

<https://github.com/hubinitp/CHAM>

CHAM: Fast modelling non-linear matter spectrum for the modified gravity



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sCreened HAlo Model

“Screening” mechanisms:

The fifth force should act only on large scales and it should be hidden on small scales

Chameleon mechanism

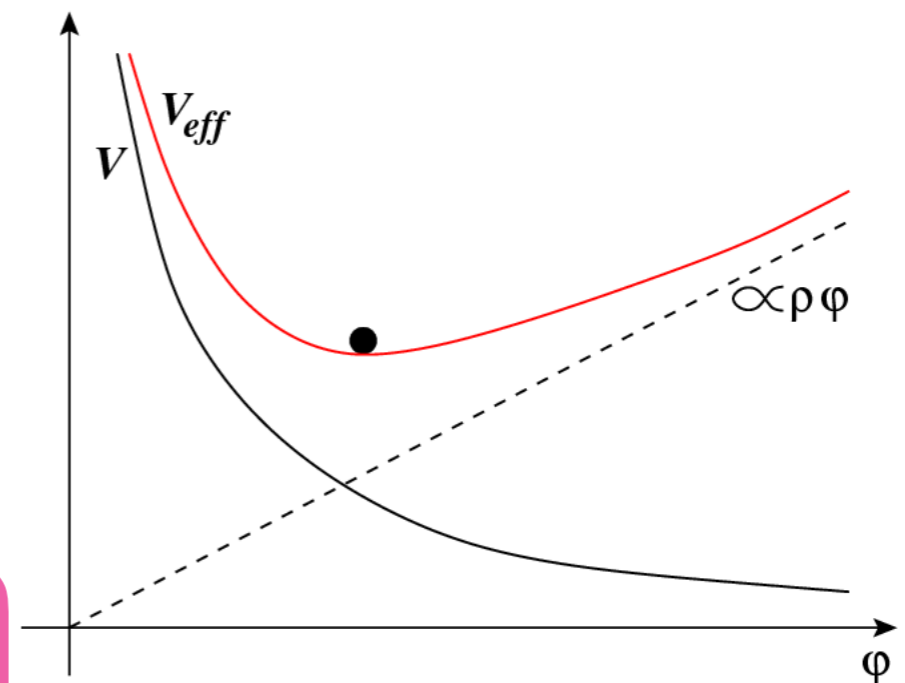
Effective Mass

Vainshtein mechanism

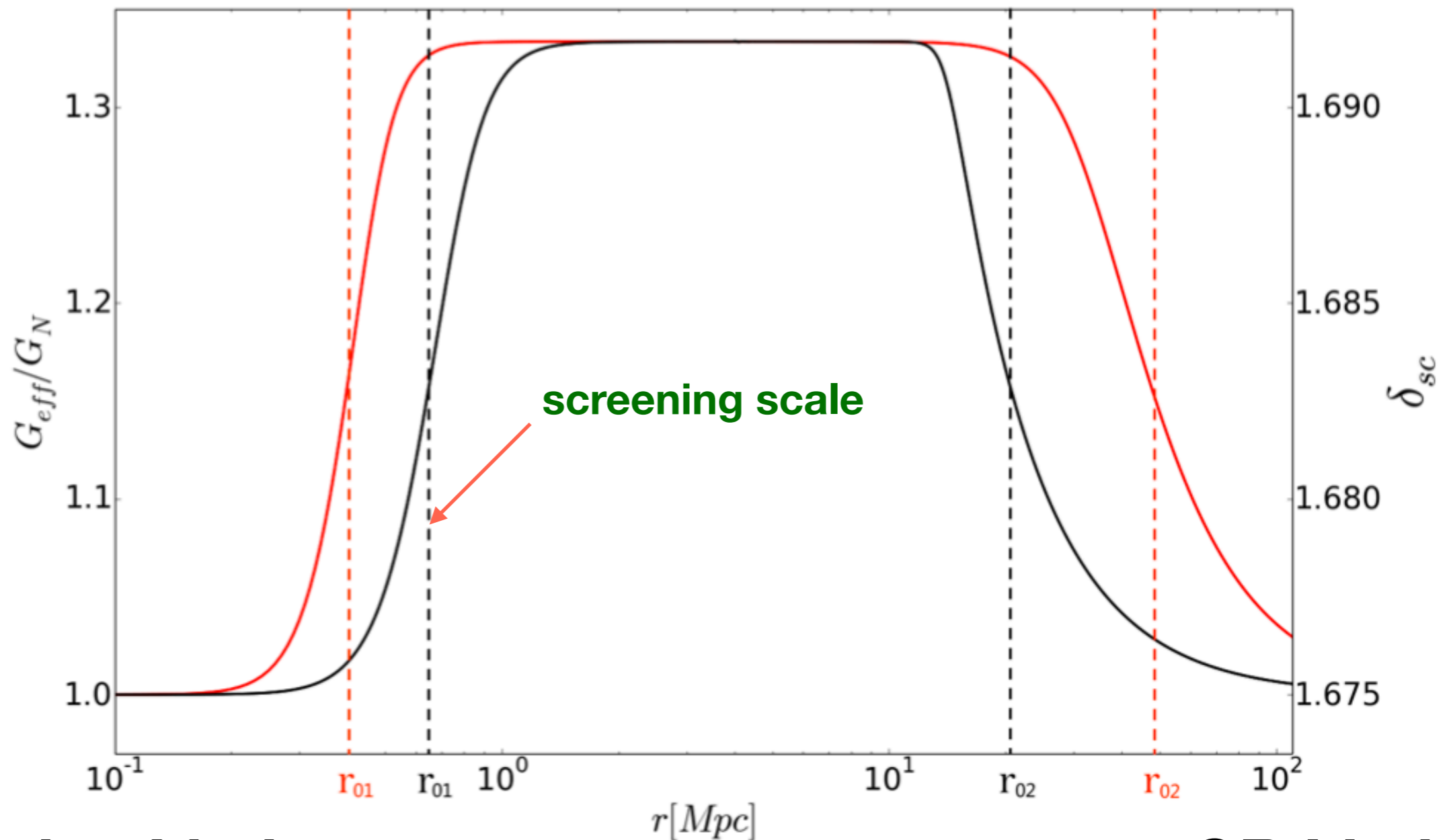
Higher order Kinetic interaction

Symmetryon mechanism

Symmetry transition in environment



Phenomenologically, MG effect shows as a **scale dependent** Newton constant



Newtonian Limit

GR Limit

Unified Parametrisation

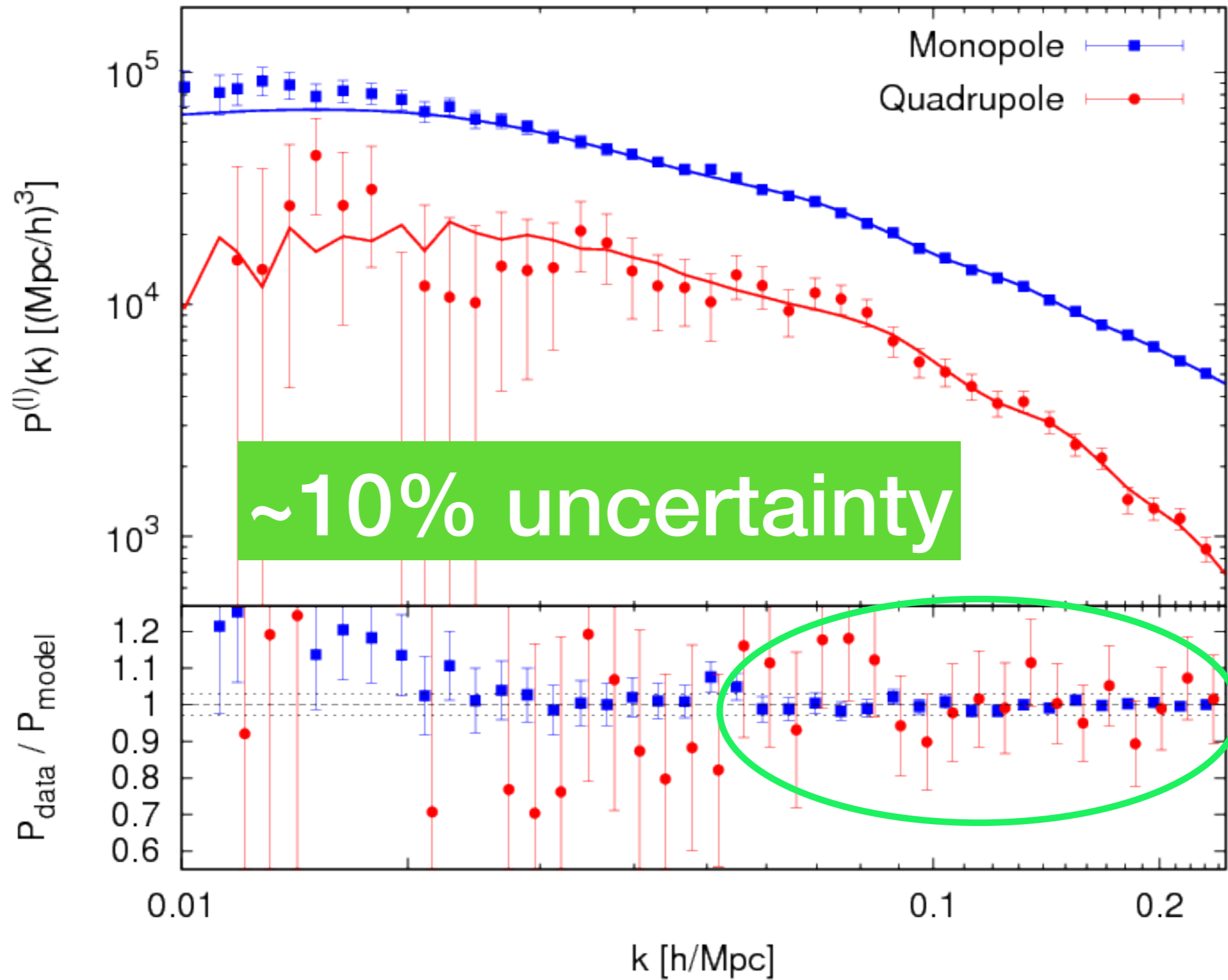
[Lucas Lombriser 1608.00522]

$$\frac{G_{eff}}{G} = A + \sum_i^{N_0} B_i \prod_j^{N_i} b_{ij} \left(\frac{r}{r_{0ij}} \right)^{a_{ij}} \left\{ \left[1 + \left(\frac{r_{0ij}}{r} \right)^{a_{ij}} \right]^{1/b_{ij}} - 1 \right\},$$

