CMB physics

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3. Secondary anisotropy

Key concept

3.1 ISW effect & reionisation

- optical depth
- gravitational potential decay during dark energy dominated epoch
- Lensing mixing different ell. In another word, mixing the light ray from different direction.
- Lensing smearing out the T mode.
- Lensing convert E into B

ISW effect

在上帝的第一推动之后，宇宙经历的各个历史时期中，
虽然宇宙的大小（size）不断在变大 / 膨胀。但是，
其膨胀速度是慢慢变小的，即减速膨胀。

这是由于，引力是一个吸引力，使各种物质组分聚集到一起。
从而，阻碍着宇宙膨胀。

而在，大约红移为0.3的时候，（大约当前宇宙的一半大小），
宇宙开始**加速**膨胀，进入到**暗能量**为主时期。

$$a=1/(1+z)$$

在物质为主时期，引力势 Ψ 不随时间变化，$\dot{\Psi}=0$

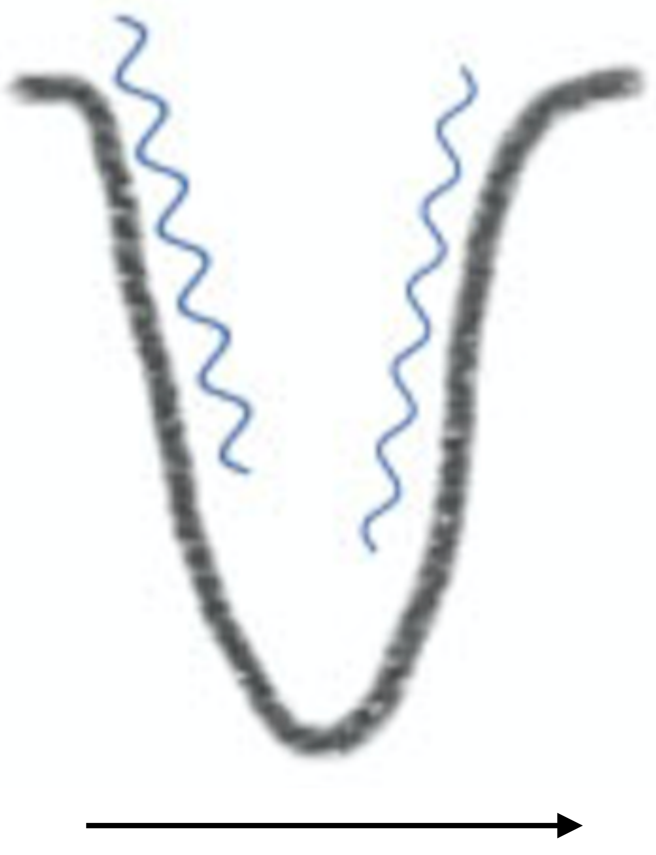
这是由于，引力不稳定性所产生的对引力势的增加，
被背景膨胀所抵消。

泊松方程 $-k^2\Psi = 4\pi G a^2 \bar{\rho}\delta$

积分Sachs-Wolfe效应 / Integrated Sachs-Wolfe Effect

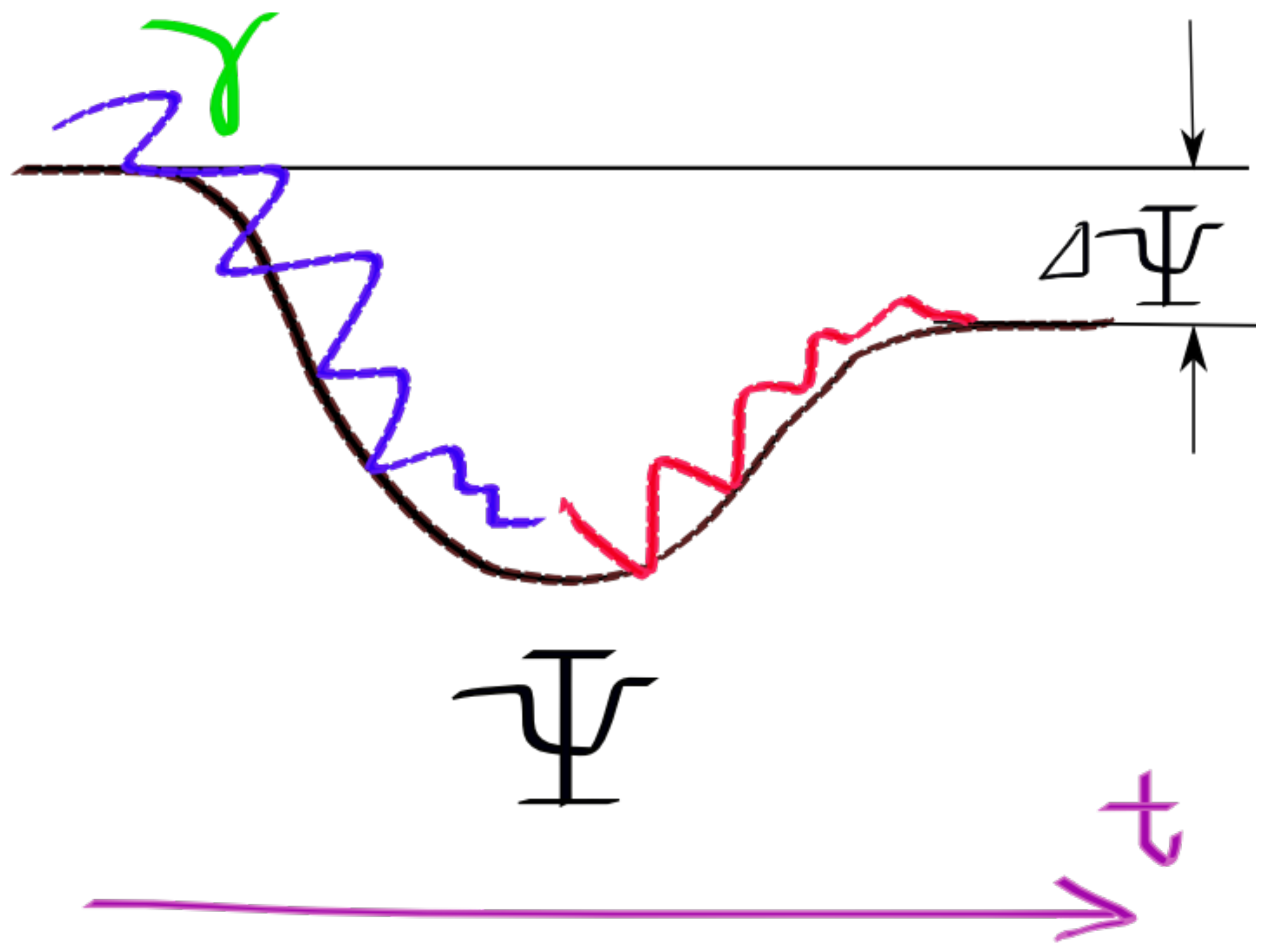
在暗能量为主时期，引力不稳定性不足以抵消背景膨胀，
Ψ 开始衰减， $\dot{\Psi} < 0$

由于光子运动速度很快，对于窄势阱而言，势阱来不及反应



窄势阱

而对于，
视界尺度上的宽势阱而言，
即便是以光速运动，
光子也要在
势阱中运动很久，
所以，有时间
来感受到
势阱深度的变化



宽势阱

Reionization

红移10，恒星大气被加热到10万K左右，
中性氢原子重新被电离。当 CMB光子穿过
由这些恒星组成的星系时，再次与其自由电子发生
Thomson散射。原初信息被擦除，CMB光子温度
再一次被均匀化，90%的原初信息还得以保留。

