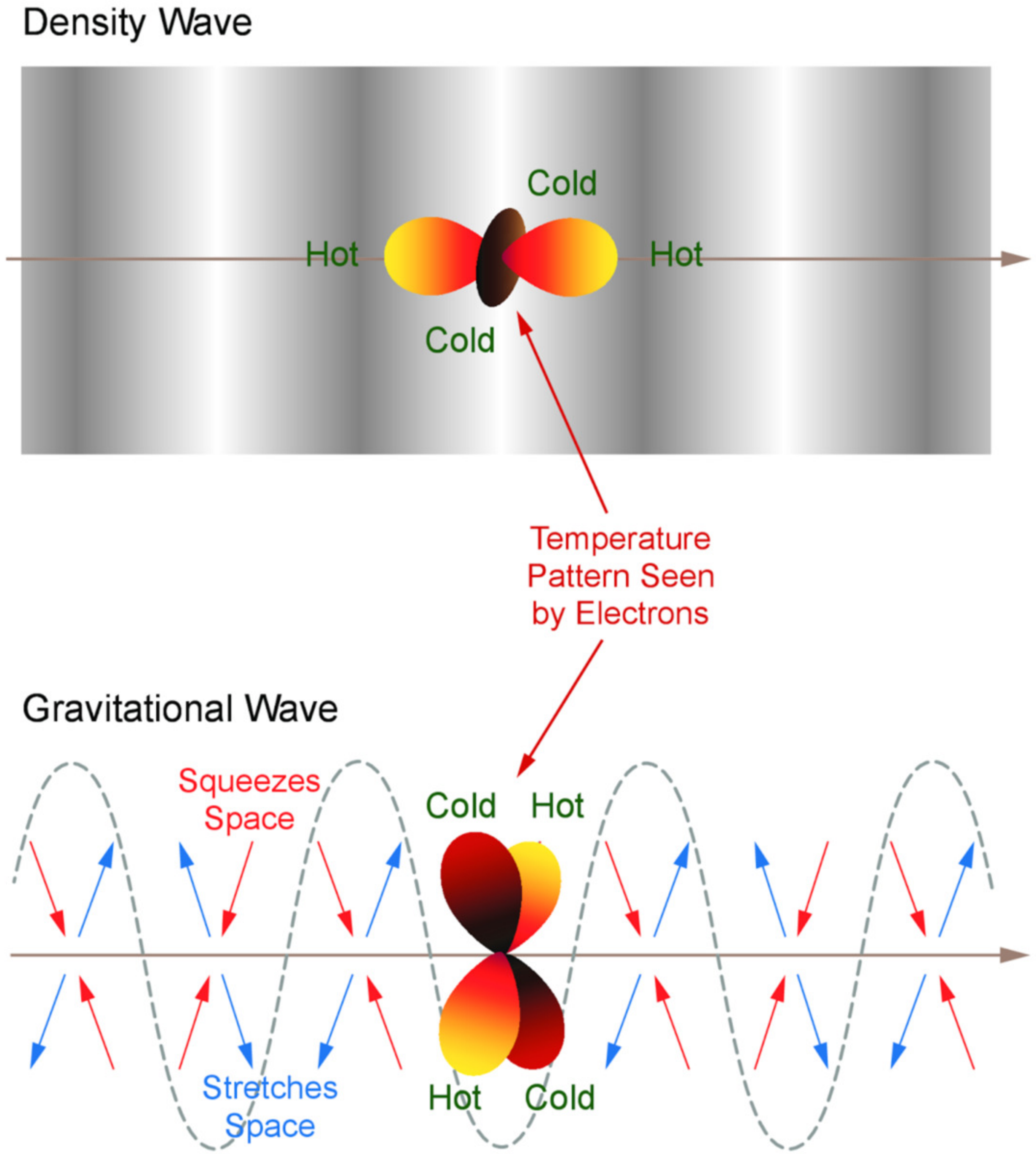
Transverse and traceless perturbations

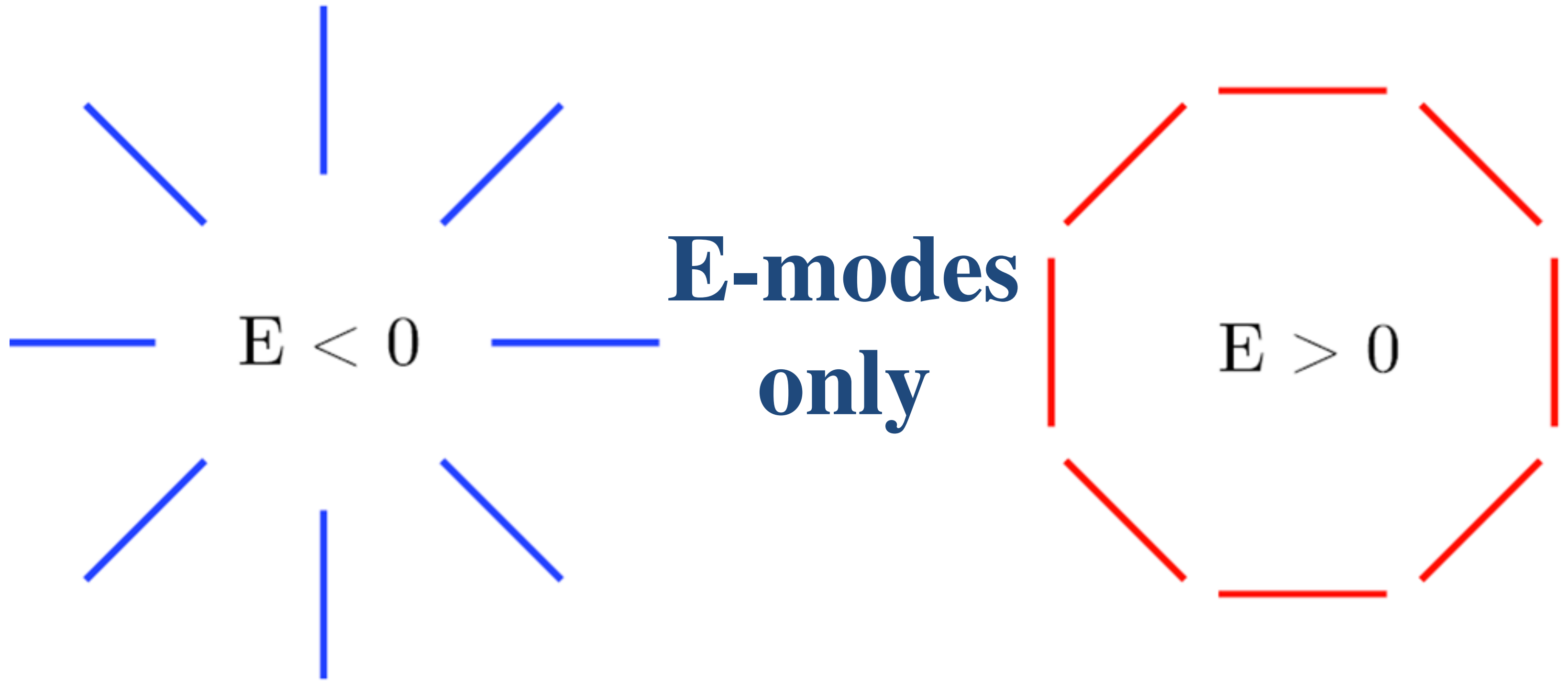
$$h_{ij} = h_+ \varepsilon^+{}_{ij} + h_\times \varepsilon^\times{}_{ij}$$