- (i) **Screening at large field values** such as in **chameleon** [4] or **symmetron** [6] models: This screening effect operates in regions where the Newtonian gravitational potential exceeds a given threshold, $|\Psi_N| > \Lambda_T$. The mapping of this mechanism to Eq. (2.12) will be described in Sec. 3.1.
- (ii) **Screening with first derivatives** such as in **k-mouflage** [5]: This screening effect is activated when the local gravitational acceleration passes a given threshold value, $|\nabla\Psi_N| > \Lambda_T^2$. The mechanism can be mapped to Eq. (2.12) as described in Sec. 3.2.
- (iii) **Screening with second derivatives** such as in the **Vainshtein mechanism** [3]: This screening mechanism operates when curvature or local densities become large, $|\nabla^2\Psi_N| > \Lambda_T^3$. The mapping of this effect onto Eq. (2.12) is presented in Sec. 3.3.
- (iv) **Linear suppression effects** such as the **Yukawa suppression** or linear shielding mechanism [8]: These effects become important when separations cross the scale set by the linearised mass or sound speed of the field. The mapping to Eq. (2.12) is provided in Sec. 3.4.

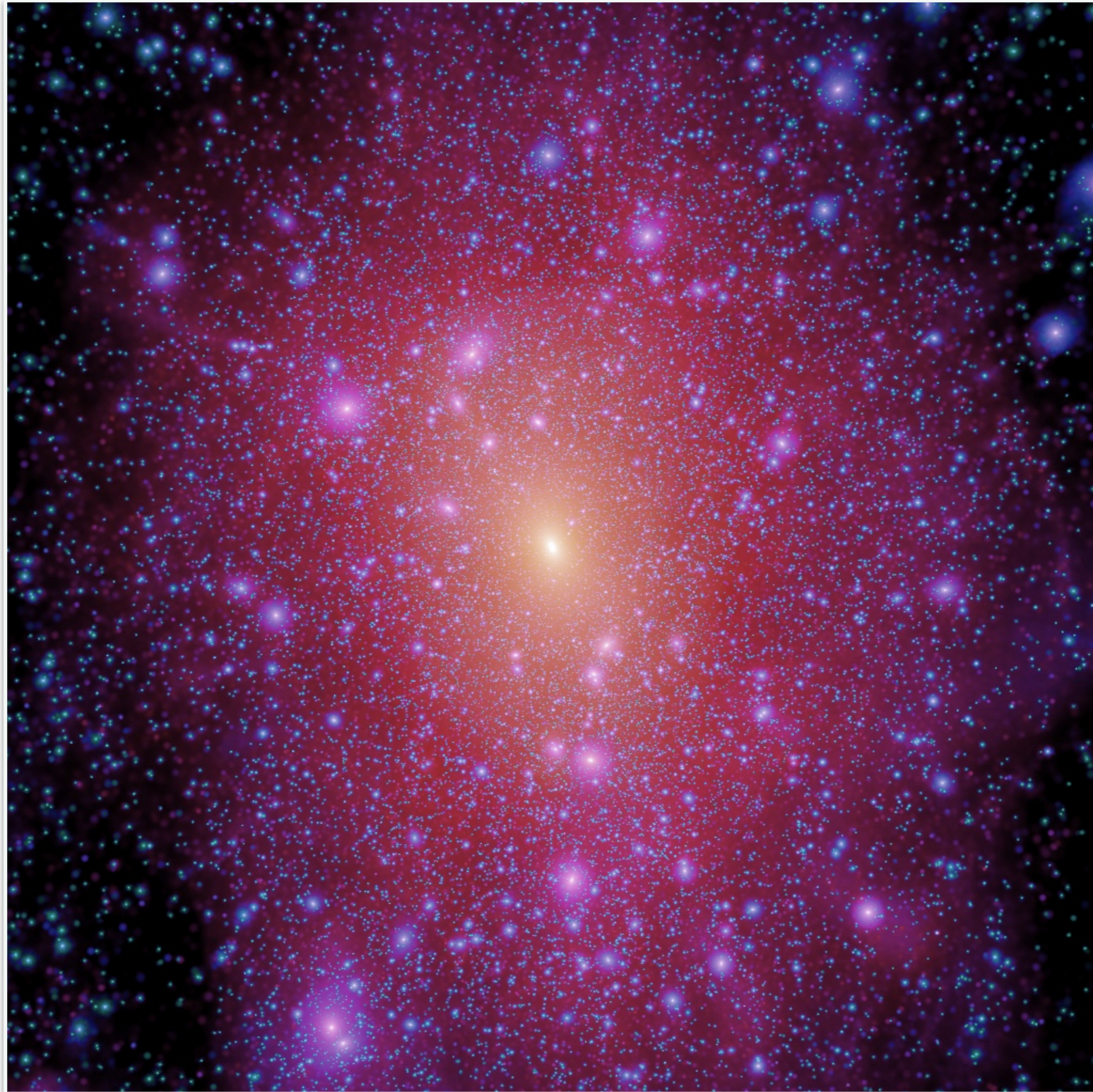
**Where we stand for
testing GR with LSS?**

SDSS LOWZ ($z_{\text{eff}}=0.32$)



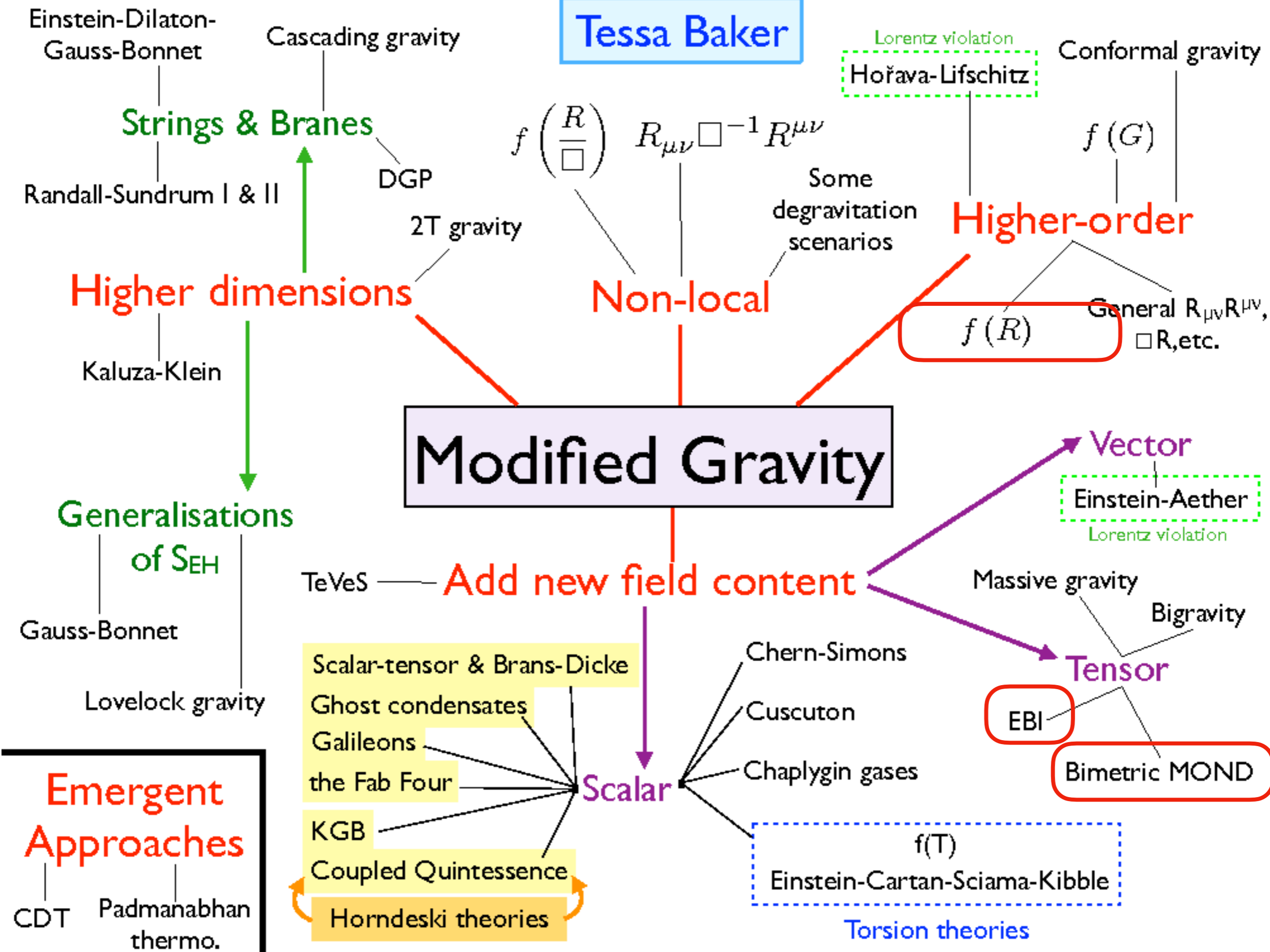
N-body simulation of MG is too much expensive: **hundreds of CPU**, **several weeks run**