因此，经过再电离过程之后的有效CMB光子温度为

$$e^{-\tau}\delta T\,/\,T$$

再电离过程的光学深度

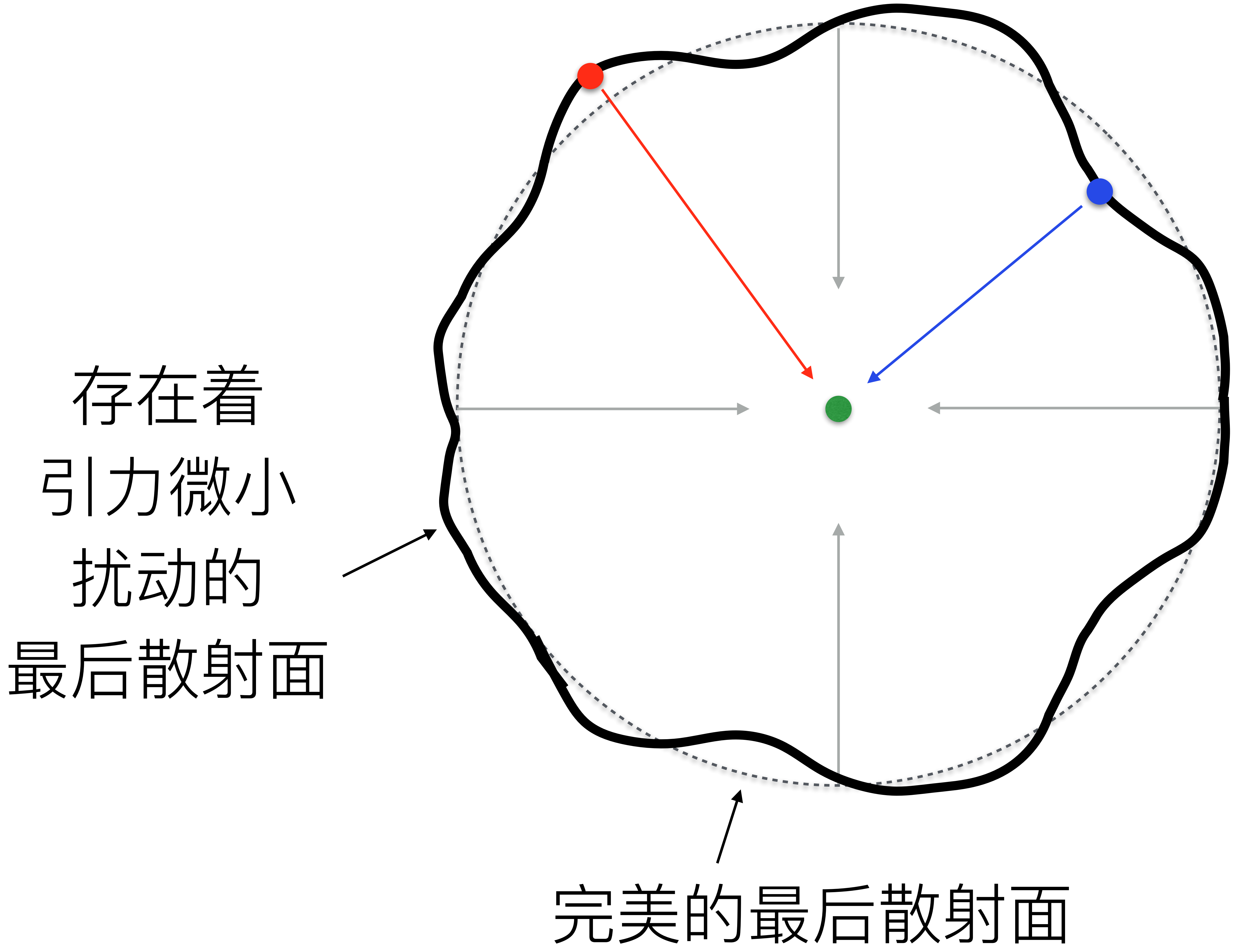
3. Secondary anisotropy

Key concept

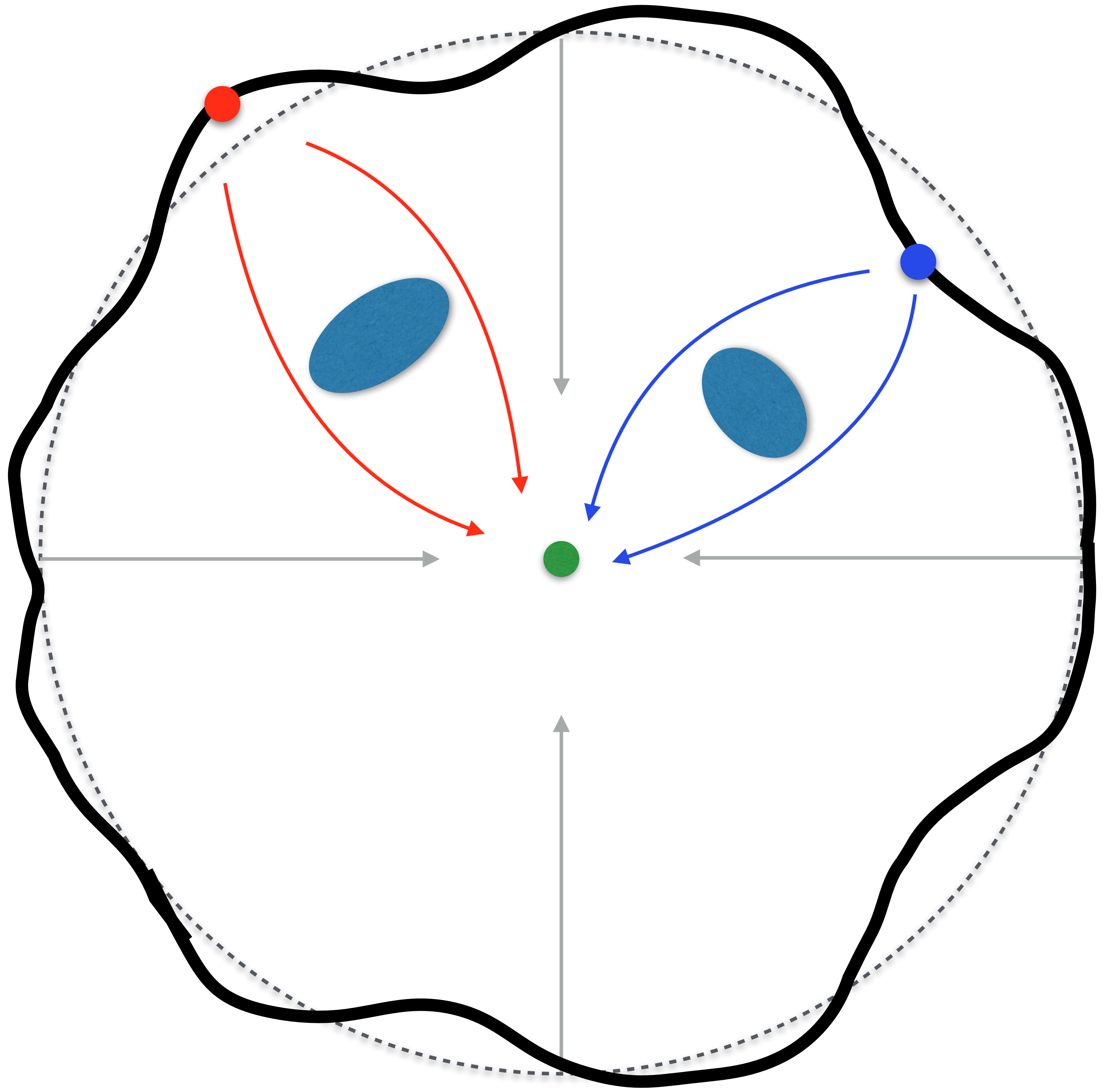
3.2 CMB Lensing

- 2 arcmin deflection angle of CMB lensing
- mixing light rays from different direction
- Lensing magnification
- Brightness conservation
- mode mixing