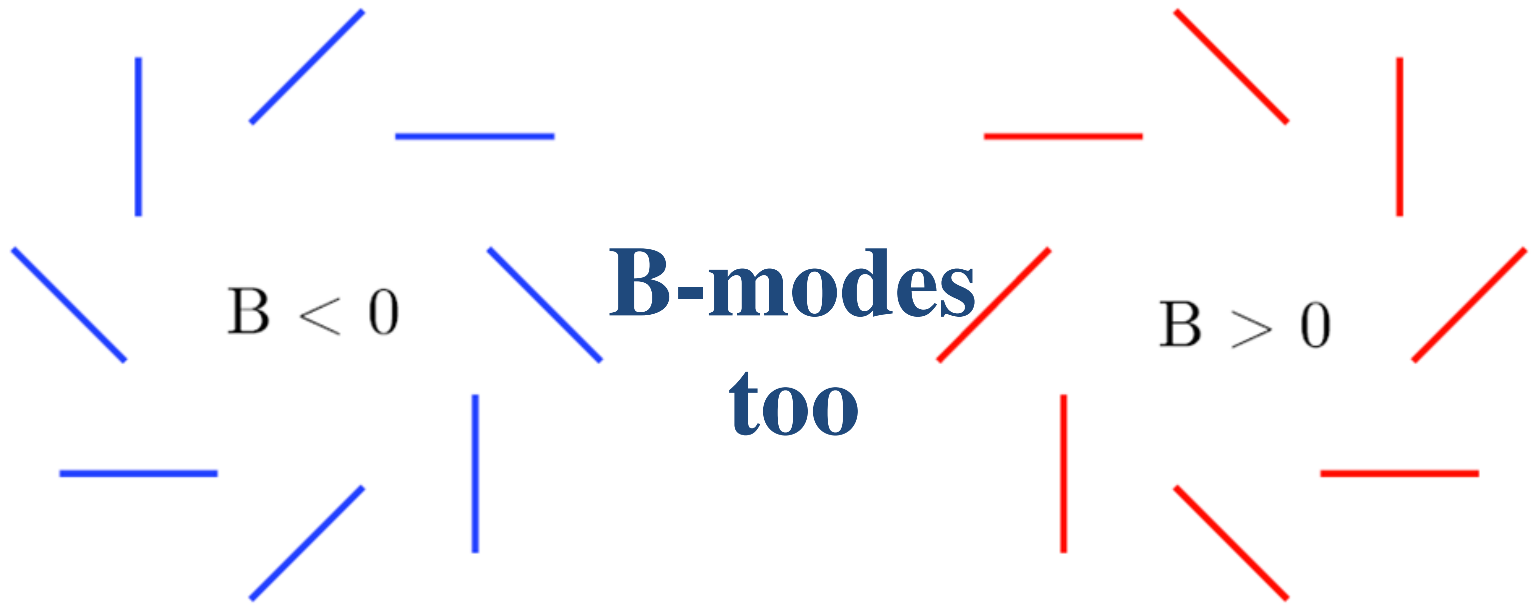
CMB E- and B-modes

[credit: Carbone]





The exchange symmetry {Q,U} ↔ {U,−Q} as E↔B indicates that E-modes and B-modes represent polarizations rotated by 45°

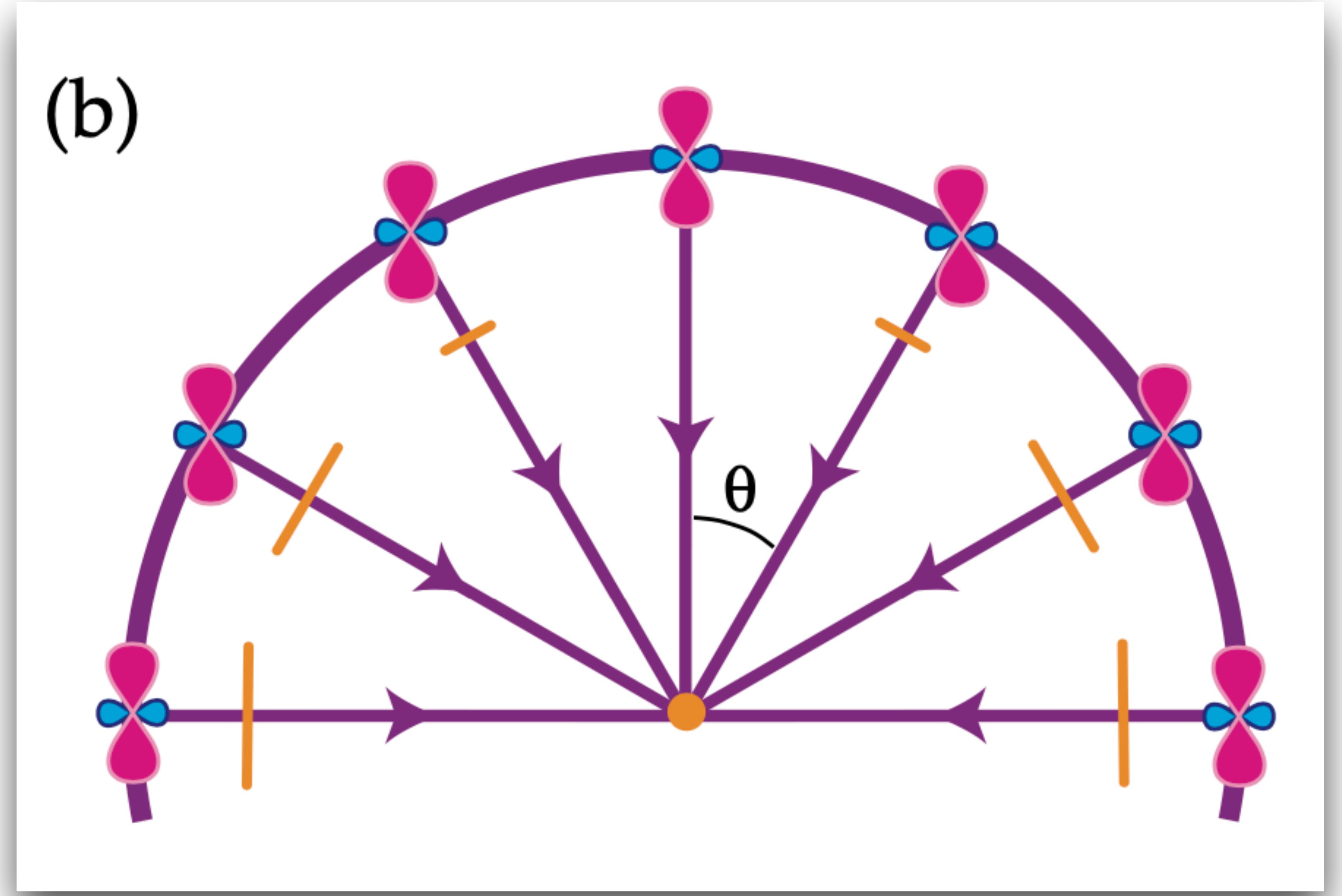


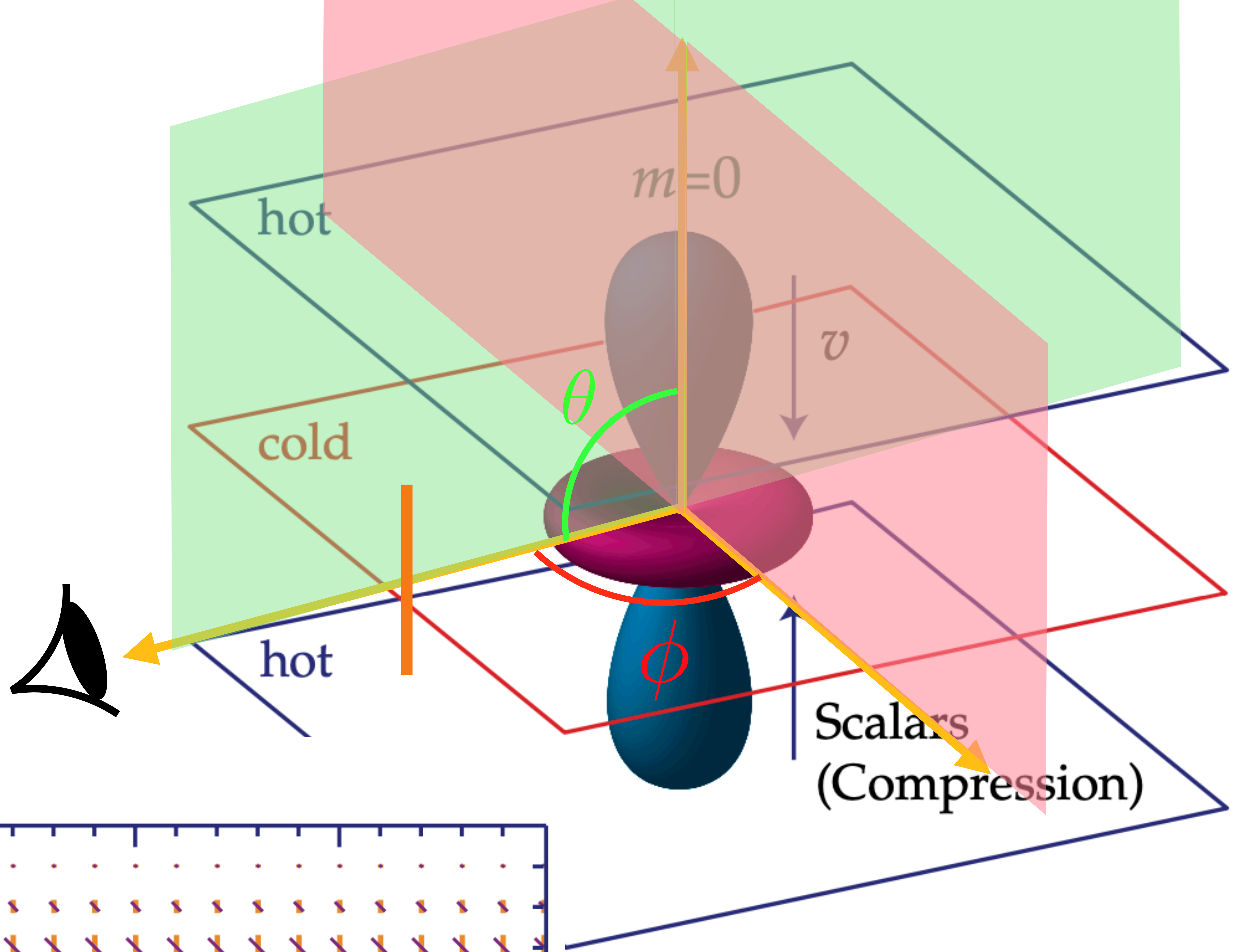
"Electric-modes", since an electric field can be written as the gradient of a scalar. "Magnetic-modes" since they are the curl of a vector field. Heuristically, scalar perturbations have no handedness so they cannot produce any "curl", whereas vector and tensor perturbations do have a handedness and therefore can.