For **ONE** model, **ONE** cosmology



Tessa Baker

Modified Gravity



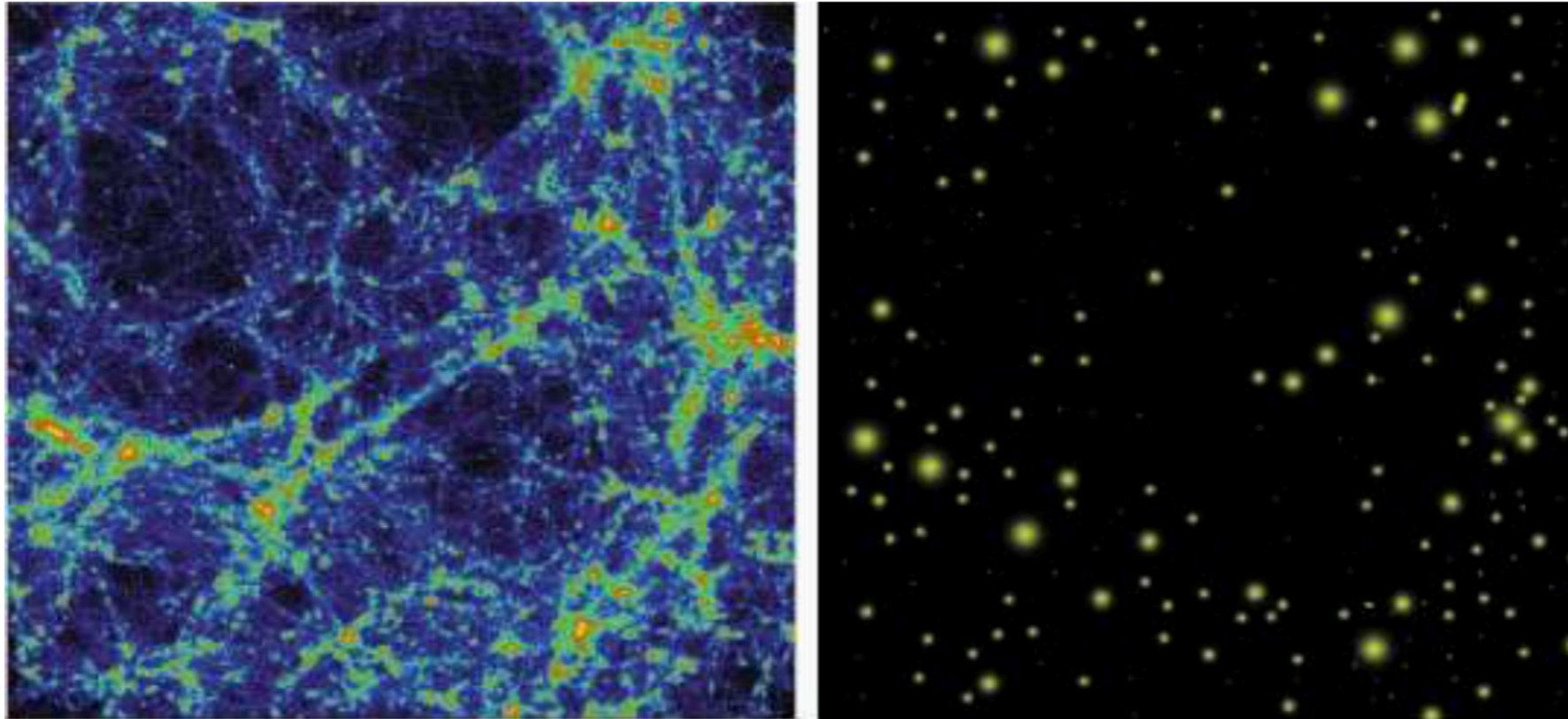
For $P(K)$:

**Is there another way
beside N-body?**

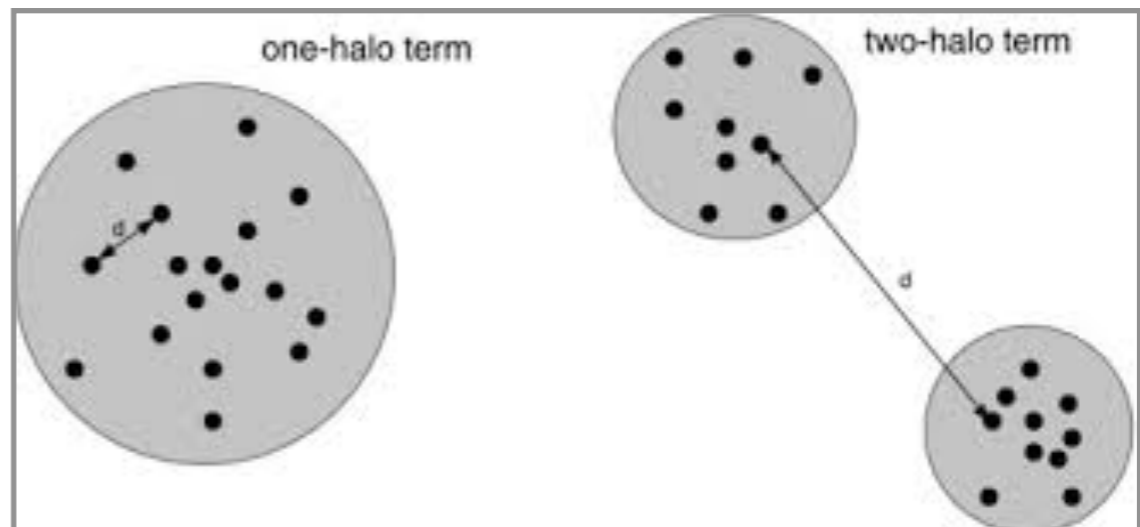
FASTER but **LESS** ACCURATE

Halo Model

discretise the continuous **matter** distribution into **halo** (bounded state)