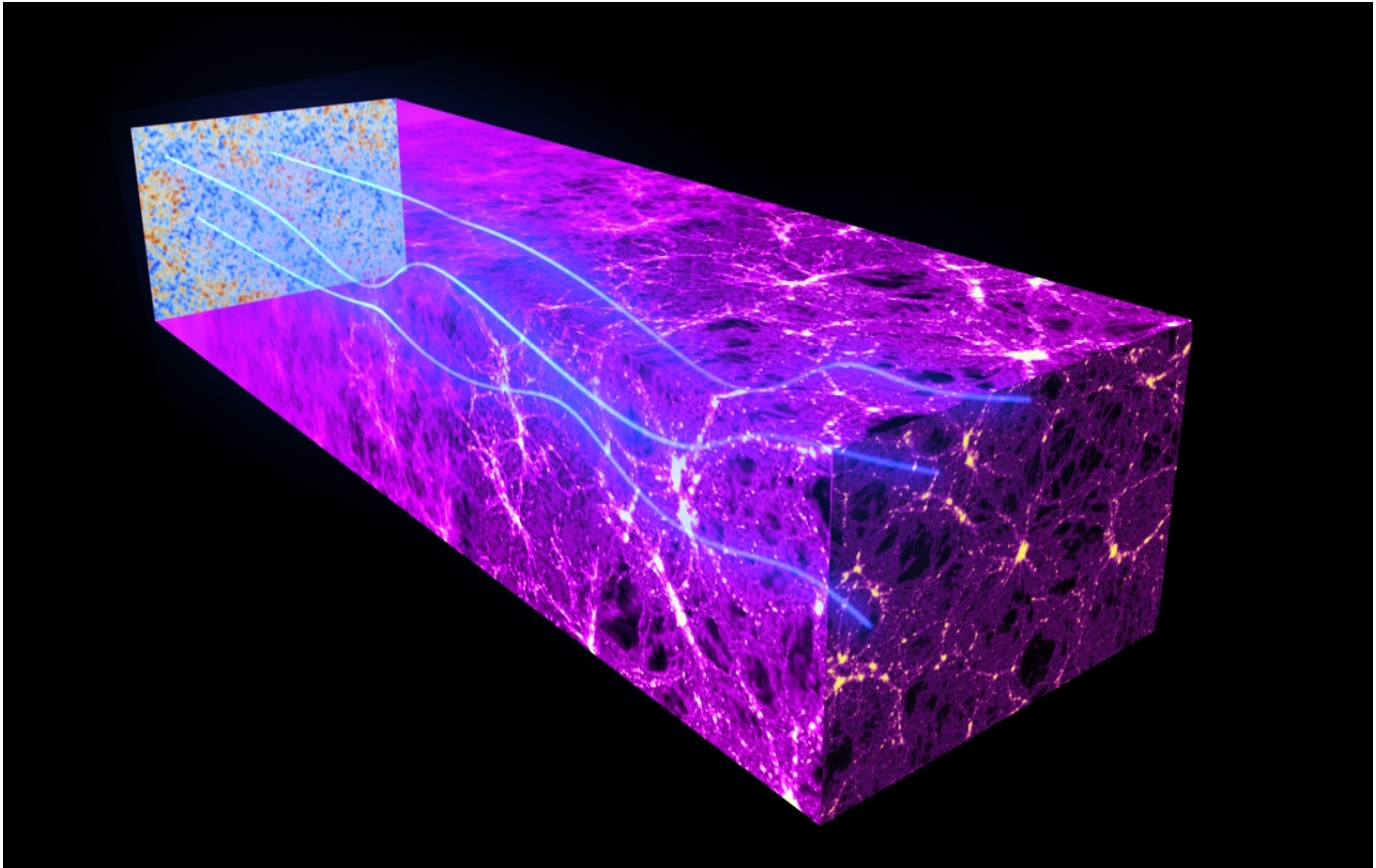
原初CMB各向异性之Sachs－Wolfe 效应



CMB Lensing效应

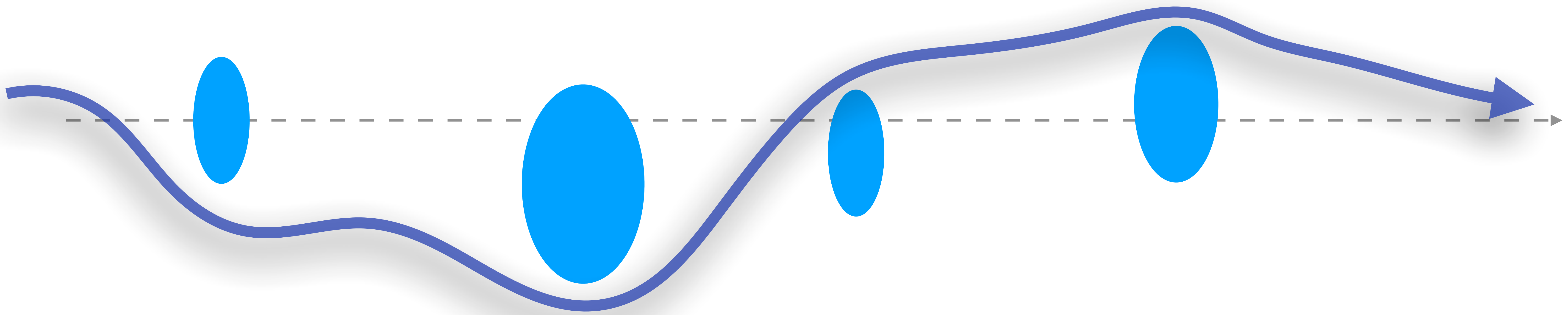


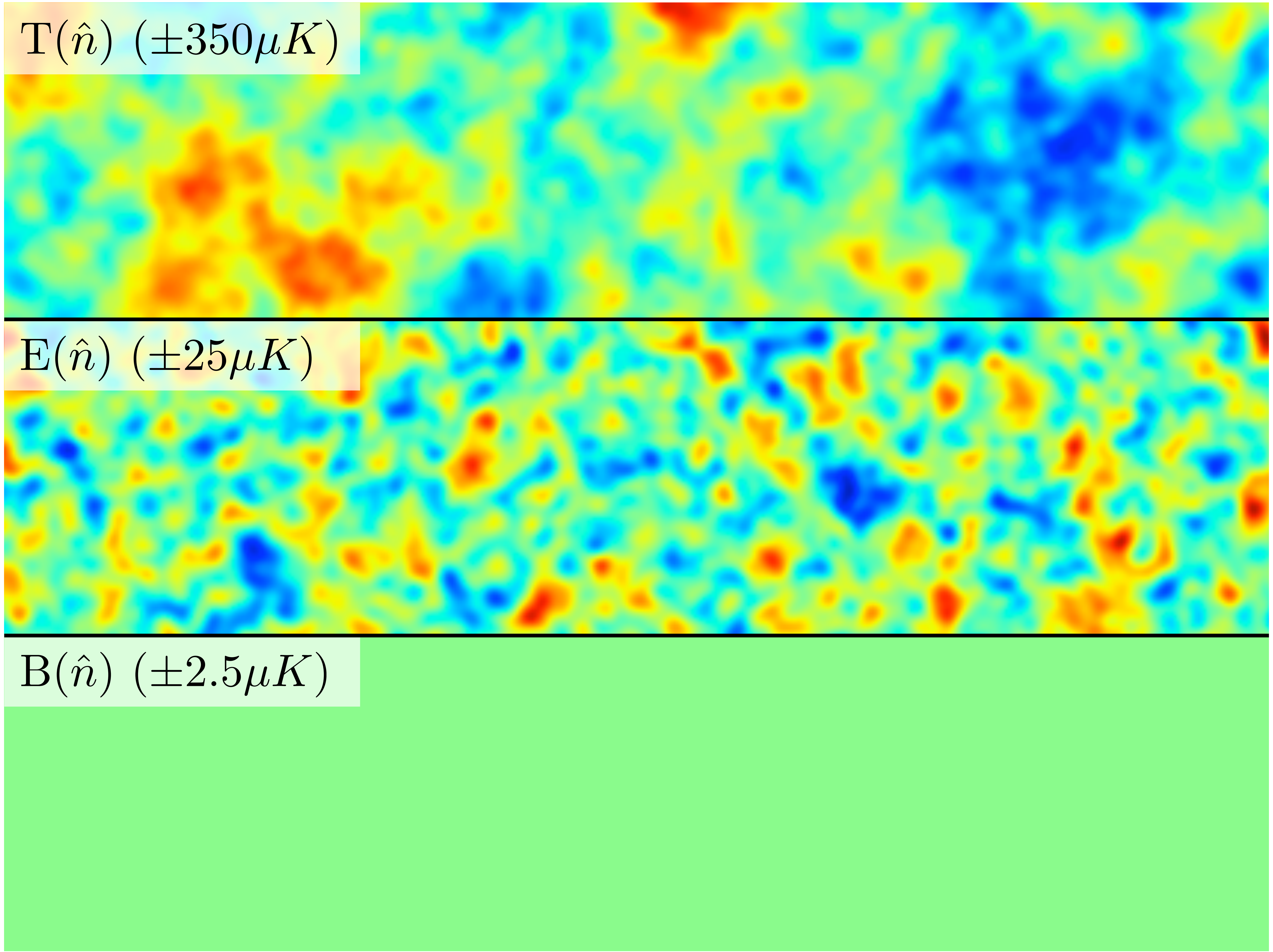
混合不同方向来的光



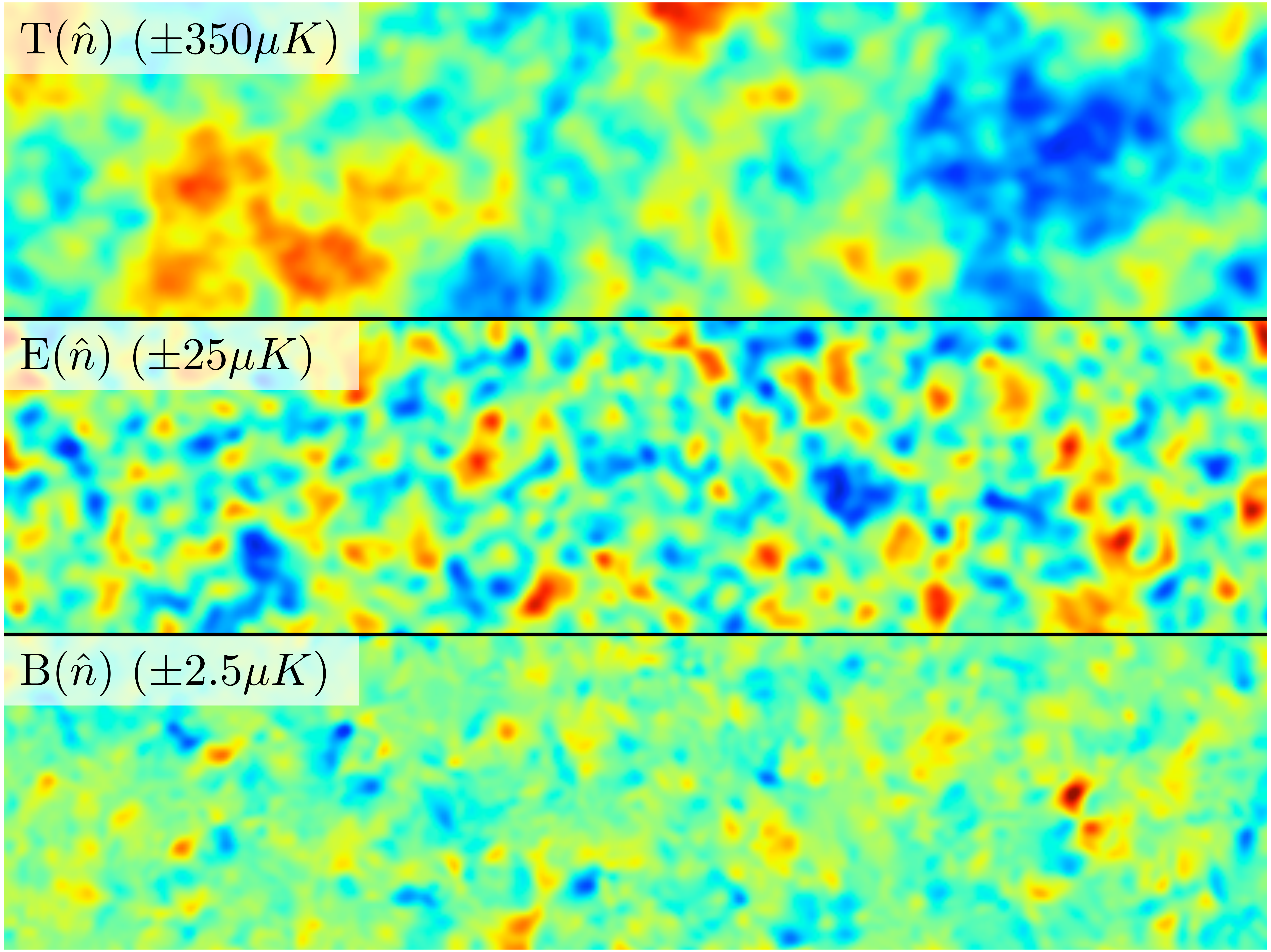
[Pb]

where in matter domination the potentials due to these perturbations are constant in the linear regime. The depth of the potentials is $\sim 2 \times 10^{-5}$, so we might expect each potential encountered to give a deflection $\delta\beta \sim 10^{-4}$. The characteristic size of potential wells given by the scale of the peak of the matter power spectrum is $\sim$ 300Mpc (comoving), and the distance to last scattering is about 14000Mpc, so the number passed through is ~ 50. If the potentials are uncorrelated this would give an r.m.s. total deflection $\sim 50^{1/2} \times 10^{-4} \sim 7 \times 10^{-4}$, corresponding to about ~ 2 arcminutes. We might therefore expect the lensing to become an order unity effect on the CMB at $l \gtrsim 3000$. In fact the unlensed CMB has very little power on