Electric mode: gradient of scalar
Magnetic mode: curl of vector

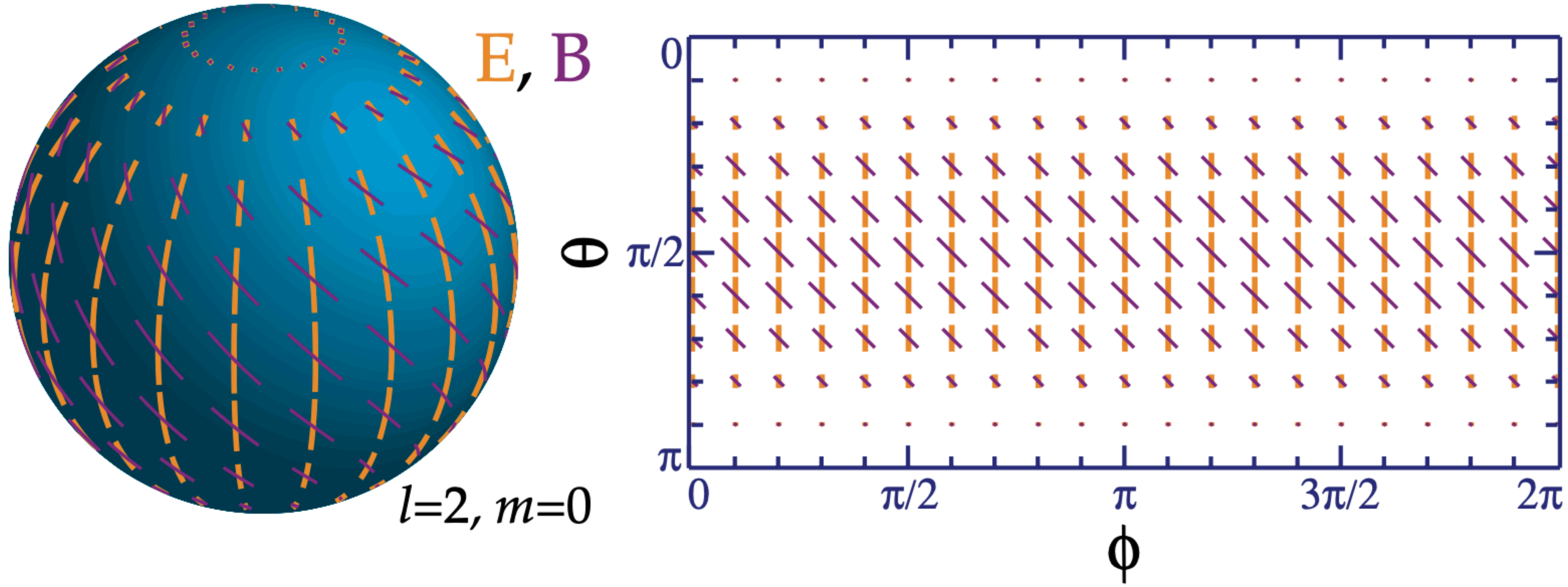
Scalar pert.
Vector pert.
Tensor pert.

scalar pert.