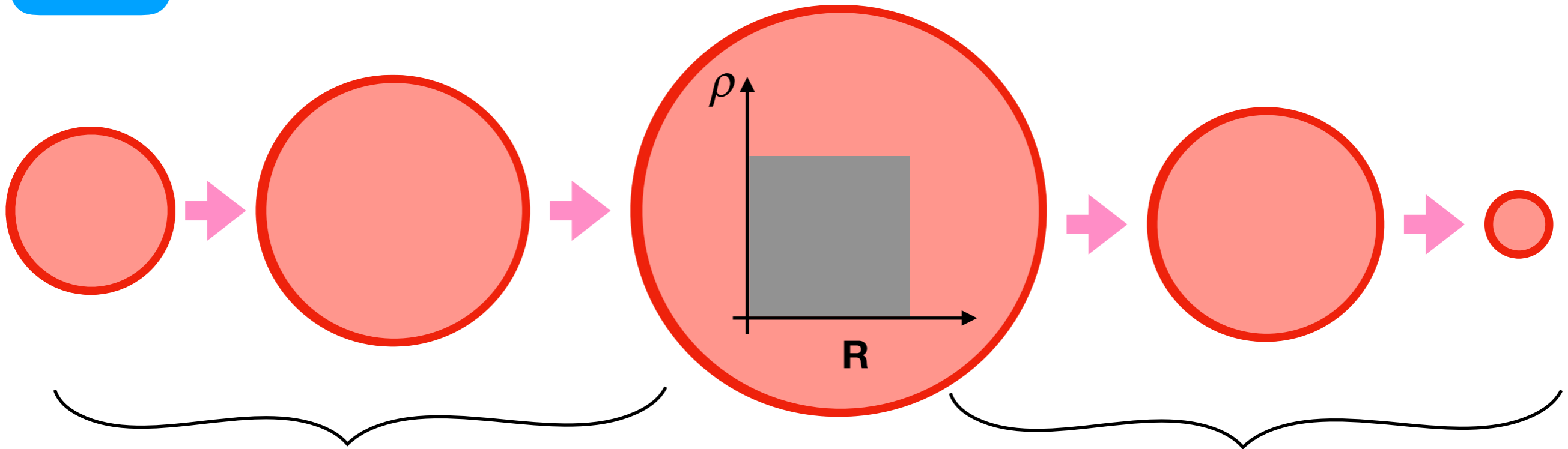


$$P_m(k) \sim P_{2halo}(k) + P_{1halo}(k)$$



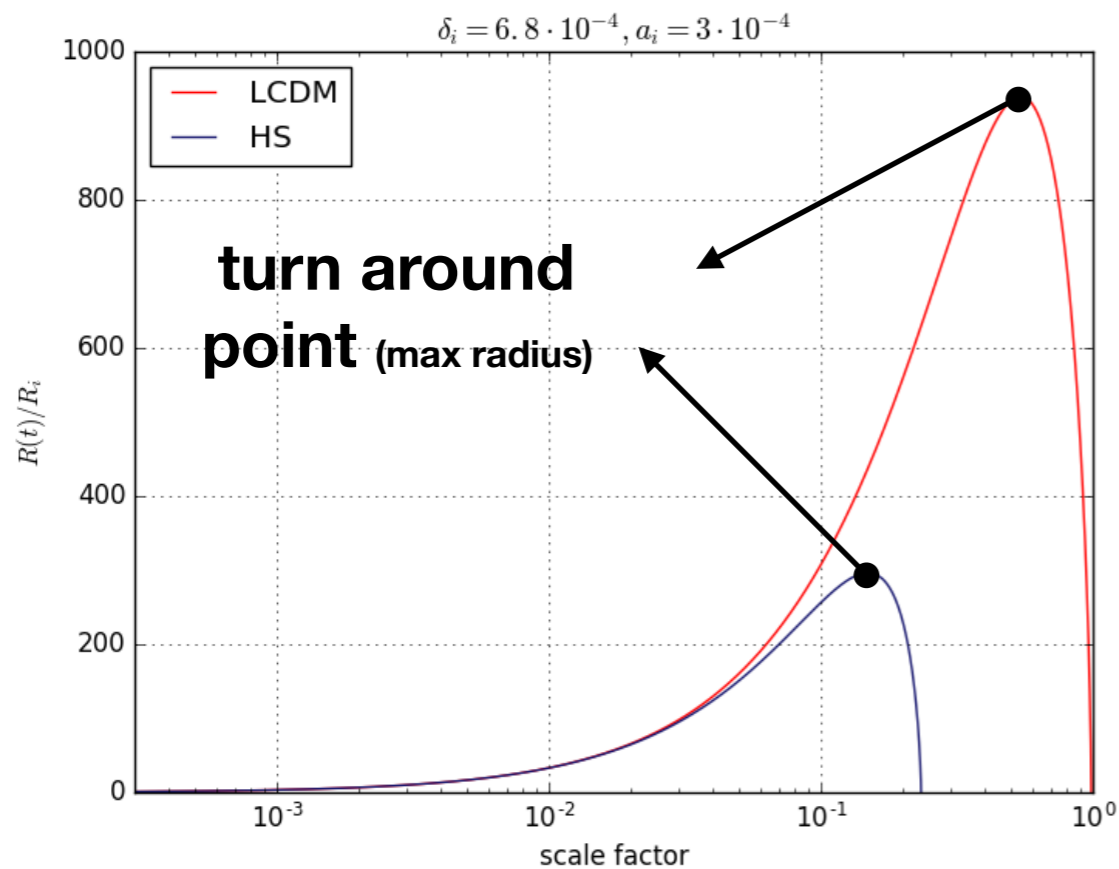
Step-1:

Halo formation – Spherical Collapse



expand with background

decouple from background expansion



$$\frac{1}{2} \left(\frac{dr}{dt} \right)^2 - \frac{GM}{r} = \mathcal{E}$$

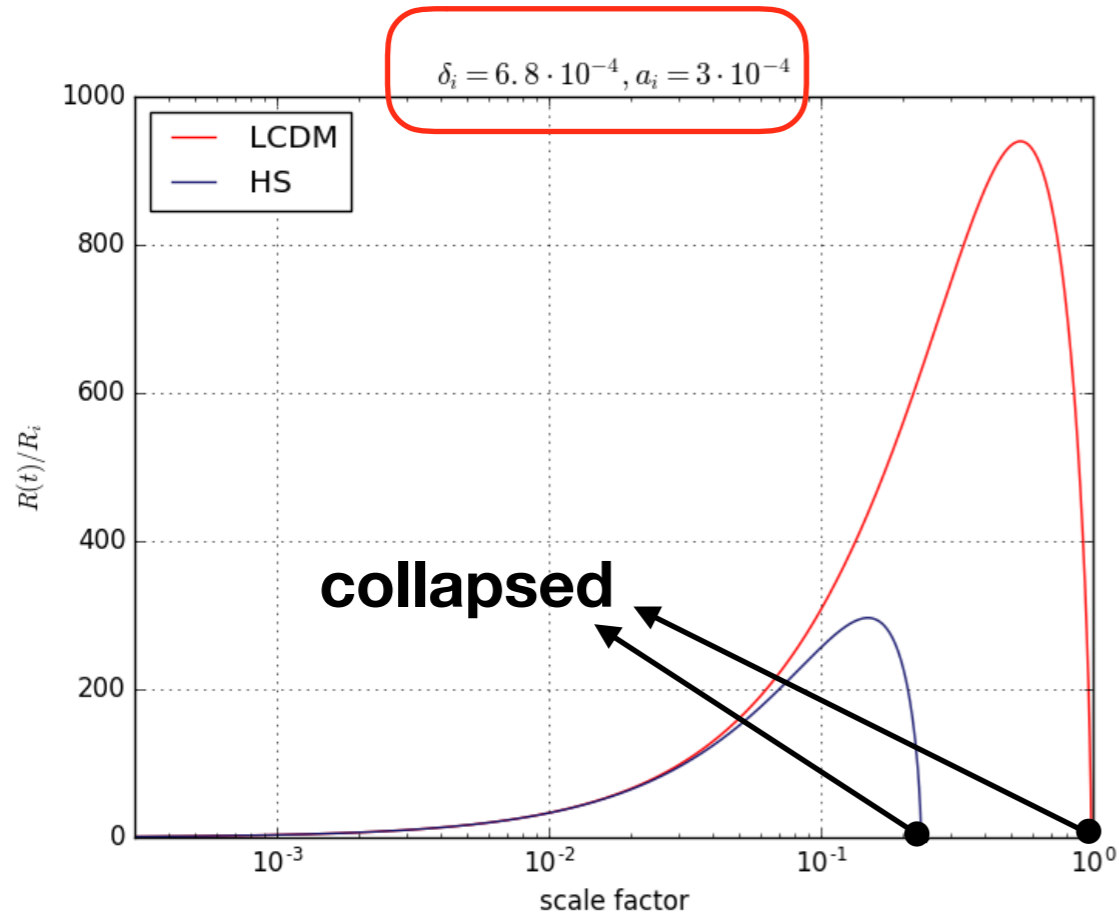
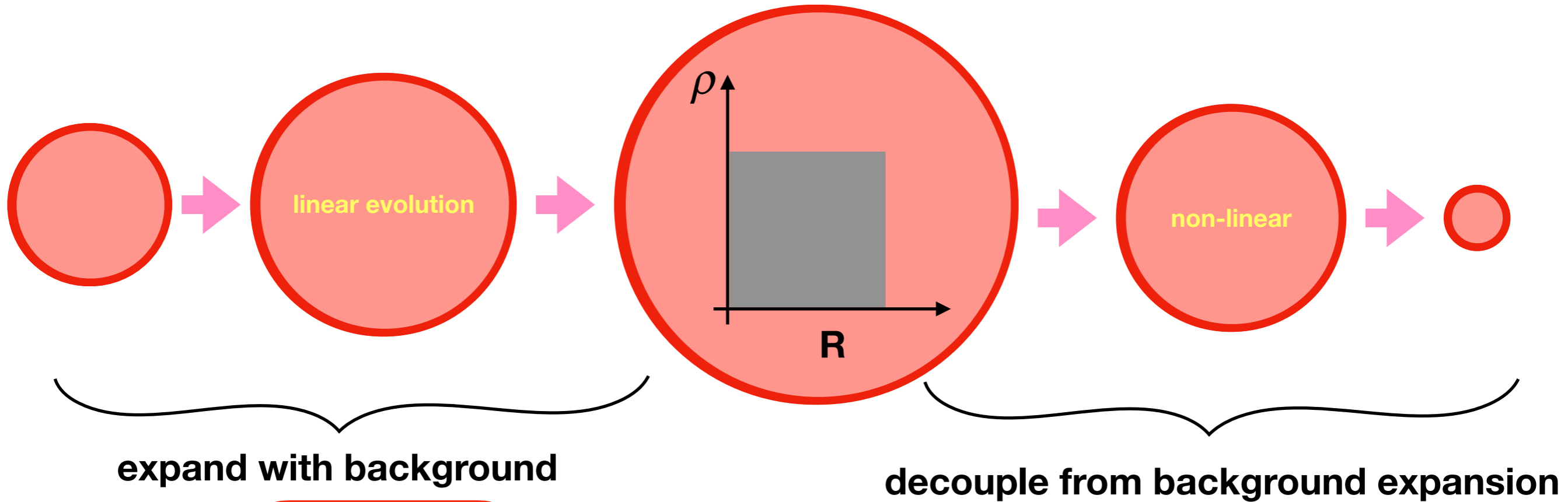
assuming initially, shell is co-moving

$$\dot{r} = Hr$$

One can prove that, in EdS universe, for arbitrary over density regime, it will always collapse!
 early collapse ~ higher density; later collapse ~ lower density