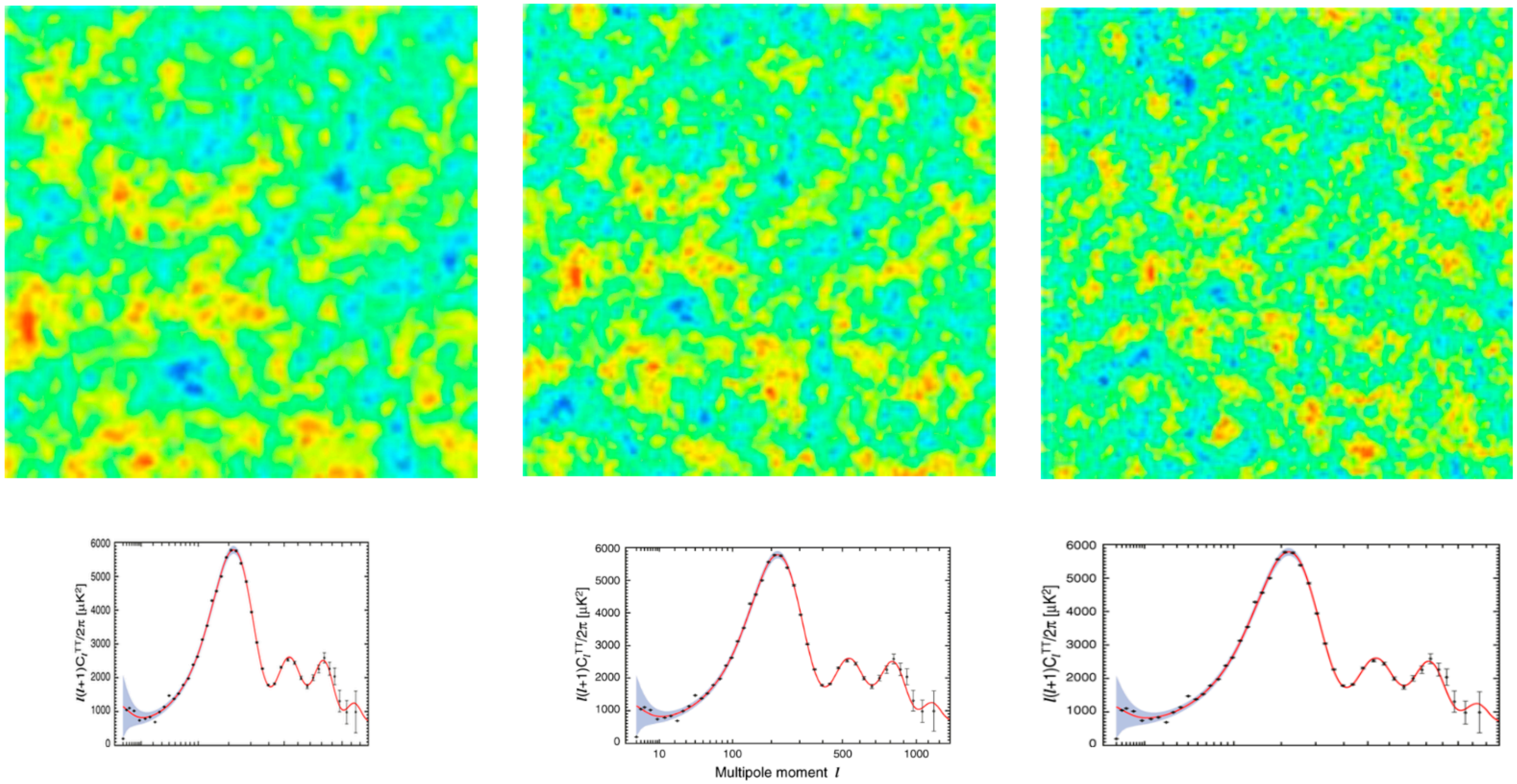


[credit: Lewis]



[credit: Lewis]

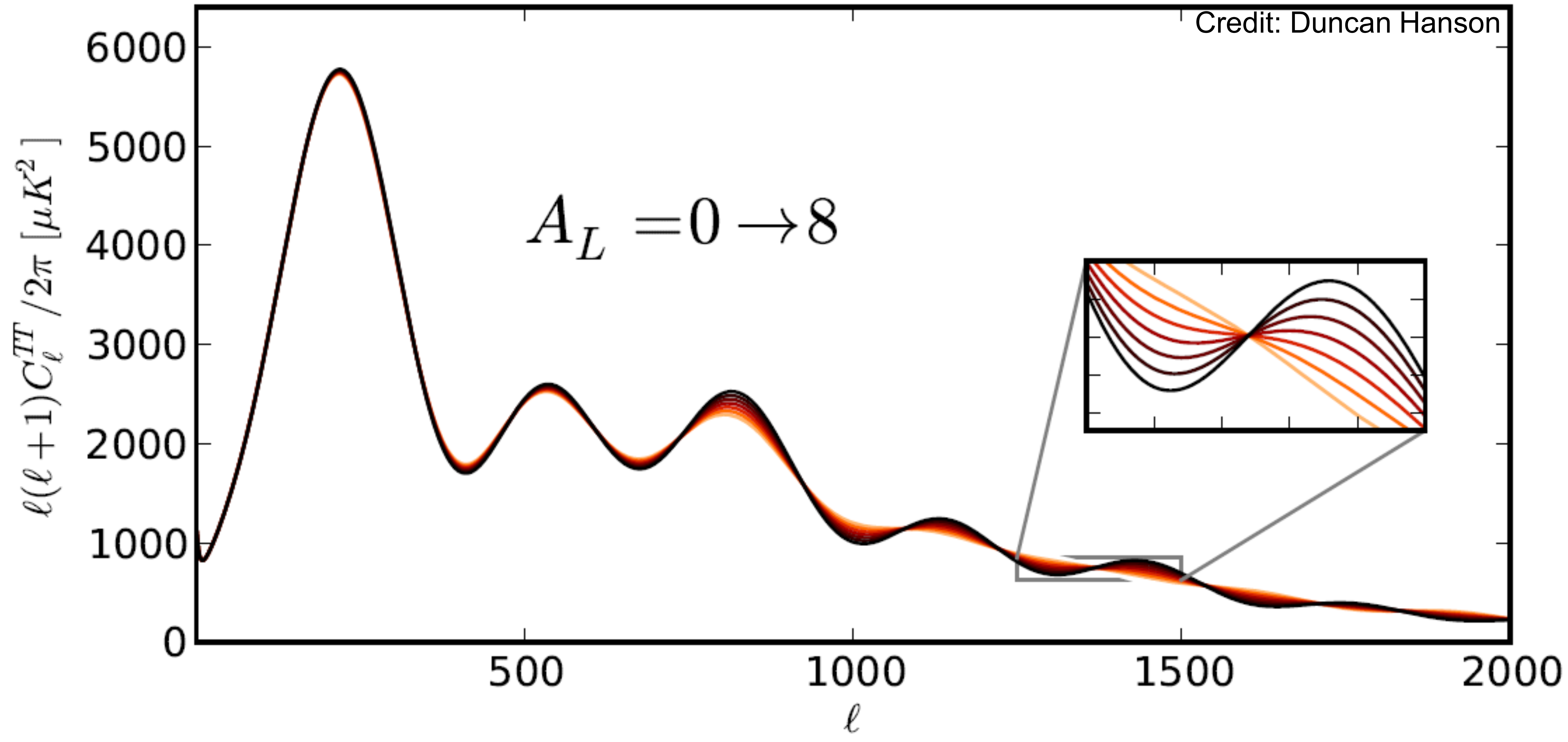
Magnification | Unlensed | Demagnification



声学峰的位置发生平移!

[credit: Lewis]

Averaged over the sky, lensing smooths out the power spectrum



[credit: Lewis]

CMB Lensing: coupling the light bundles from different direction!

$$\tilde{\Theta}(\mathbf{x}) = \Theta(\mathbf{x}') = \Theta(\mathbf{x} + \boldsymbol{\nabla}\psi)$$
$$\approx \Theta(\mathbf{x}) + \nabla^a\psi(\mathbf{x})\nabla_a\Theta(\mathbf{x}) + \tfrac{1}{2}\nabla^a\psi(\mathbf{x})\nabla^b\psi(\mathbf{x})\nabla_a\nabla_b\Theta(\mathbf{x}) + \dots$$

$$\boldsymbol{\nabla}\psi(\mathbf{x}) = i\int \frac{\mathrm{d}^2\mathbf{l}}{2\pi}\,\mathbf{l}\psi(\mathbf{l})e^{i\mathbf{l}\cdot\mathbf{x}}, \qquad \boldsymbol{\nabla}\Theta(\mathbf{x}) = i\int \frac{\mathrm{d}^2\mathbf{l}}{2\pi}\,\mathbf{l}\Theta(\mathbf{l})e^{i\mathbf{l}\cdot\mathbf{x}}.$$

Taking the Fourier transform of $\tilde{\Theta}(\mathbf{x})$ and substituting we get the Fourier comp
second order in ψ

$$\tilde{\Theta}(\mathbf{l}) \approx \Theta(\mathbf{l}) - \int \frac{\mathrm{d}^2\mathbf{l}'}{2\pi}\,\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\psi(\mathbf{l}-\mathbf{l}')\Theta(\mathbf{l}')$$
$$-\frac{1}{2}\int \frac{\mathrm{d}^2\mathbf{l}_1}{2\pi}\int \frac{\mathrm{d}^2\mathbf{l}_2}{2\pi}\,\mathbf{l}_1\cdot[\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}]\,\mathbf{l}_1\cdot\mathbf{l}_2\Theta(\mathbf{l}_1)\psi(\mathbf{l}_2)\psi^*(\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}).$$

Lensing will introducing non-gaussianity after many realization average over lensing potentials, ie. ell modes coupling

On the other hand, for a fixed lens distribution, lensing will introduce statistical anisotropy, ie. m modes coupling.