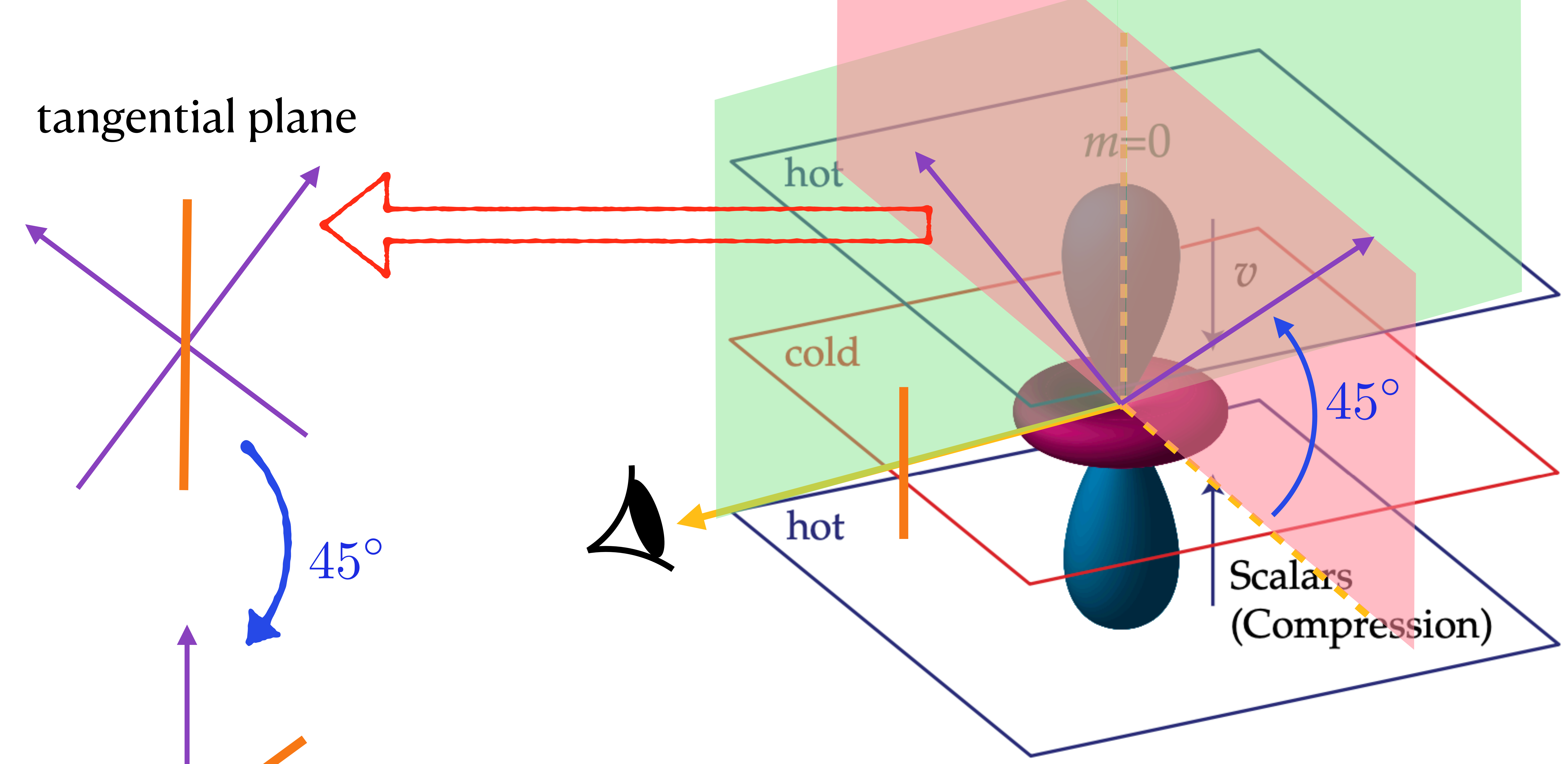




See from diff angle



constant along \phi direction,
maximum at \theta=\pi/2



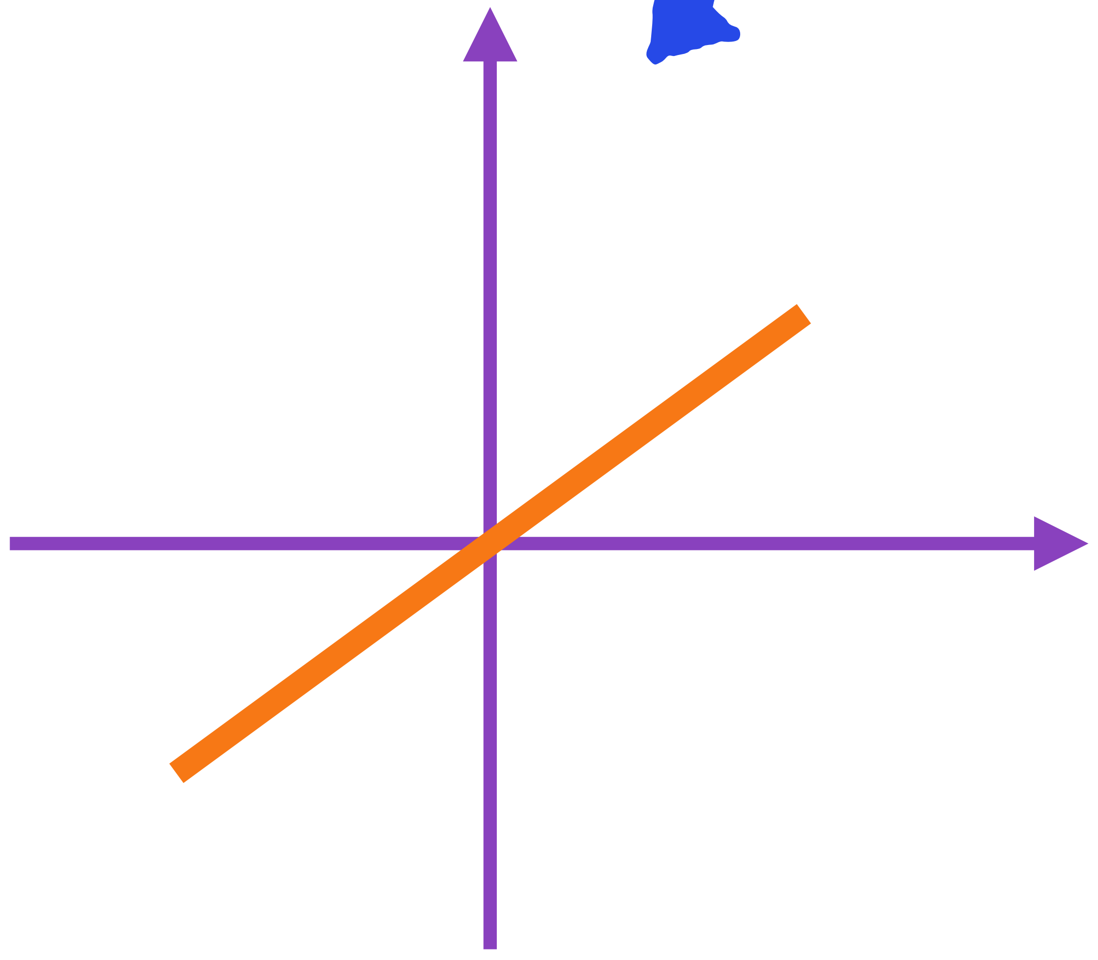


Fig. 2.— The scalar quadrupole moment ($\ell = 2, m = 0$). Flows from hot (blue) regions into cold (red), $\mathbf{v} \parallel \mathbf{k}$, produce the azimuthally symmetric pattern Y_2^0 depicted here.

$Y^{l=2}_{m=0}$ can be both pure Q or pure U field, it depends on the coordinate frame on the tangential plane

$$(Q \pm iU)'(\hat{n}) = e^{\mp 2i\phi}(Q \pm iU)(\hat{n})$$ [Q/U are frame dependent!]

If pure Q, it means pure E.

$$Q = \sin^2\theta, \qquad U = 0.$$

If pure U, it means pure B.

$$\phi = 45^\circ$$

$$Q' = 0, U' = sin^2\theta$$

Q: If so, why do we normally say, scalar mode can not generate B?

A: Thomson Scattering is a parity conserved process. In quadrupole temperature

anisotropy, ell=2.

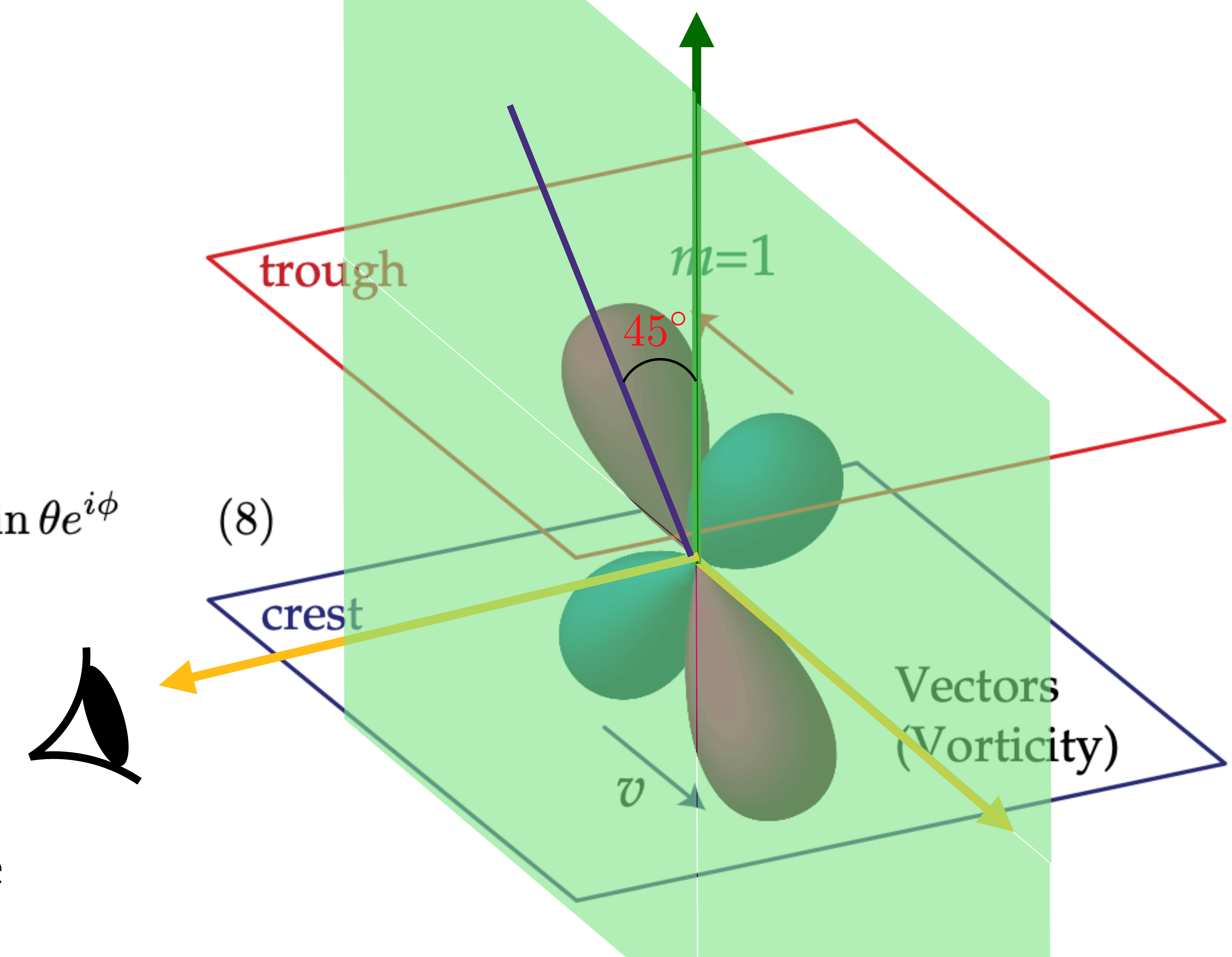
This system is parity even $(-1)^{ell}$.

Hence, TS process only generate E mode, not B mode!

vector pert.

The full $\ell = 2$, $m = 1$ pattern,

$$Q = -\sin\theta\cos\theta e^{i\phi}, \qquad U = -i\sin\theta e^{i\phi} \tag{8}$$



view from equator, the U mode dominate over Q mode.

Fig. 5.— The vector quadrupole moment ($\ell = 2$, $m = 1$). Since $\mathbf{v} \perp \mathbf{k}$, the Doppler effect generates a quadrupole pattern with lobes 45° from $\mathbf{v}$ and $\mathbf{k}$ that is spatially out of phase (interplane peaks) with $\mathbf{v}$.

tensor pert.

Tensor mode peaks at poles (not equator).