$$\mathcal{E} = -\frac{1}{2} H_i^2 r_i^2 \delta_i < 0$$

Halo formation – Spherical Collapse



linear grow in MD

$$\delta_m \sim a$$

$$\delta_{Lin}(col) \sim 1.68$$

LCDM

$$\delta_{Lin}(col) > 1.68$$

gravity enhanced $G_{eff} > G_N$

$$\delta_{Lin}(col) < 1.68$$

gravity suppressed $G_{eff} < G_N$

Compton Wavelength Scale

Poisson

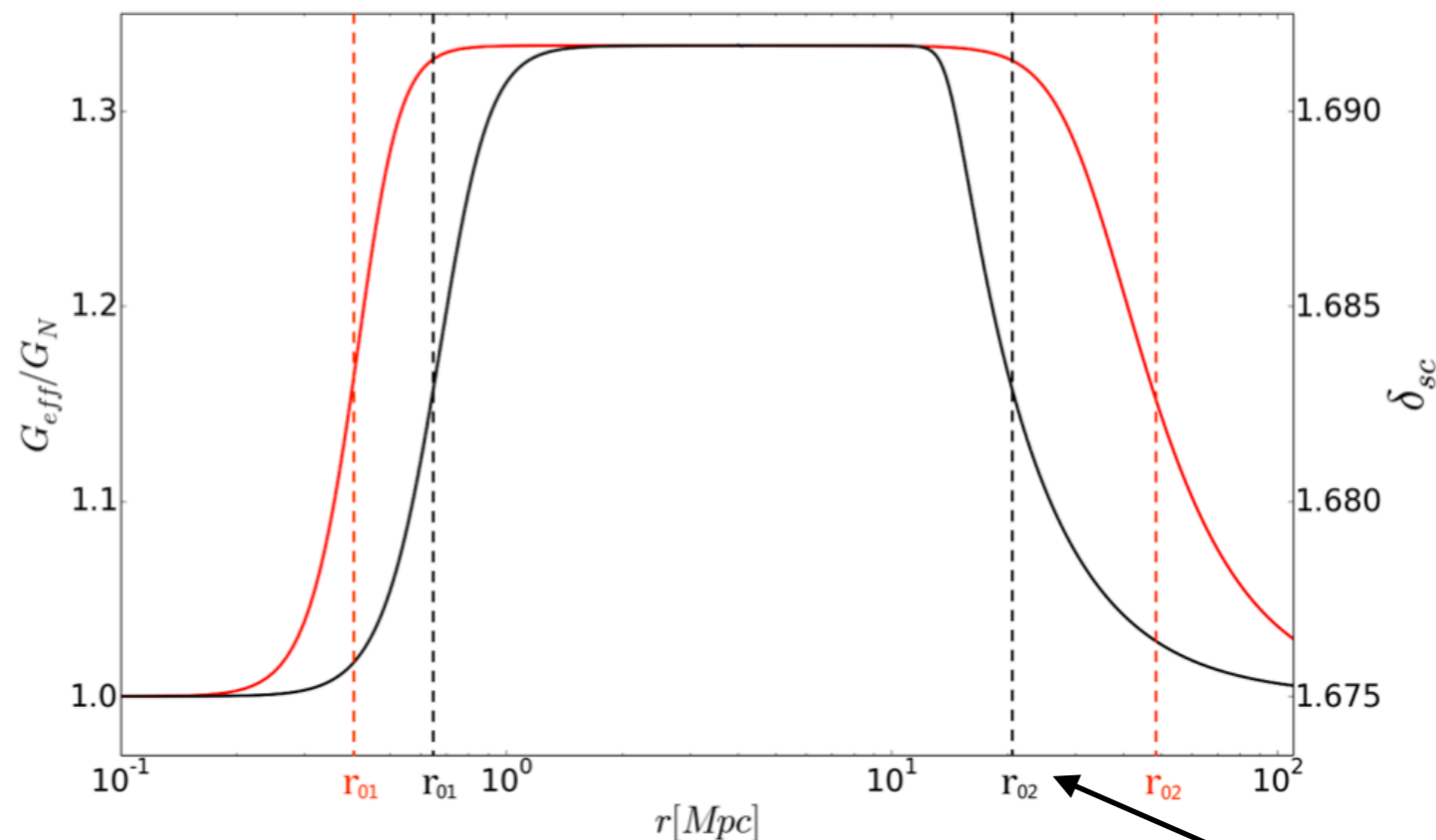
$$k^2 \Psi(\mathbf{k}) = -4\pi G \left(\frac{4}{3} - \frac{1}{3} \frac{\mu^2 a^2}{k^2 + \mu^2 a^2} \right) a^2 \delta\rho_m(\mathbf{k}),$$

$\mu^{-1} \longrightarrow$ Compton wavelength

$k^2 \gg \mu^2 a^2$ below Compton wavelength G_{eff} is enhanced by a factor **1/3**