(Normally, we assume primary CMB is gaussian and statistical isotropy)

Idea of reconstruction: using the mode-coupling!

$$< \tilde{\Theta}(\mathbf{l_1})\tilde{\Theta}(\mathbf{l_2}) > \neq 0 \qquad \text{for} \qquad \mathbf{l_1} \neq \mathbf{l_2}$$

do some calculation:

1. different primary CMB map lensed by A fixed lensing field

2. estimate the 1pt function of the lensing potential

3. calculate the 2pt function of the lensing potential
(understand the noise nature of the lensing potential)

https://arxiv.org/abs/astro-ph/0601594v4

Lensing reconstruction

1. Lensing noise level estimation

2. Lensing reconstruction

3. Delensing

primary CMB

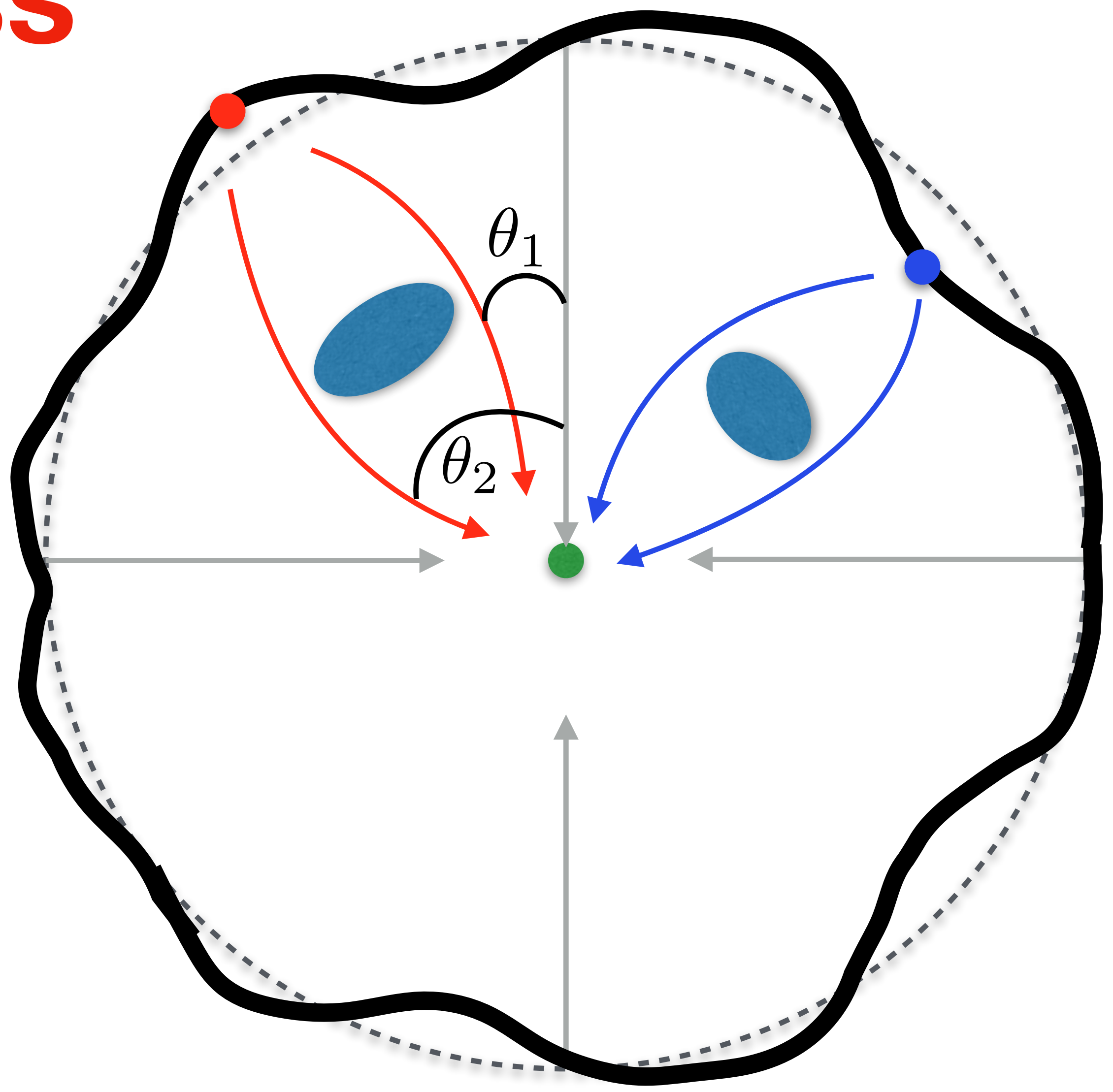
unavoidable

$$\langle d^*_\alpha(\mathbf{L}) d_\beta(\mathbf{L}')\rangle = (2\pi)^2 \delta(\mathbf{L}-\mathbf{L}')[C_L^{dd} + N_{\alpha\beta}(L)]$$

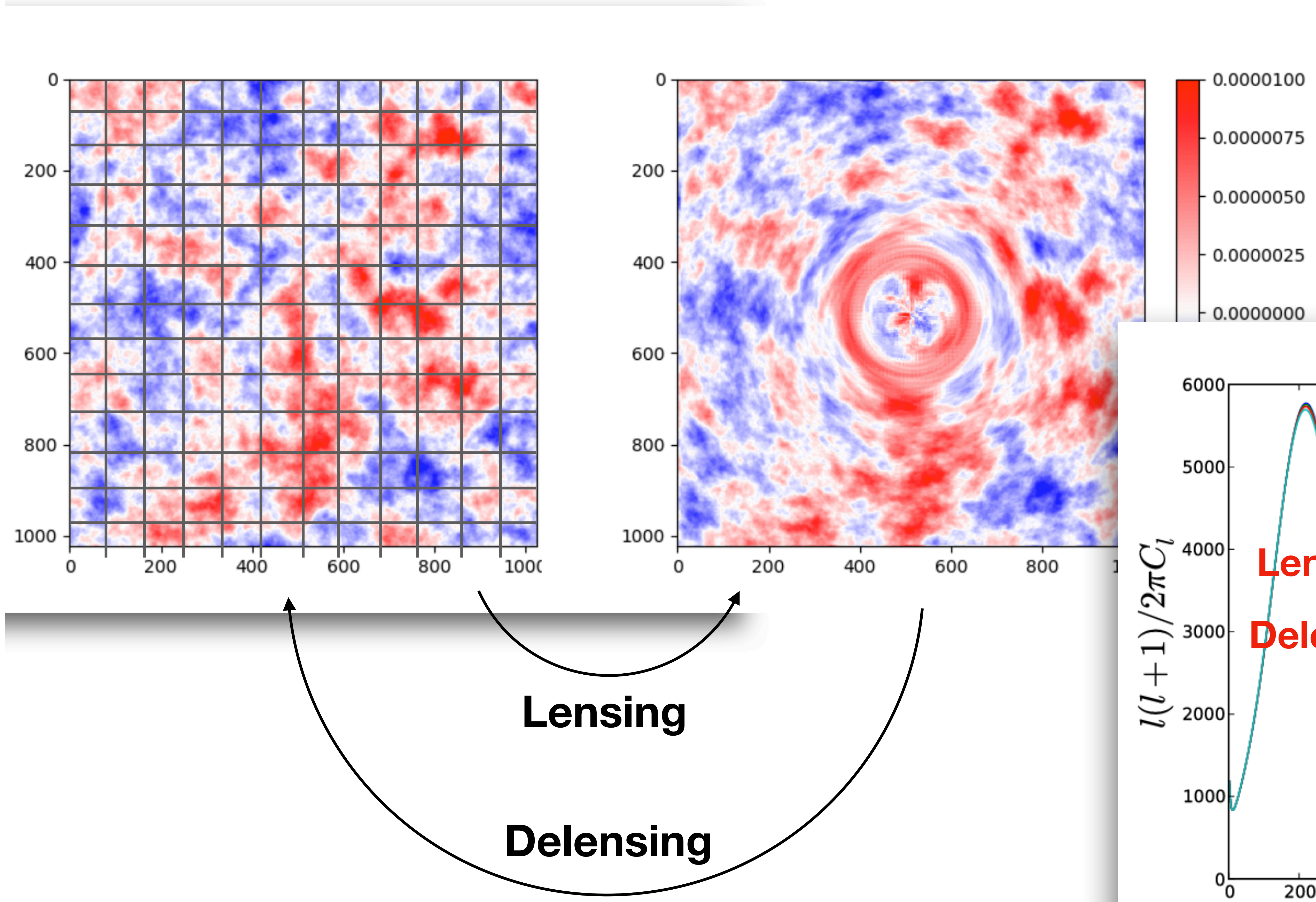
$$\hat{C}_L^{\phi\phi} = \frac{1}{(2L+1)f_{sky}} \sum_M |\hat{\phi}_{LM}|^2 - \Delta C_L^{\phi\phi}|_{N_0} + \cdots$$