$$Q = (1 + \cos^2\theta)e^{2i\phi}, \qquad U = -2i\cos\theta e^{2i\phi},$$

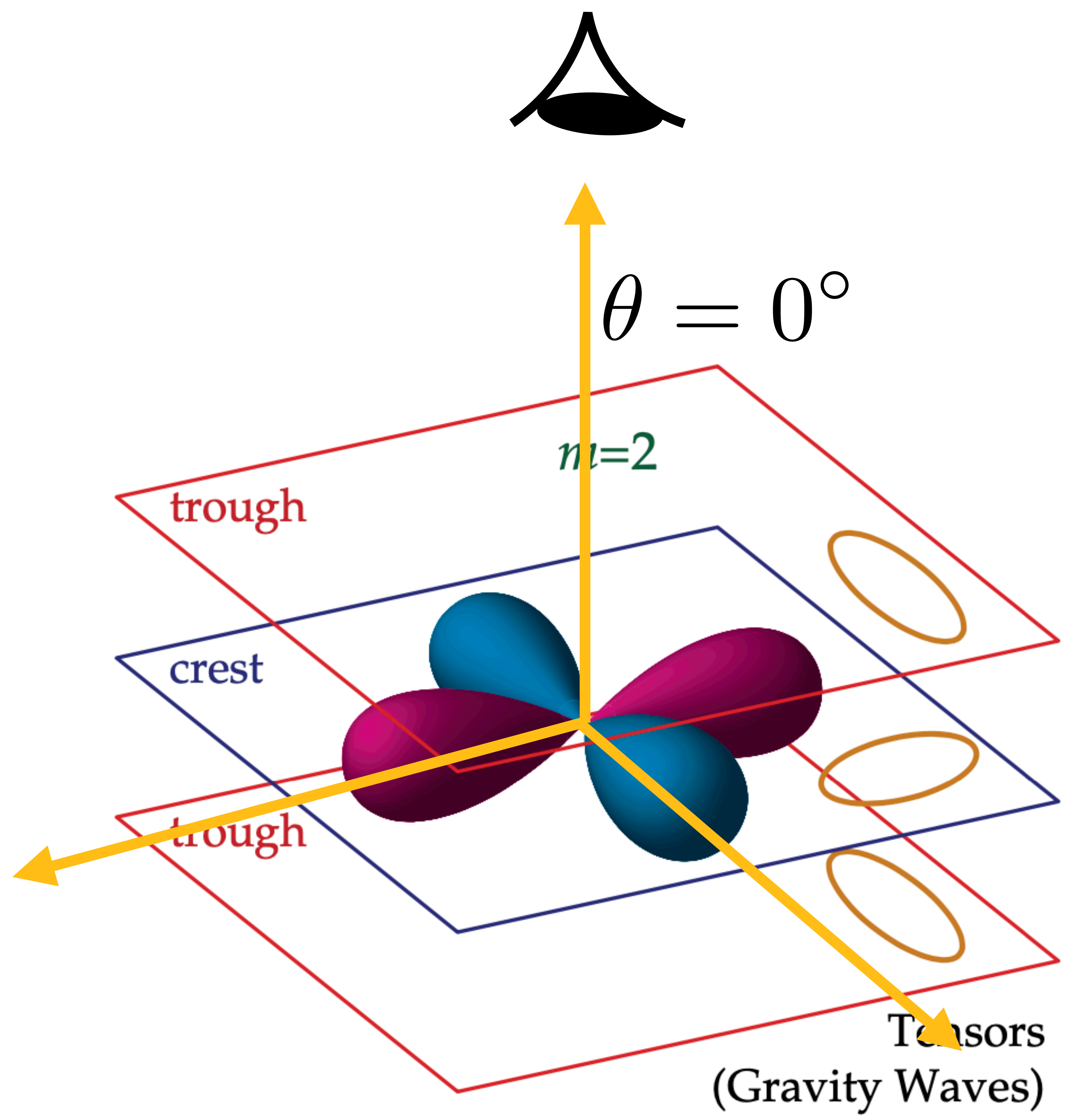
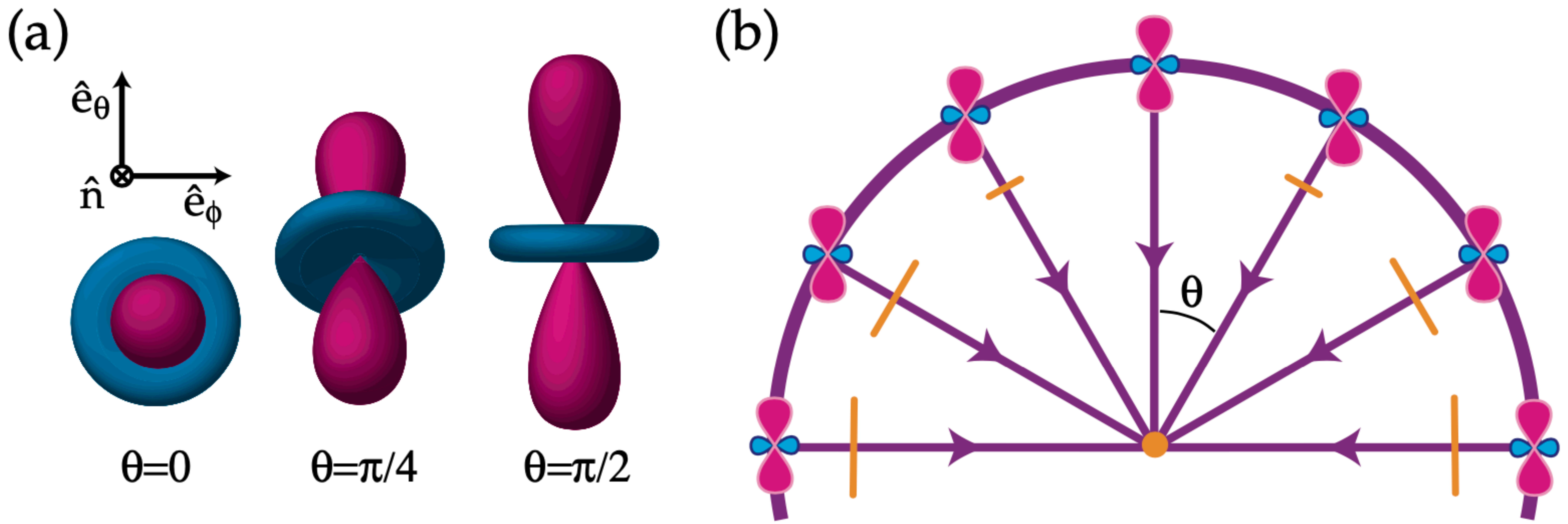


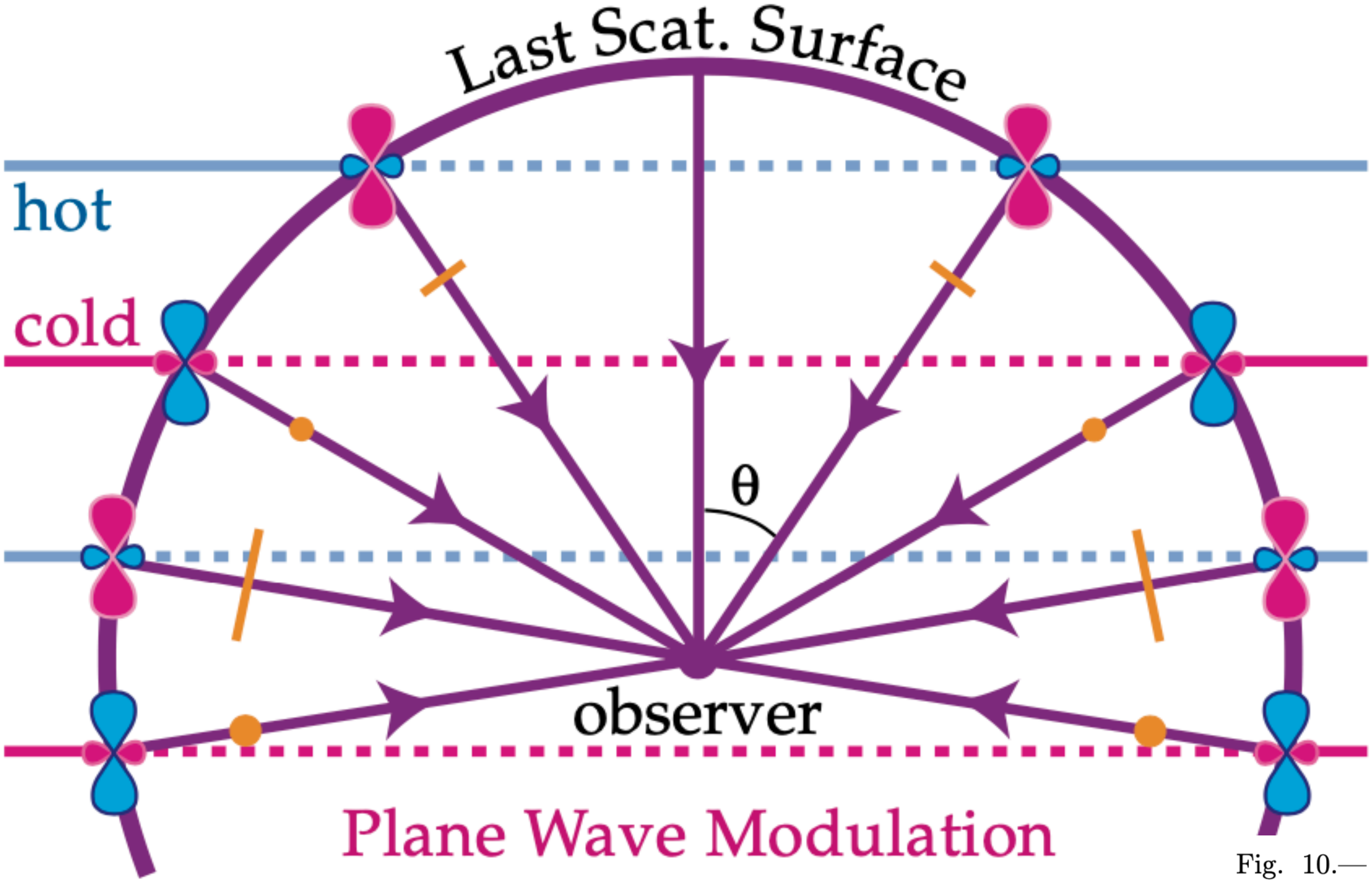
Fig. 7.— The tensor quadrupole moment ($m = 2$). Since gravity waves distort space in the plane of the perturbation, changing a circle of test particles into an ellipse, the radiation acquires an $m = 2$ quadrupole moment.