$k^2 \ll \mu^2 a^2$ above Compton wavelength **Restore LCDM limit**

$$|f_{R0}| \sim \frac{\mu^{-1}}{H_0^{-1}}$$

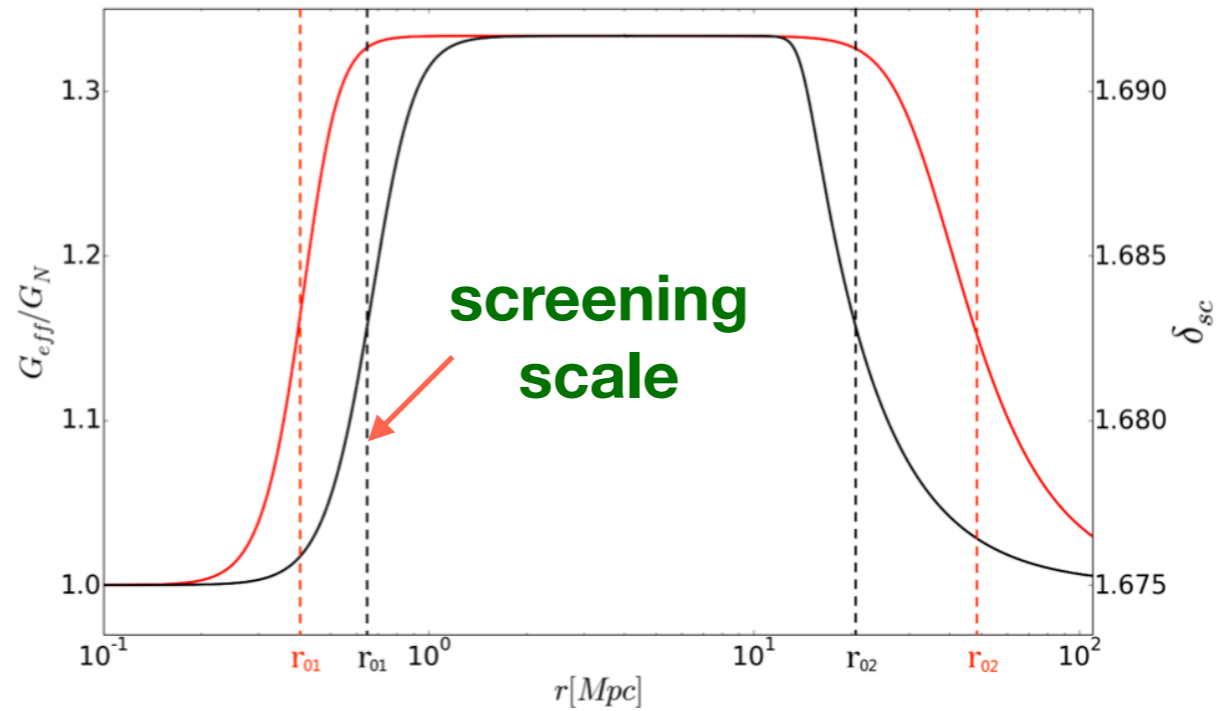


Compton scale



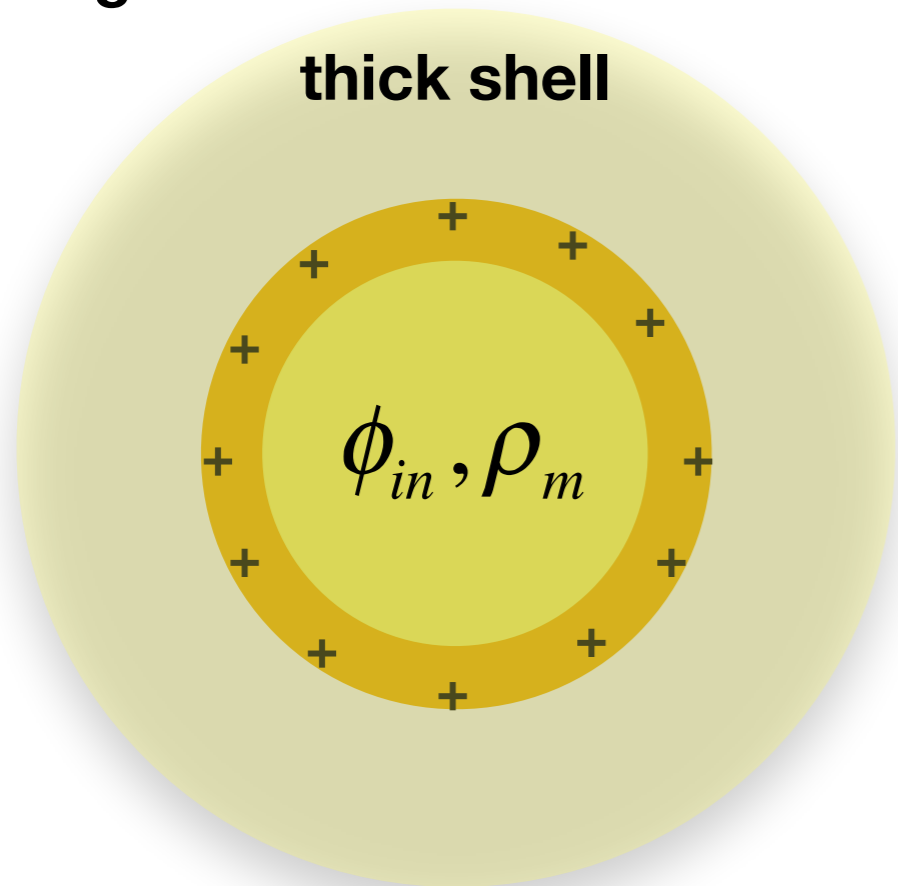
Chameleon Mechanism

Screening Scale



ξM

Charge



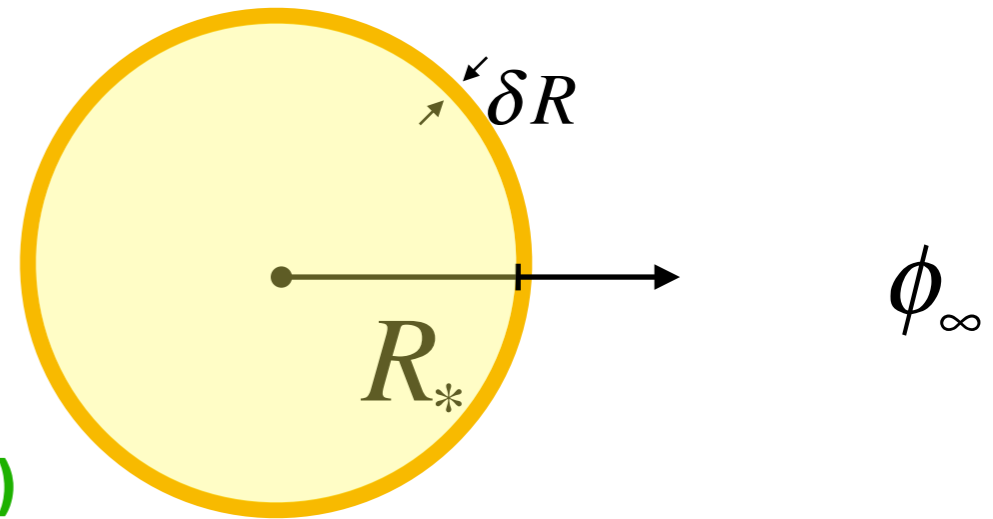
thick shell

Scalar charge is only distributed on the surface! R^2

V.S.

gravity charge (mass) is distributed in the whole volume! R^3

thin shell

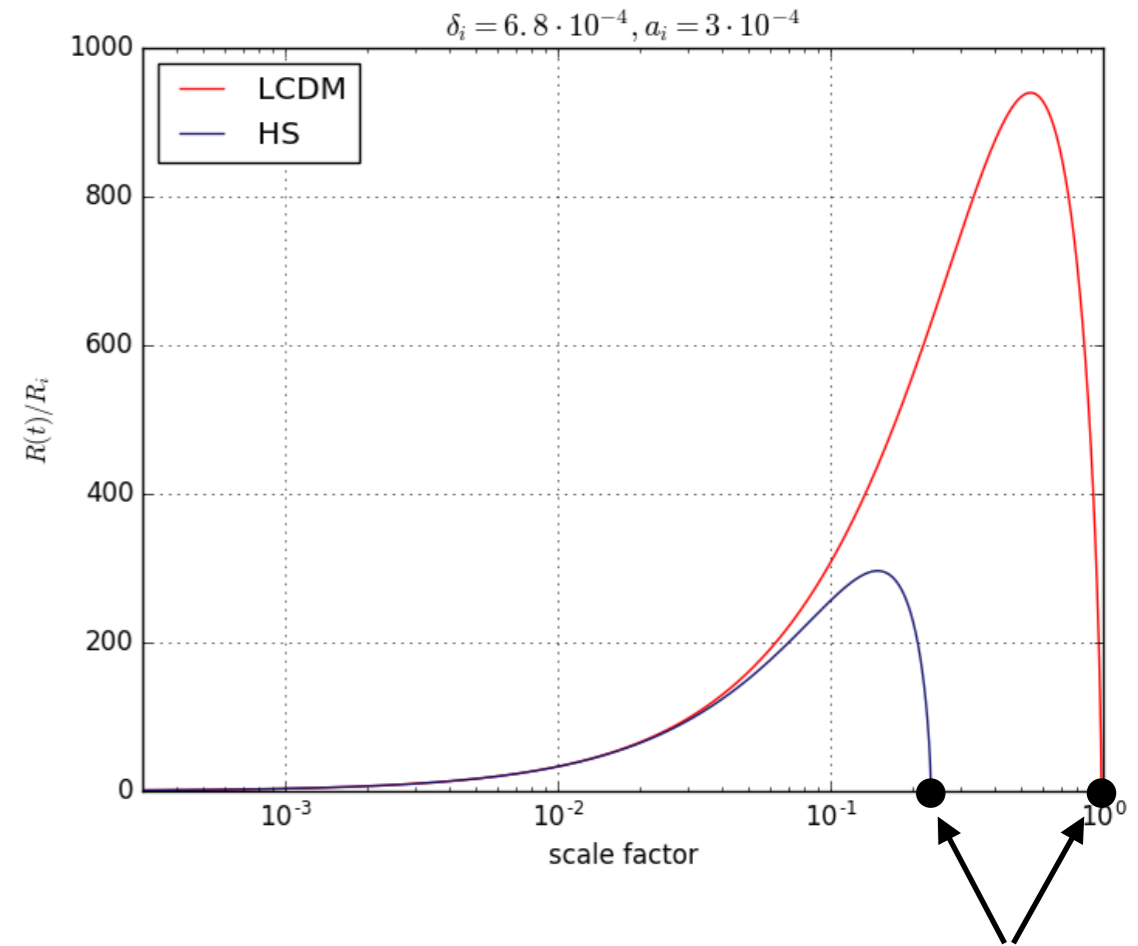
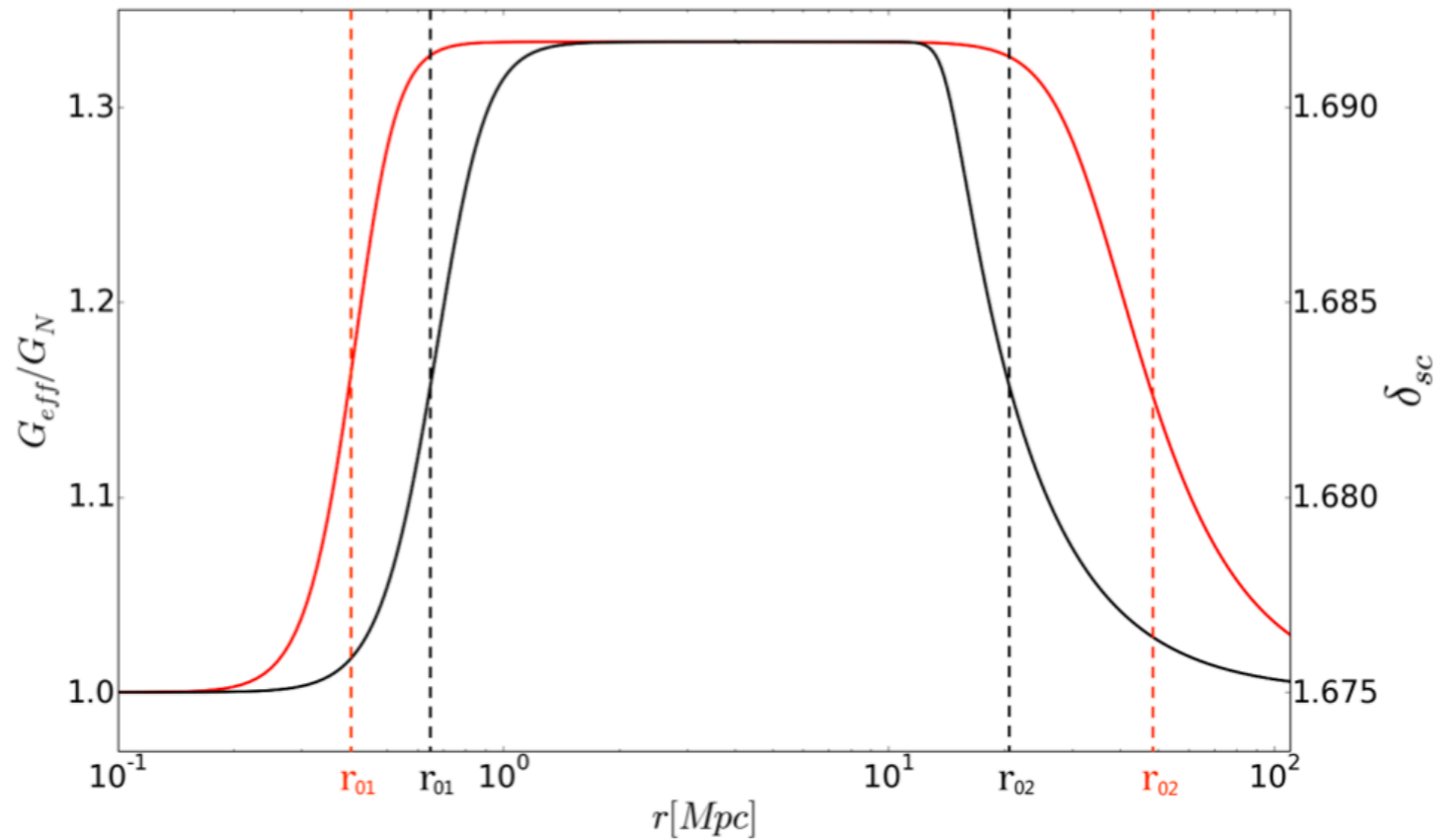


$$\frac{\delta R}{R_*} \sim \frac{\phi_{in} - \phi_{\infty}}{\Psi}$$

$$\phi(r > R_*) \sim \frac{\delta R}{R_*} \frac{M}{r}$$

The 5th force is screened in the **high density** regime!