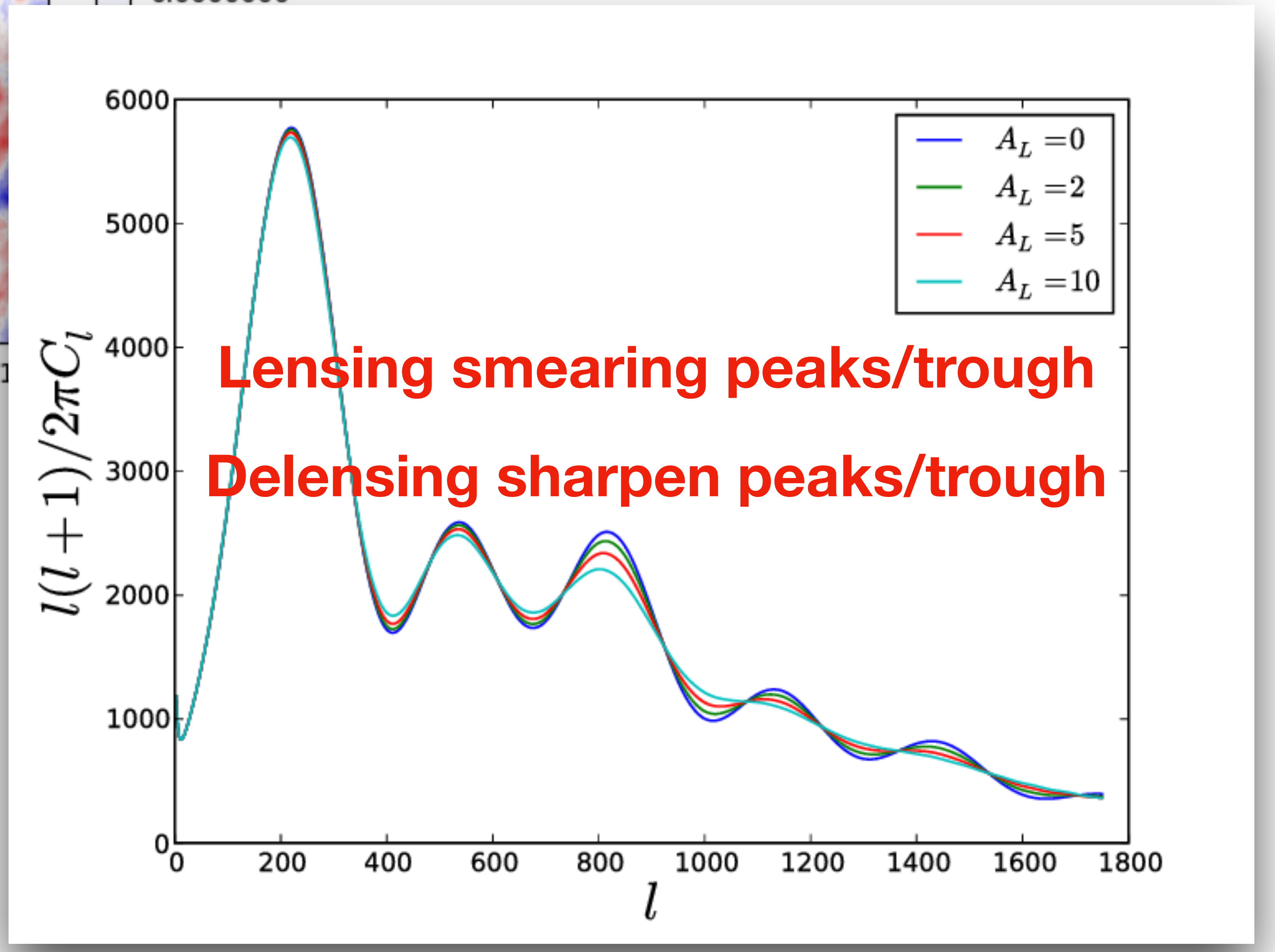
conservation of surface brightness

$$\tilde{\Theta}(\mathbf{x}) = \Theta(\mathbf{x}') = \Theta(\mathbf{x} + \boldsymbol{\nabla}\psi)$$