Thompson scattering only E mode;
modulation producing B mode

w/o plane wave modulation

[arxiv: 9706147]





[arxiv: 9706147]

Fig. 10.— Modulation of the local pattern Fig. 3b by plane wave fluctuations on the last scattering surface. Yellow points represent polarization out of the plane with magnitude proportional to sign. The plane wave modulation changes the amplitude and sign of the polarization but does not mix Q and U. Modulation can mix E and B however if U is also present.

Example of local m=2 quadrupole $Q = (1+\cos^2\theta)e^{2i\phi}, \qquad U = -2i\cos\theta e^{2i\phi},$

$$E_{lm} + iB_{lm} = \int d\Omega\ {}_2Y^*_{lm}(\hat{n})(Q+iU)(\hat{n})$$

m	Y_2^m	${}_2Y_2^m$
2	$\frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$	$\frac{1}{8}\sqrt{\frac{5}{\pi}}(1-\cos\theta)^2\, e^{2i\phi}$
1	$\sqrt{\frac{15}{8\pi}}\sin\theta\,\cos\theta\, e^{i\phi}$	$\frac{1}{4}\sqrt{\frac{5}{\pi}}\sin\theta\,(1-\cos\theta)\, e^{i\phi}$
0	$\frac{1}{2}\sqrt{\frac{5}{4\pi}}(3\cos^2\theta-1)$	$\frac{3}{4}\sqrt{\frac{5}{6\pi}}\sin^2\theta$
-1	$-\sqrt{\frac{15}{8\pi}}\sin\theta\,\cos\theta\, e^{-i\phi}$	$\frac{1}{4}\sqrt{\frac{5}{\pi}}\sin\theta\,(1+\cos\theta)\, e^{-i\phi}$
-2	$\frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{-2i\phi}$	$\frac{1}{8}\sqrt{\frac{5}{\pi}}(1+\cos\theta)^2\, e^{-2i\phi}$

TAB. 1: Quadrupole ($\ell = 2$) harmonics for spin-0 and 2.

derive that B_{2m} is vanishing.

Adding Plane-wave modulation

$$E_{lm} + iB_{lm} \;=\; \int d\Omega \; {}_2Y^*_{lm}(\hat{n})(Q + iU)(\hat{n})\, e^{i\vec{k}\cdot\vec{x}}$$

Modulation only change the sign and magnitude of Q & U, independently.

However, it DOES convert E to B!

Because, plane-wave does not have definite parity, hence it mix E and B.

The Q and U Stokes parameters, which describe the linear polarization, depend on the reference frame. If $\vec{e}_1$ and $\vec{e}_2$ are rotated by an angle θ around $\vec{k}$ then Q and U rotate to Q' and U' by an angle 2θ:

$$\begin{aligned} Q' &= Q\cos 2\theta + U\sin 2\theta, \\ U' &= -Q\sin 2\theta + U\cos 2\theta, \end{aligned} \quad \text{or} \quad Q \pm \mathrm{i}U \to Q' \pm \mathrm{i}U' = \mathrm{e}^{\mp 2\mathrm{i}\theta}(Q \pm \mathrm{i}U). \tag{2}$$

This is the transformation of a spin 2 object.[2] The V parameter describes the circular polarization and is invariant under

$$(Q \pm iU)(\hat{n}) = \sum_{lm} (E_{lm} \pm iB_{lm})\ {}_{\pm 2}Y_{lm}(\hat{n}),$$

$$(Q \pm \mathrm{i}U)(\vec{n}) = \sum_{l\geqslant 2,\ |m|\leqslant l} a_{\pm 2lm\,\pm 2}Y_l^m(\vec{n}). \tag{3}$$

From these spin 2 objects, one can construct 2 real scalar quantities with opposite behavior under parity transformations:[4]

$$\begin{aligned} E(\vec{n}) &= \sum_{l\geqslant 2,\ |m|\leqslant l} a^E_{lm} Y_l^m(\vec{n}), \quad \text{with positive parity} \\ B(\vec{n}) &= \sum_{l\geqslant 2,\ |m|\leqslant l} a^B_{lm} Y_l^m(\vec{n}), \quad \text{with negative parity,} \end{aligned} \quad \text{where} \quad \begin{aligned} a^E_{lm} &= -\frac{a_{2lm} + a_{-2lm}}{2}, \\ a^B_{lm} &= \mathrm{i}\frac{a_{2lm} - a_{-2lm}}{2}. \end{aligned} \tag{4}$$

Q/U are components of spin-2 vector, hence their values changes accordingly between frames

with $\hat{\chi}$ representing the unit vector in the direction of the polarization. The Q and U parameters depend on the reference frame and transform like a spin-2 object, i.e. if the reference frame $(\hat{x}, \hat{y})$ is rotated by an angle ψ around $\hat{n}$, then Q and U rotate to Q' and U' by an angle 2ψ

$$(Q \pm iU)'(\hat{n}) = e^{\mp 2i\psi}(Q \pm iU)(\hat{n}). \tag{4}$$

E/B are scalar (Psudo-scalar), their values does not change (sign flip)

$$\bar{\eth}^2(Q + iU)'(\hat{n}') = \eth^2(Q - iU)(\hat{n}) \tag{22}$$

Substituting Eqn (22) into (19), we will have $E'_{lm}(\hat{n}') = E_{lm}(\hat{n})$ and $B'_{lm}(\hat{n}') = -B_{lm}(\hat{n})$. Therefore E and B have even and odd parities respectively. It is useful to define two quantities in real space

[Challinor's lecture]

Why modulation mix E&B?