Critical density threshold in f(R) gravity



We think DM halo is stable formed at this point

NON-LINEAR density at this point approaches singular at this point,
 $\delta_{non-linear}$ is **NOT GOOD** indicator of halo!



LINEAR density at this point δ_c is regular, hence a **GOOD** proxy!



$\delta_{lin} > \delta_c$ **formed!**

$\delta_{lin} < \delta_c$ **not yet!**

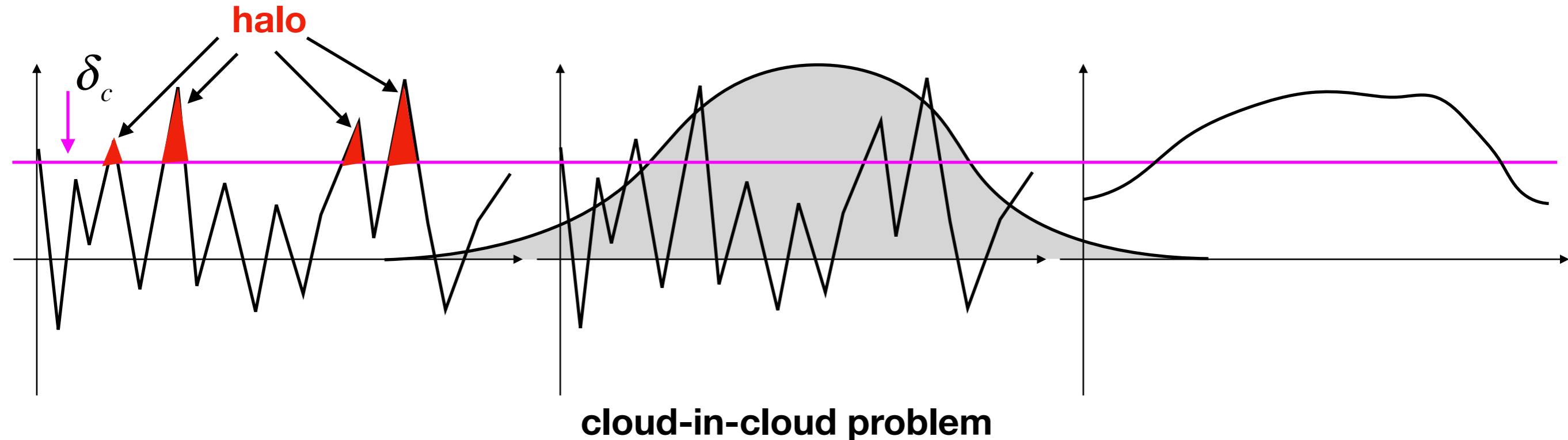
UP to now, we answered how a single spherical halo form.

Next question, **how the halo distributed in the large scales?**

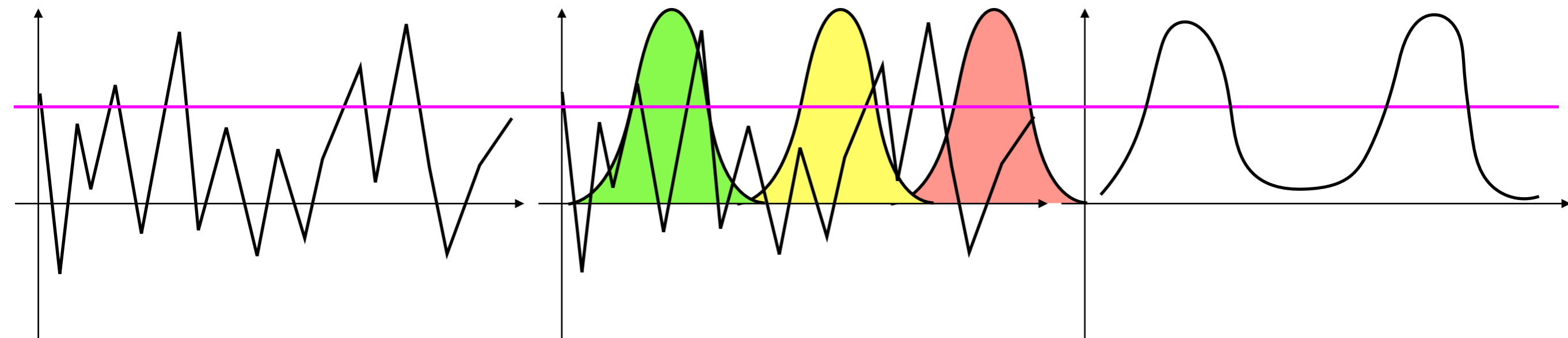
smoothed density field on scale R_W

peak-background split

$$\delta(\vec{x}; R_W) \equiv \int d^3x' W(|\vec{x}' - \vec{x}|; R_W) \delta(\vec{x}')$$

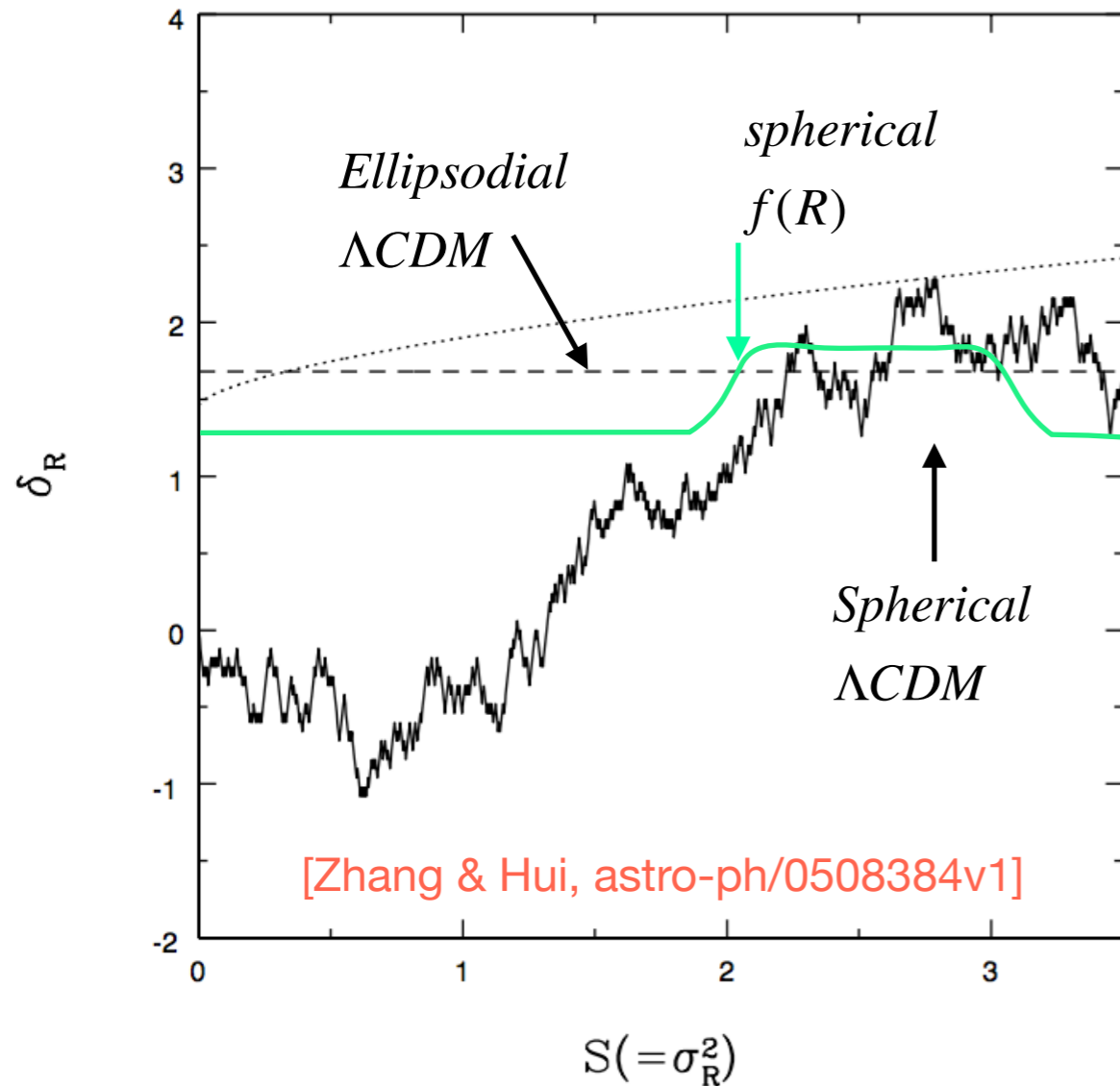


cloud-in-cloud problem



Scale dependent threshold

[Zhang & Hui, astro-ph/0508384v1]



$$P(\delta, S) = P_0(\delta, S) - \int_0^S dS' f(S') P_0(\delta - B(S'), S - S')$$

$$f(S) = -P(B(S), S) \frac{dB}{dS} - \int_{-\infty}^{B(S)} \frac{\partial P(\delta, S)}{\partial S} d\delta$$