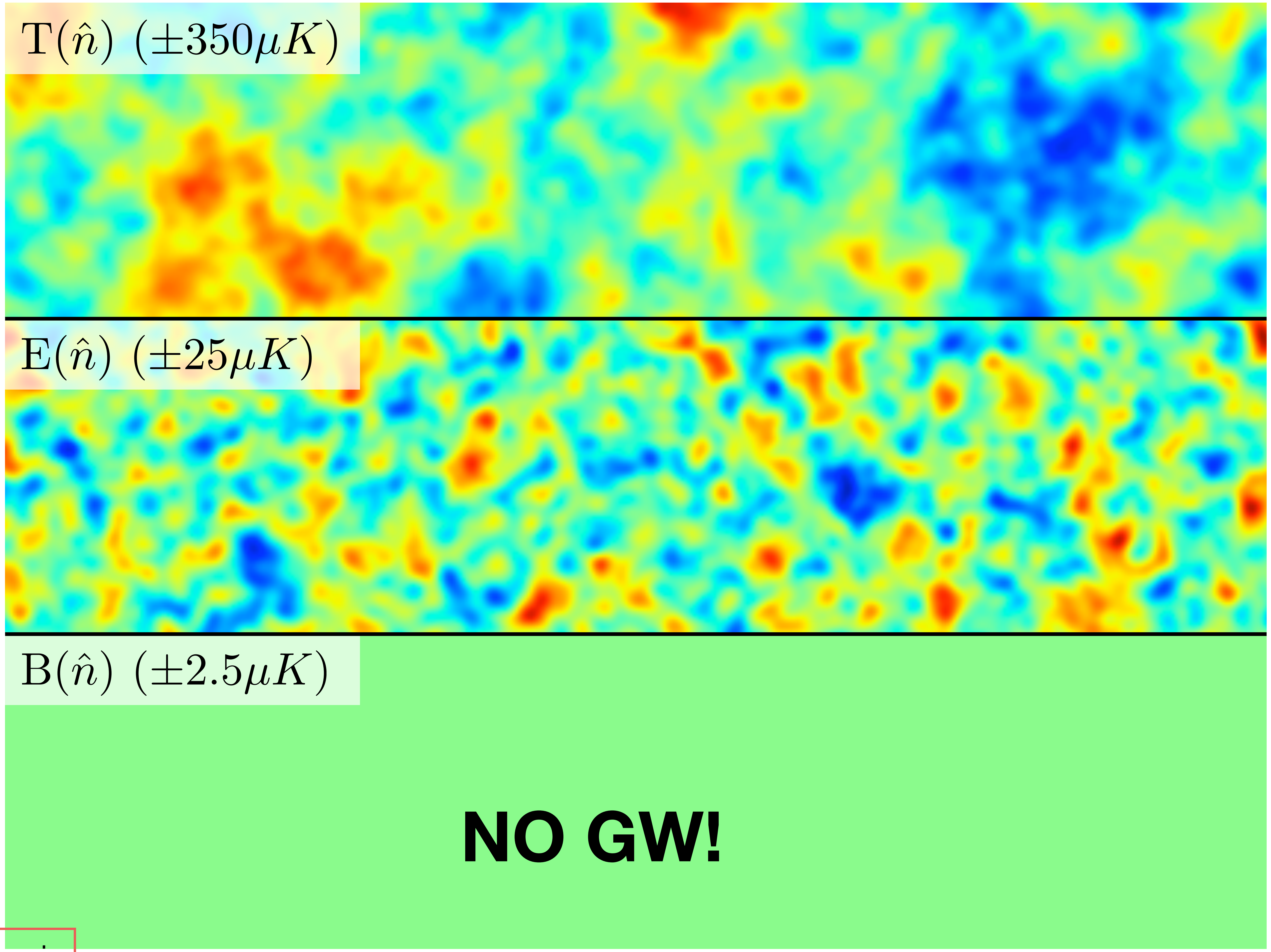


re-distribution of the primary CMB map



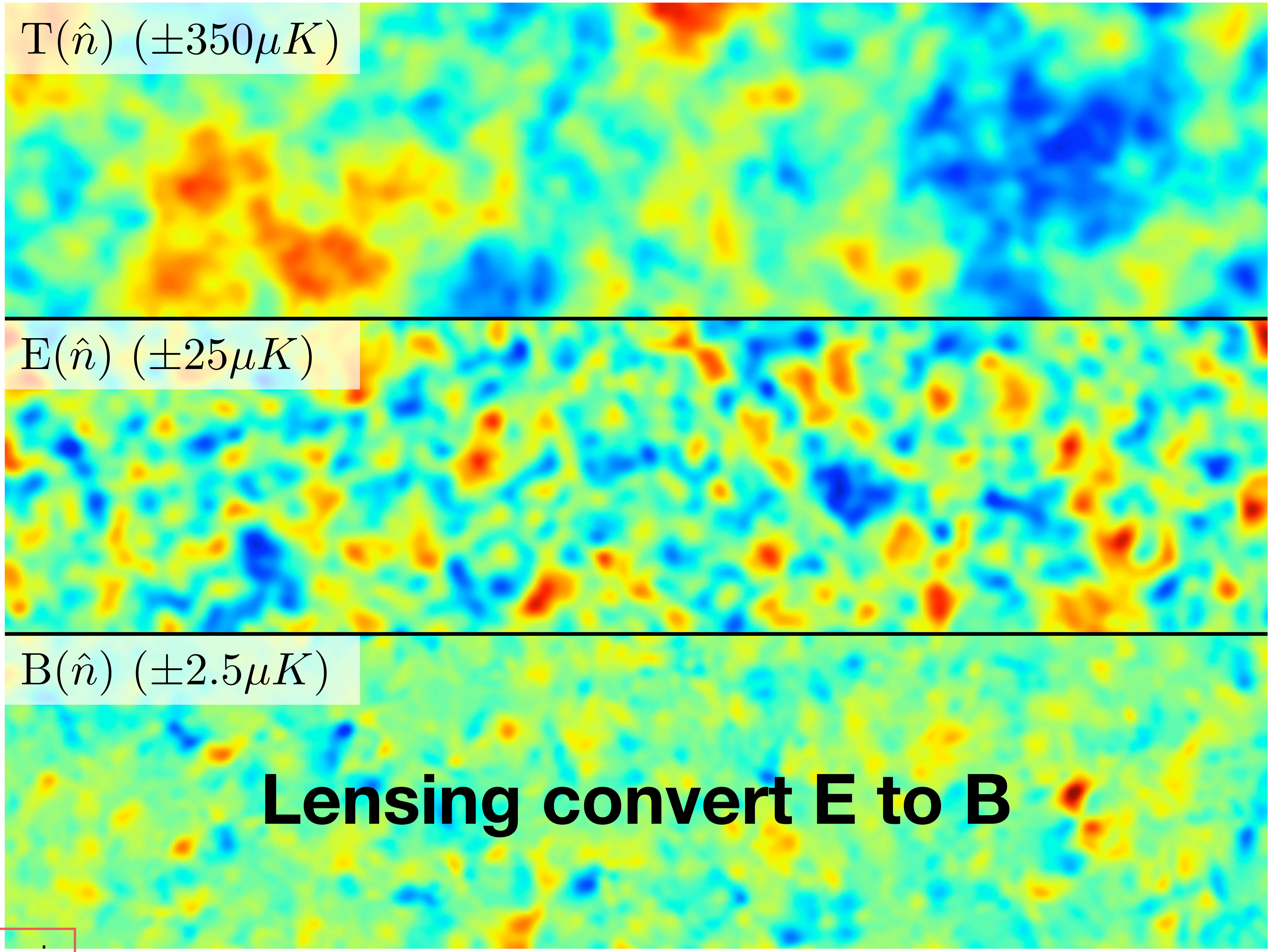


Before Lensing



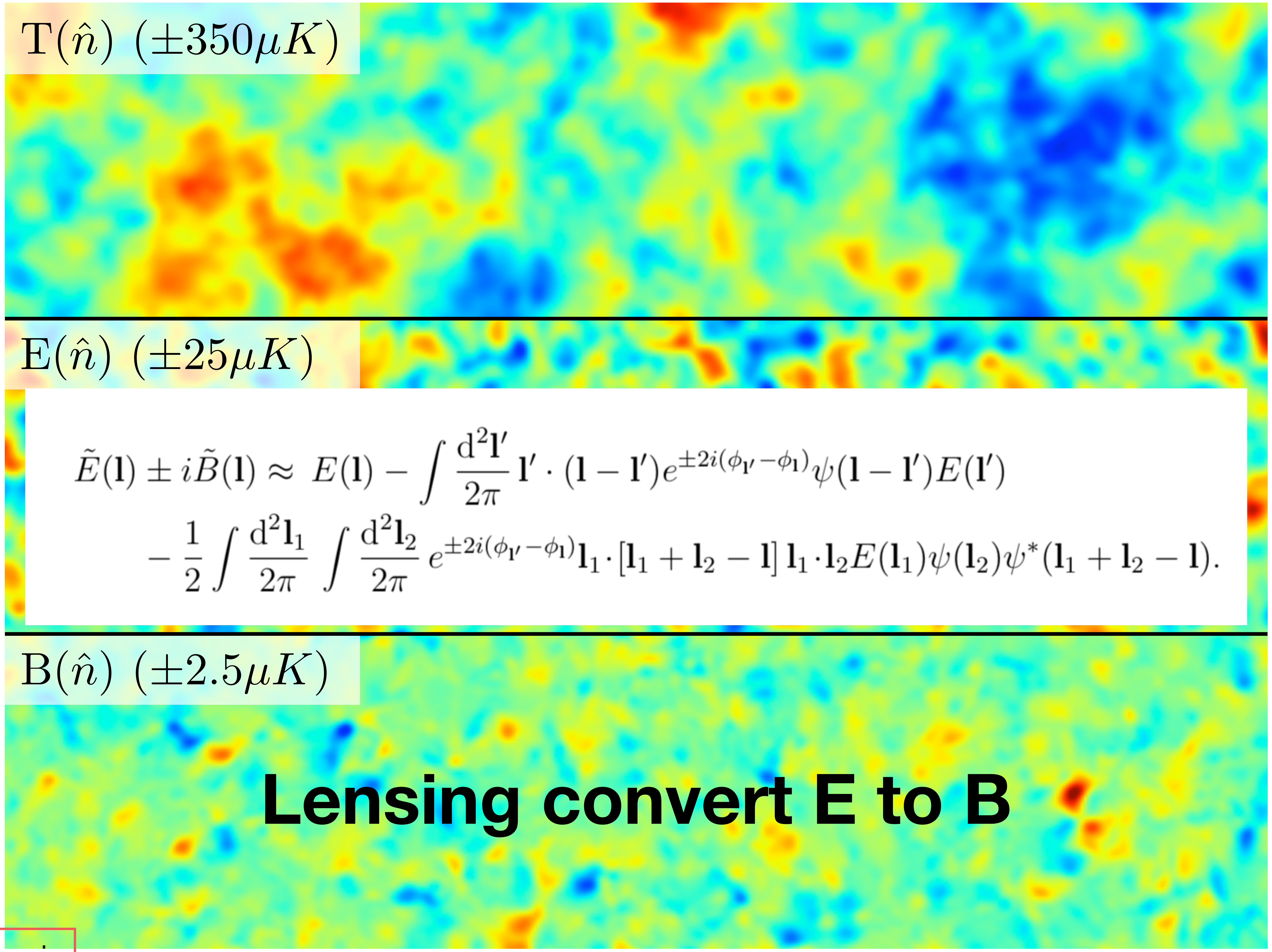
credit: A. Lewis

After Lensing



credit: A. Lewis

After Lensing

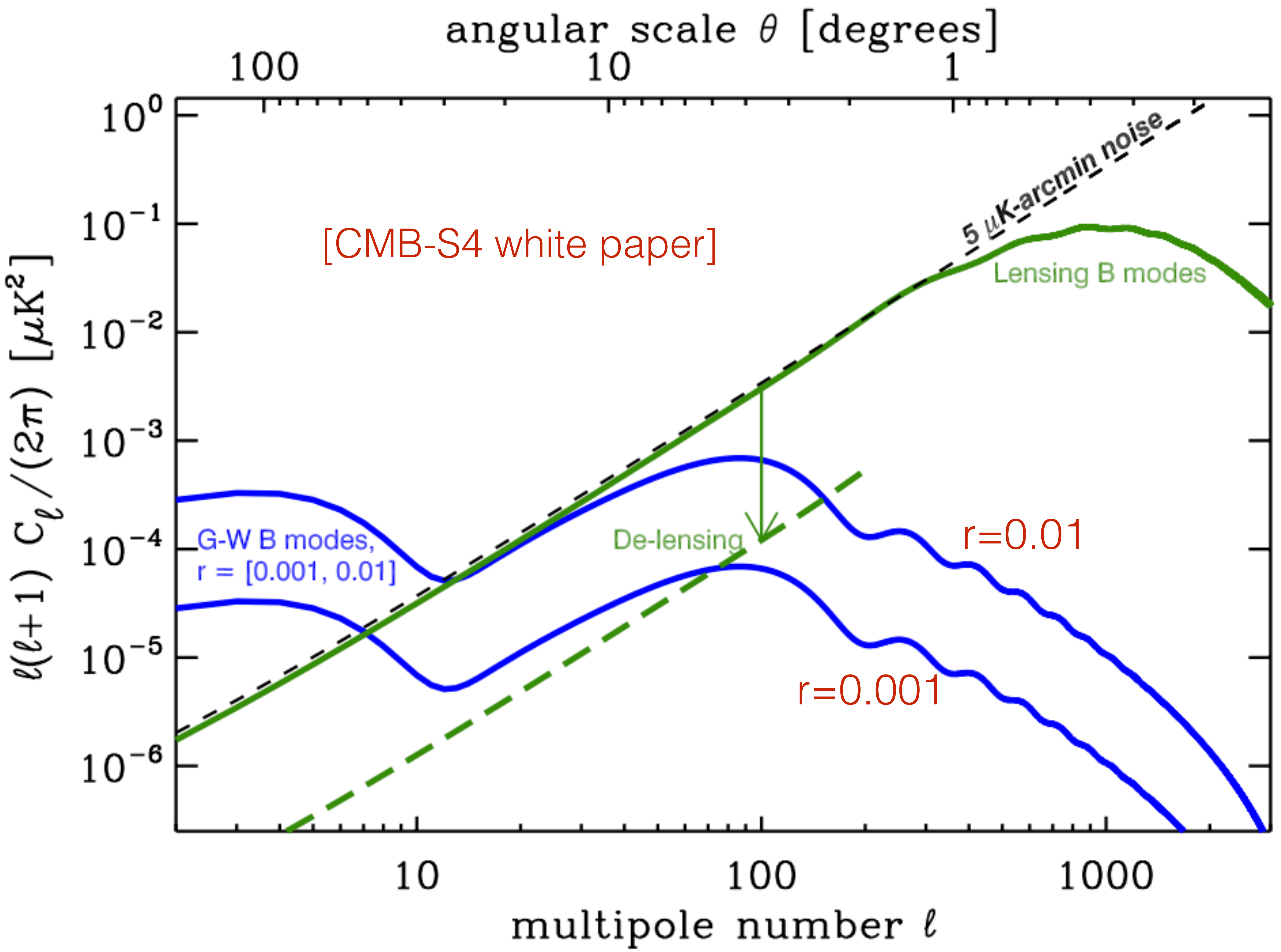


$$\tilde{E}(\mathbf{l}) \pm i\tilde{B}(\mathbf{l}) \approx E(\mathbf{l}) - \int \frac{d^2\mathbf{l}'}{2\pi} \mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}') e^{\pm 2i(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}})} \psi(\mathbf{l} - \mathbf{l}') E(\mathbf{l}')$$
$$- \frac{1}{2} \int \frac{d^2\mathbf{l}_1}{2\pi} \int \frac{d^2\mathbf{l}_2}{2\pi} e^{\pm 2i(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}})} \mathbf{l}_1 \cdot [\mathbf{l}_1 + \mathbf{l}_2 - \mathbf{l}]\, \mathbf{l}_1 \cdot \mathbf{l}_2 E(\mathbf{l}_1) \psi(\mathbf{l}_2) \psi^*(\mathbf{l}_1 + \mathbf{l}_2 - \mathbf{l}).$$