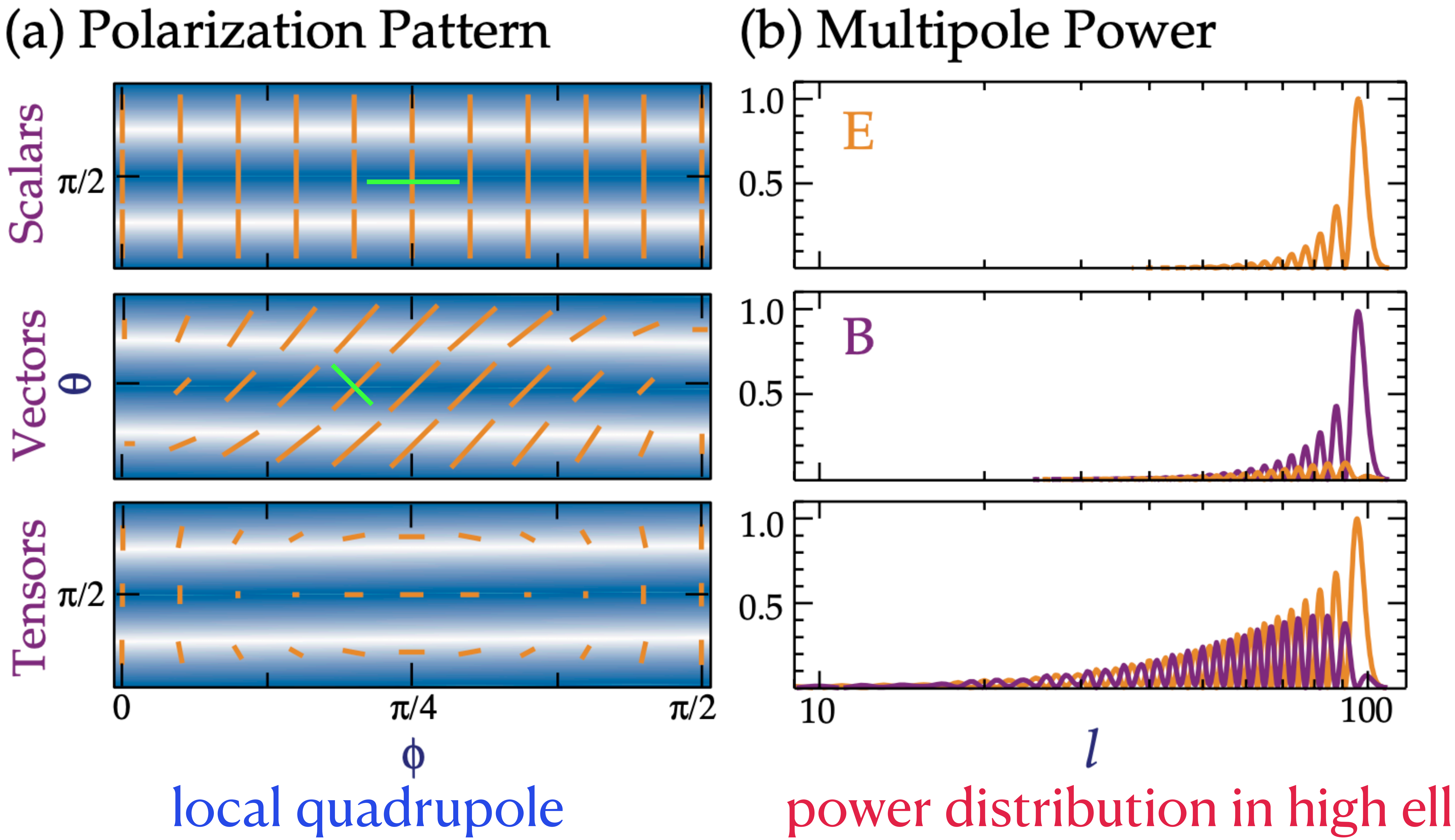
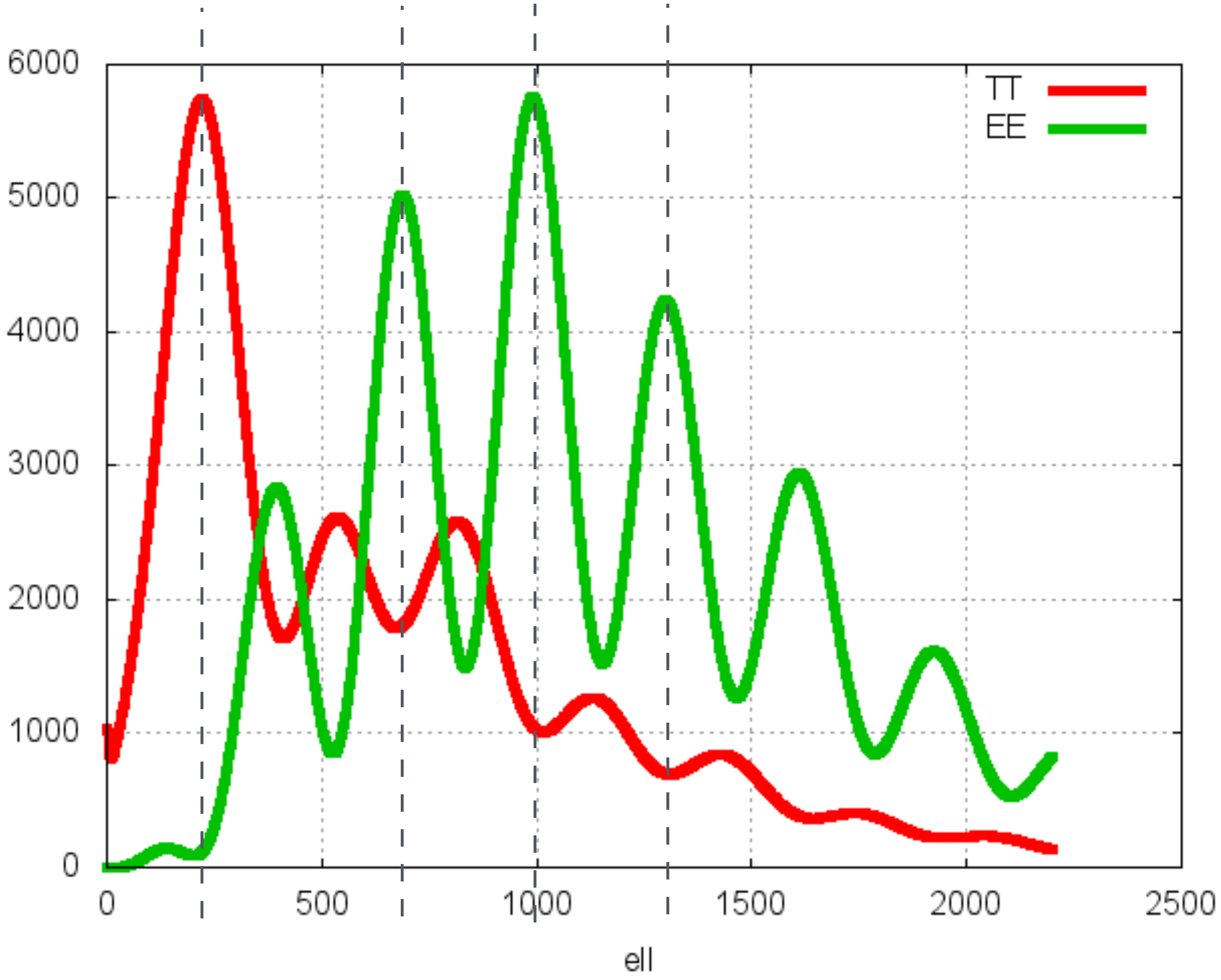


Fig. 11.— The E and B components of a plane wave perturbation. (a) Modulation of the local E-quadrupole pattern (yellow) from scattering by a plane wave. Modulation in the direction of (or orthogonal to) the polarization generates an E-mode with higher ℓ; modulation in the crossed (45°) direction generates a B-mode with higher ℓ. Scalars generate only E-modes, vectors mainly B-modes, and tensors comparable amounts of both. (b) Distribution of power in a single plane wave with $kr = 100$ in multipole ℓ from the addition of spin and orbital angular momentum. Features in the power spectrum can be read directly off the pattern in (a).

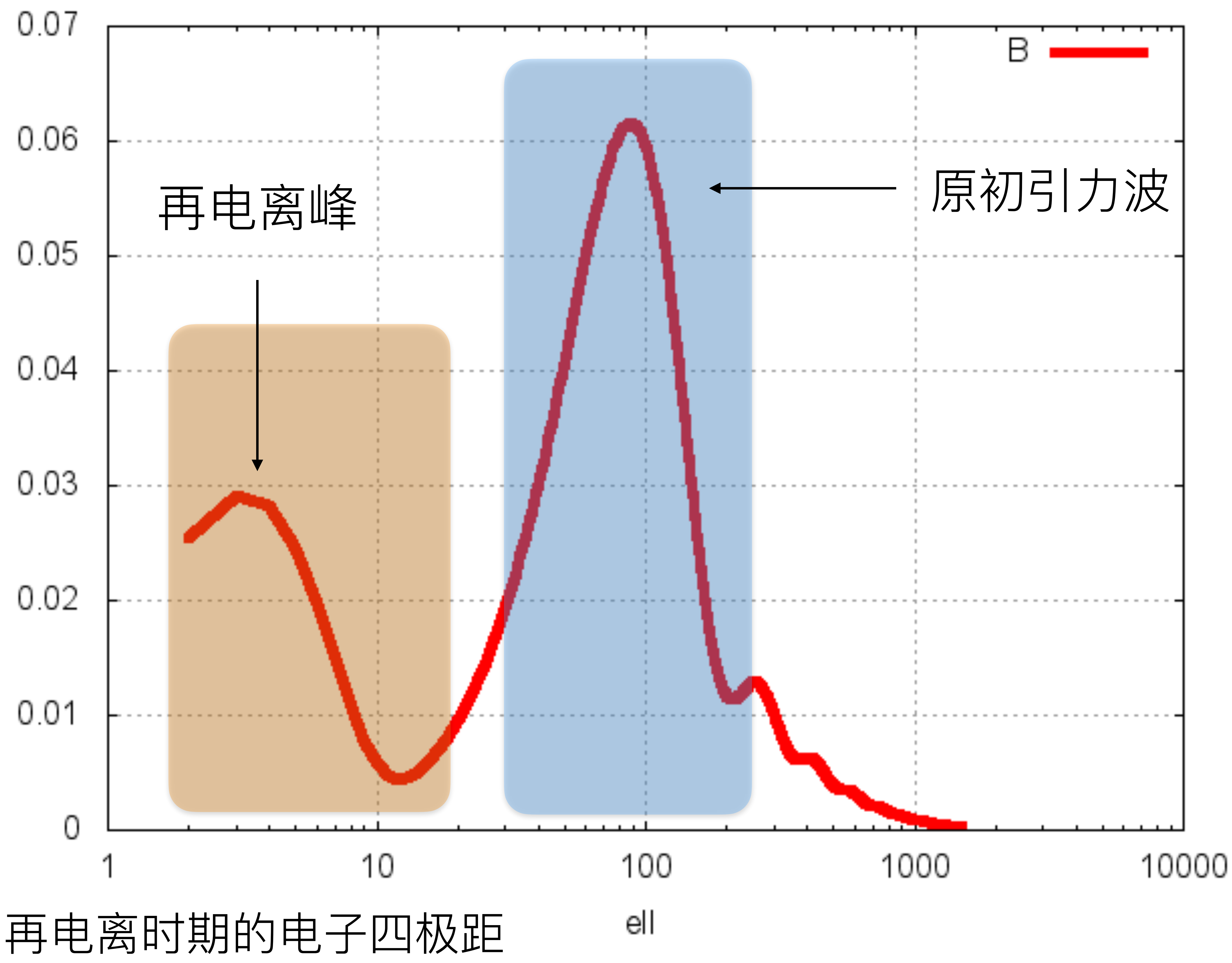
(Y_ℓ^0) with the local spin angular dependence. The result is that plane wave modulation takes the $\ell = 2$ local angular dependence to higher ℓ (smaller angles) and splits the signal into E and B components with ratios which are related to Clebsch-Gordan coefficients. At short wavelengths, these ratios are $B/E = 0, 6, 8/13$ in power for scalars, vectors, and tensors (see Fig. 11b and Hu & White 1997).



温度谱主要
response
物质密度扰动

T&E modes are out of phase!