Lensing convert E to B

credit: A. Lewis

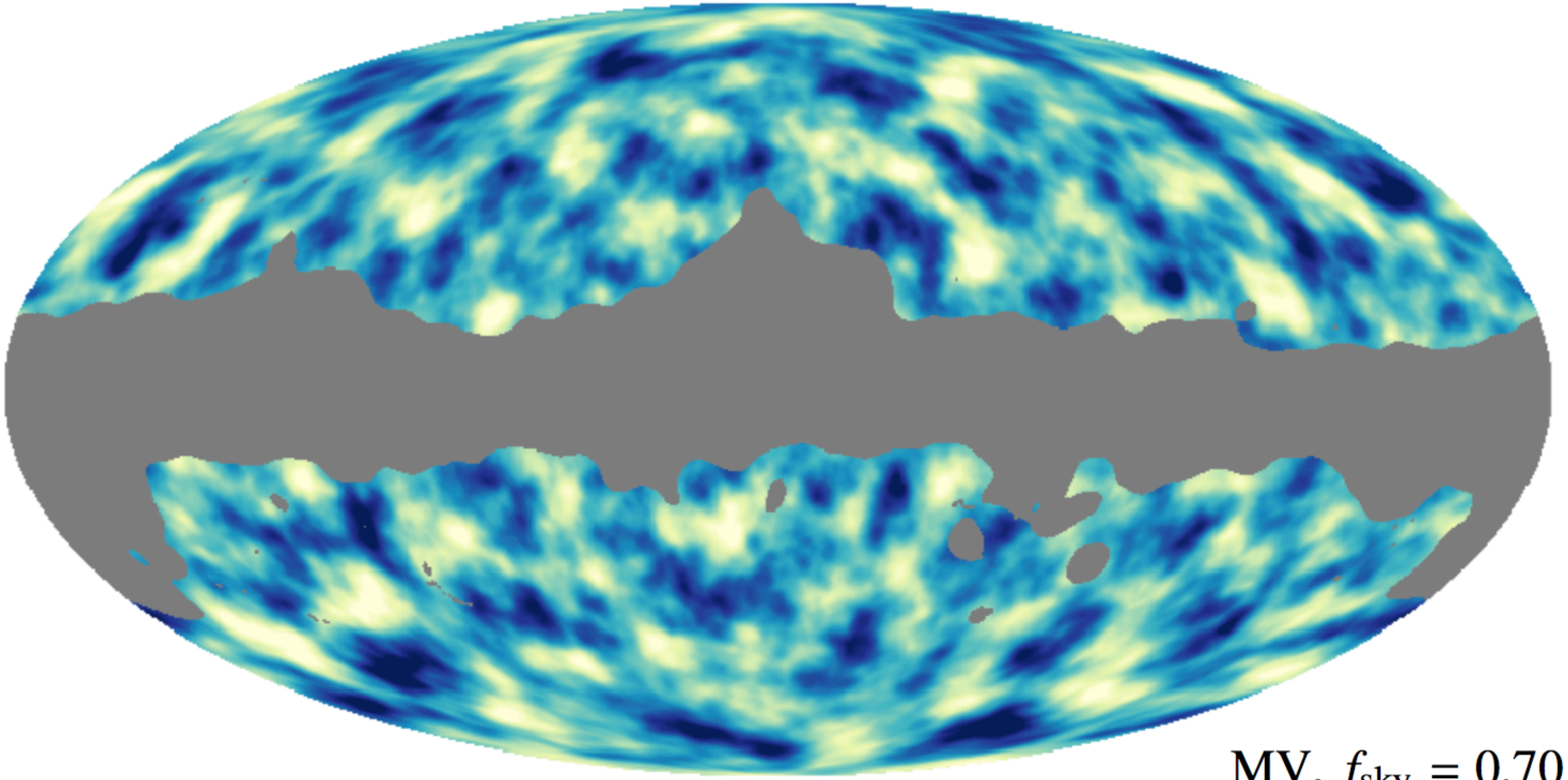
•Lensing B-mode strength: 5 uK*arcmin



r>0.01, lensing B-mode is not that much serious!

r<0.01, lensing B-mode is serious problem, need de-lensing!

However, $\hat{\phi}_{LM}$ is very very noisy!



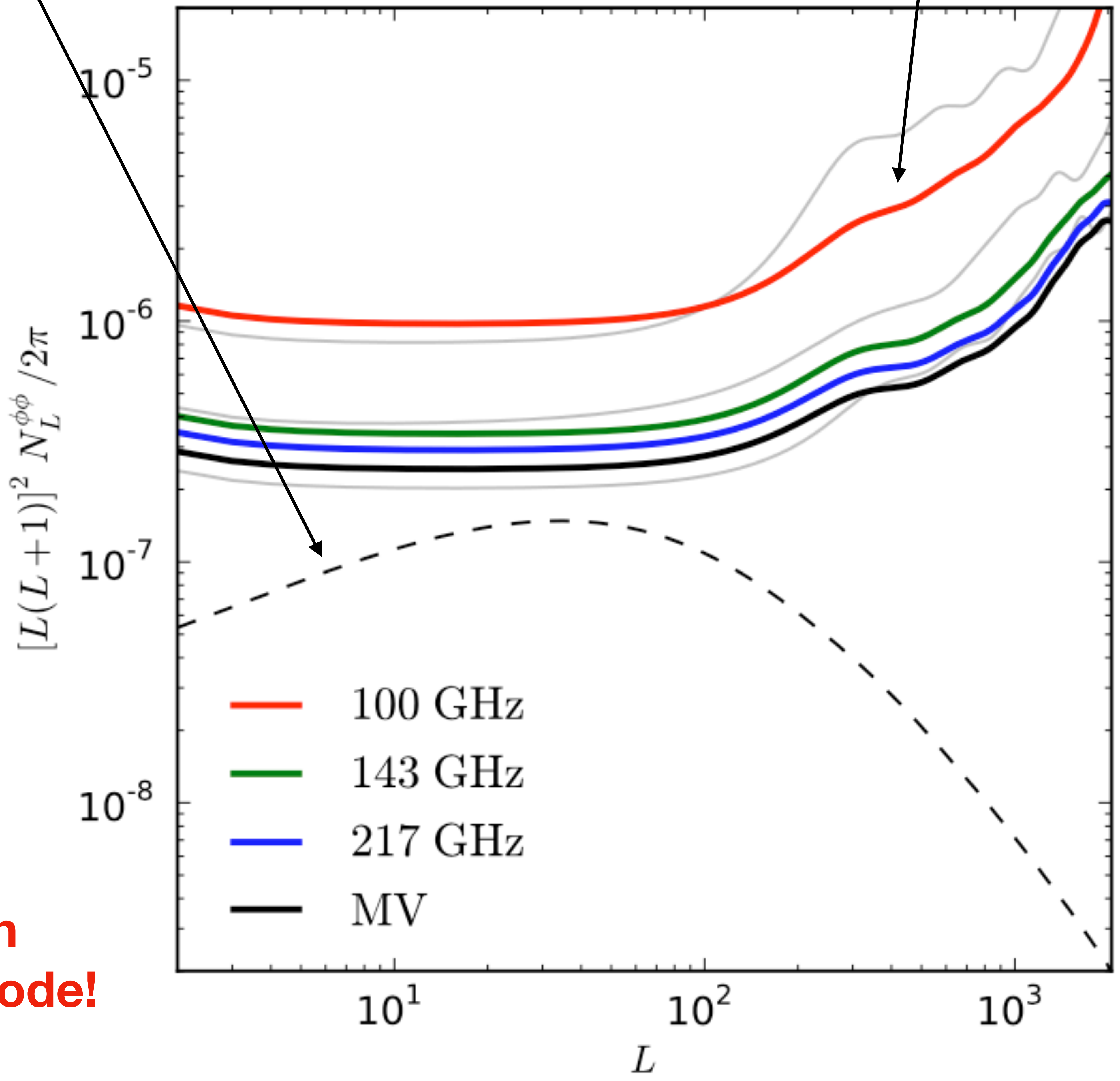
MV, $f_{\rm sky} = 0.70$

signal

estimator

unavoidable noise (from primary cmb)

$$\hat{C}_L^{\phi\phi} = \frac{1}{(2L+1)f_{sky}} \sum_M |\hat{\phi}_{LM}|^2 - \Delta C_L^{\phi\phi}|_{N_0} + \cdots$$



Even for Planck, we can NOT reach S/N >1 for each ell mode!

[Planck 2013]

Quadratic estimator

Hu-Okamoto 02'

Idea: give an optimal weight to each multiples to maximize the S/N!

$$\hat{\phi}_{LM} \sim \frac{\bar{x}_{LM}}{R_L}$$

Trispectrum (4pt)

Normalization factor

$$\bar{x}_{LM} \sim W_{\ell_1 \ell_2 L} \bar{T}_{\ell_1 m_1} \bar{T}_{\ell_2 m_2}$$

$$R_L \sim W^2_{\ell_1 \ell_2 L}$$

Window Func

$$\hat{\phi}_{LM} \sim \frac{(\bar{T}\bar{T})(\bar{T}\bar{T}\phi)}{(\bar{T}\bar{T})^2} \sim \phi$$

[credit: Jinyi Liu]