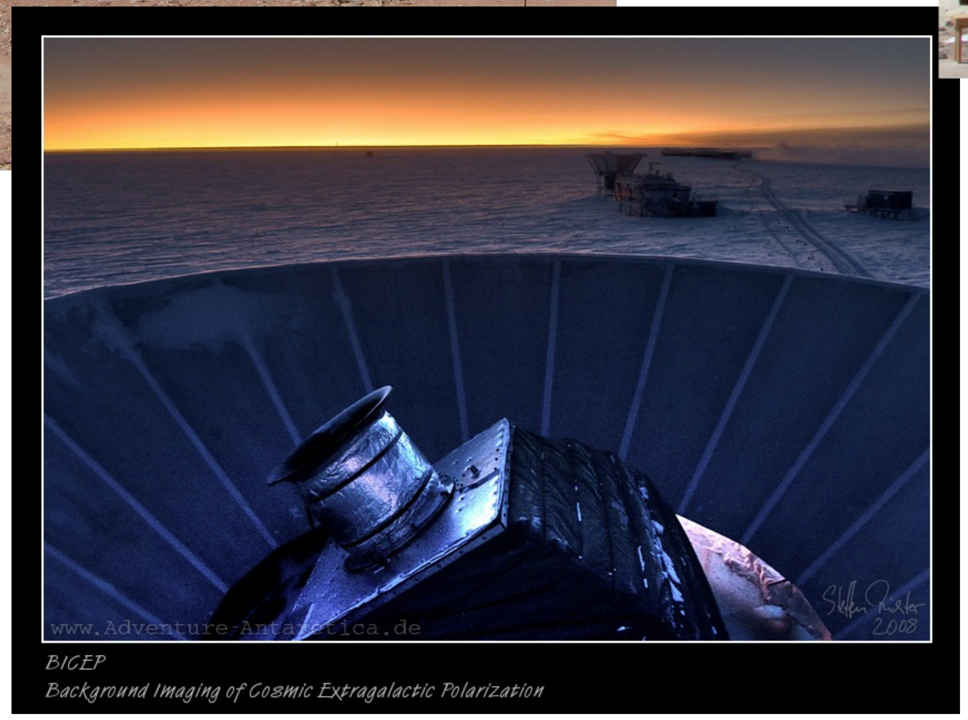
E模极化谱主要
response
速度扰动



再复合时期的电子四极距

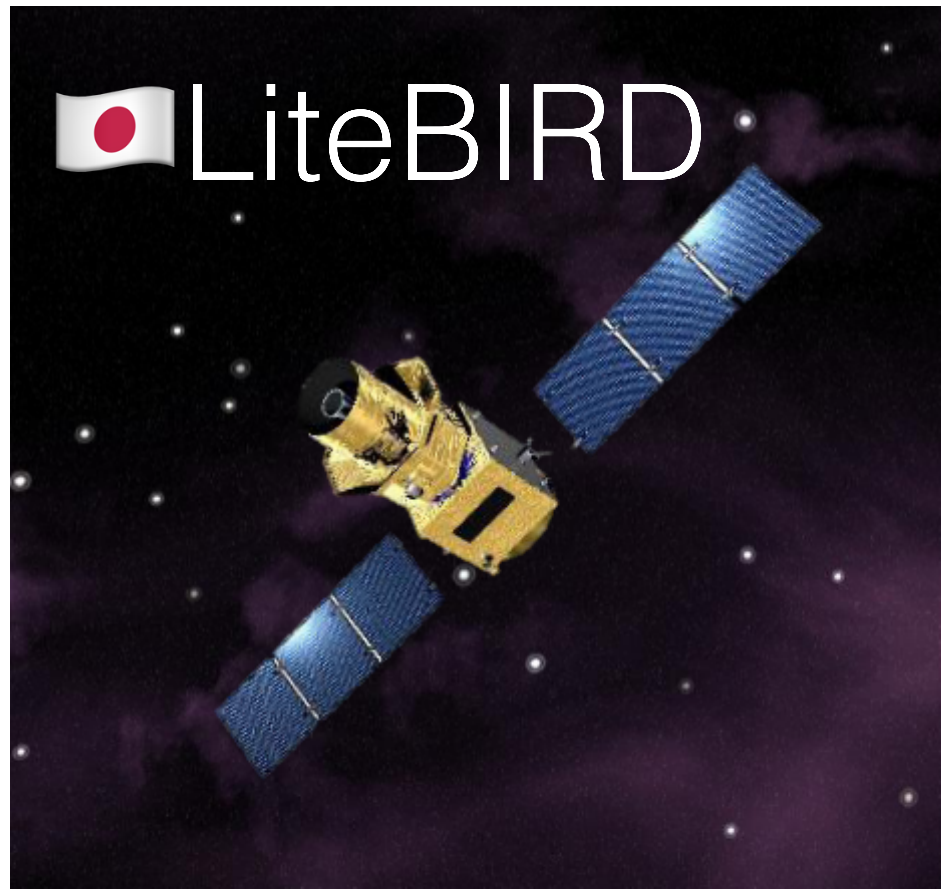
未来10～15年的CMB实验，基本全部以测量B模极化为目标

PolarBear

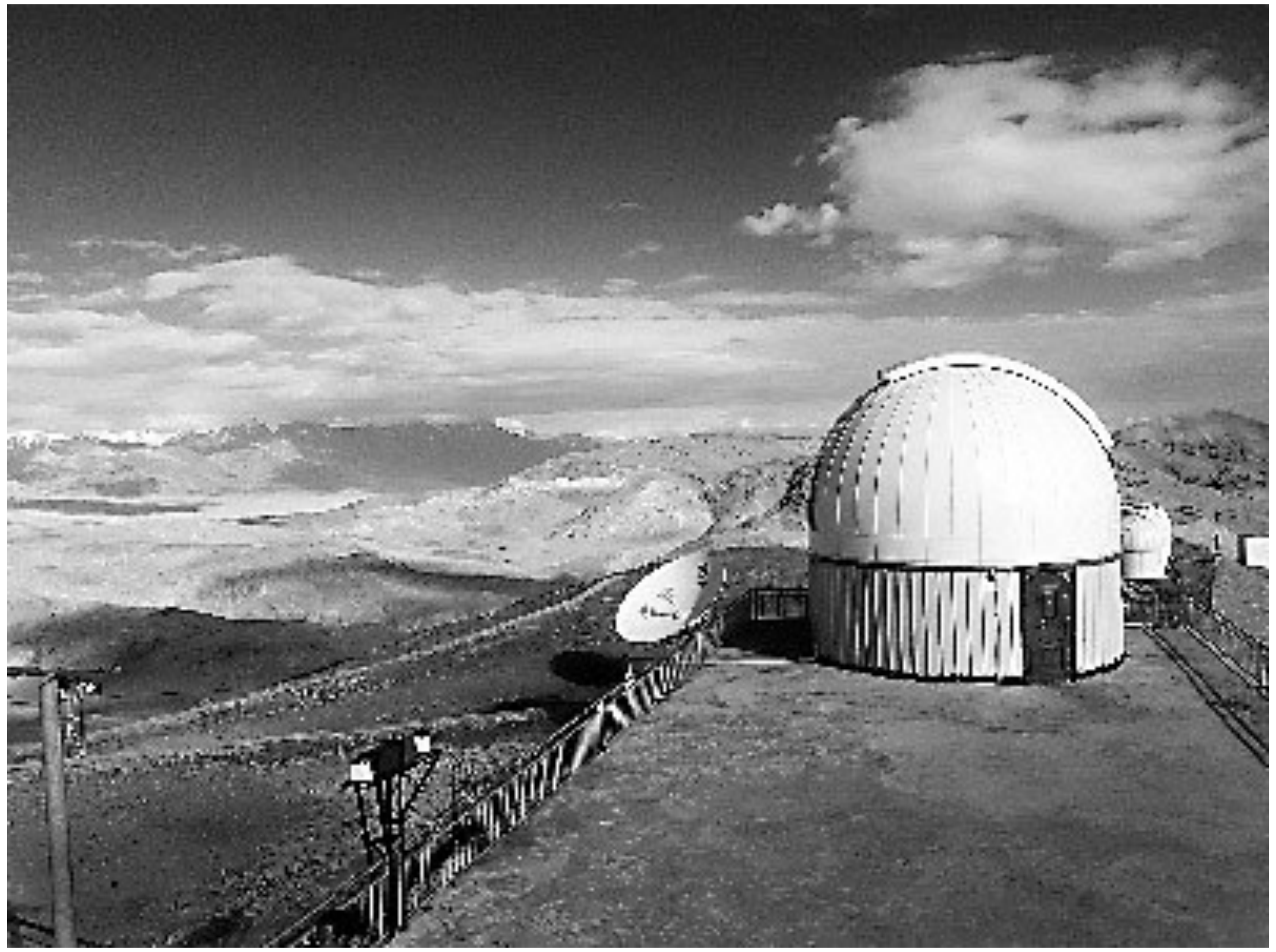


BICEP

SPT



未来10～15年内的CMB实验，以将对r的测量精度提高到**0.01**，为科学目标。我国在西藏阿里天文台也正在积极展开，相关CMB的B模测量项目。**欢迎大家积极参与其中！！！**



Further reading

- arxiv: astro-ph/9706147v1
- Challinor's lecture note