$f(S)dS$: probability of first up-crossing @S

$P(\delta, S)d\delta$: probability of cross btw δ and $\delta + d\delta$ without cross δ_c before S.

conservation:

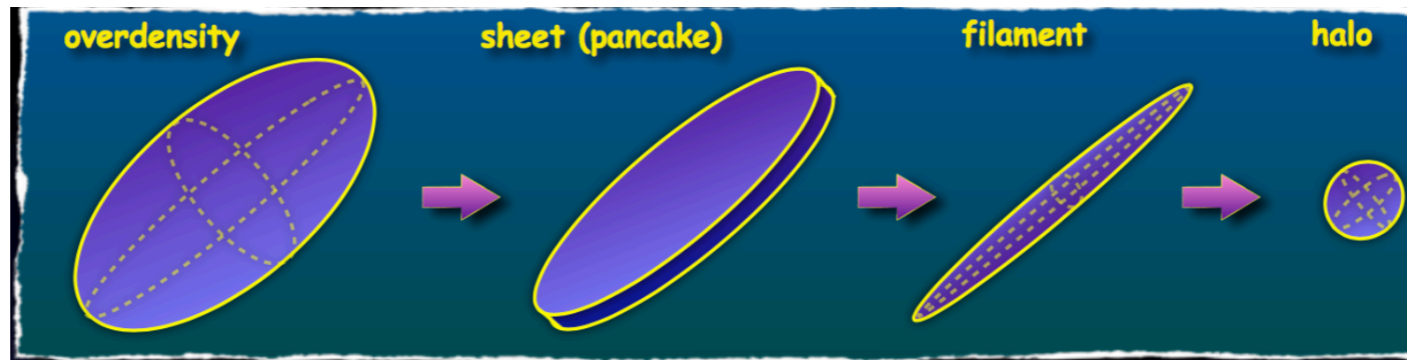
$$1 = \int_0^S f(S') dS' + \int_{-\infty}^{B(S)} P(\delta, S) d\delta$$

large scale

$$\sigma_R^2 = \langle \delta_{\text{linear}}^2(\vec{x}, R) \rangle$$

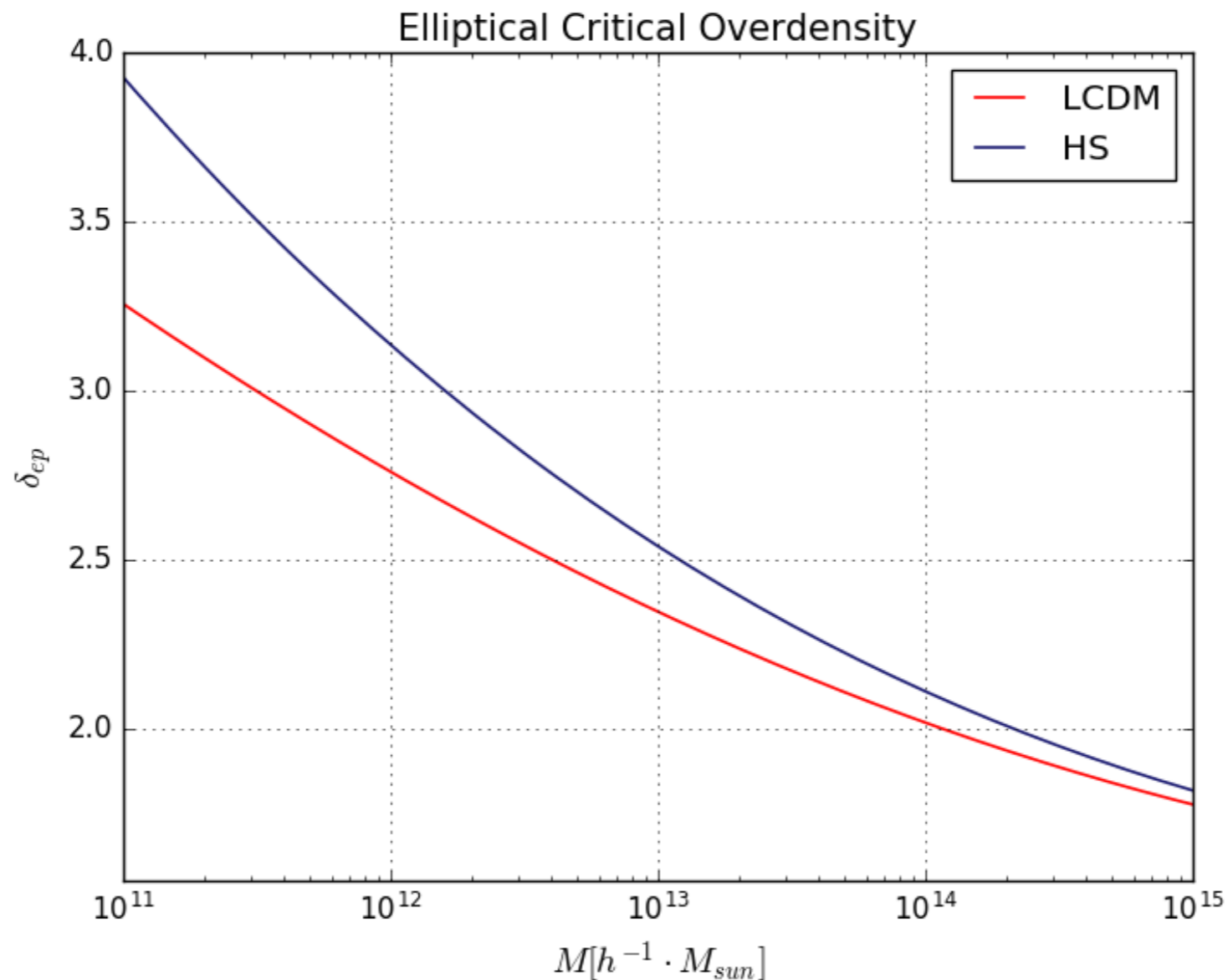
Ellipsoidal Collapse

[Sheth, Mo, Tormen, 1999]



Ansatz:

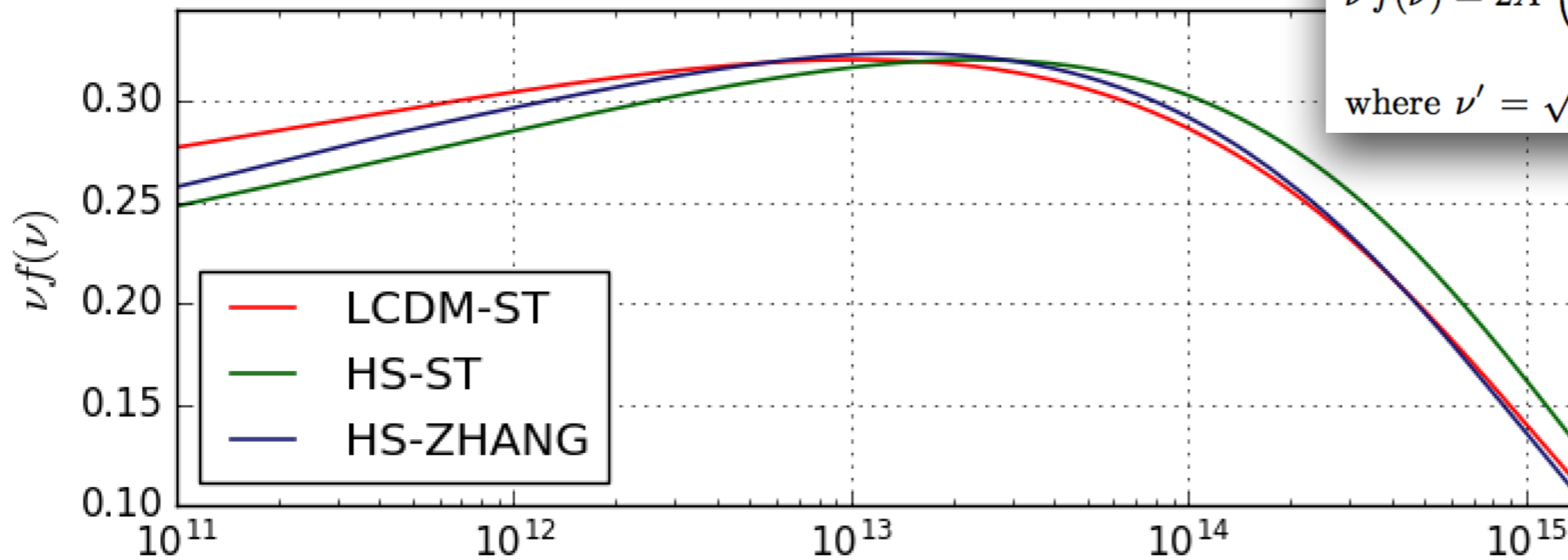
$$\delta_{sc}^{LCDM} \Rightarrow \delta_{sc}^{MG} \quad B_{GIF}(\sigma, z) = \sqrt{a} \delta_{sc}(z) \left(1 + b \left[\frac{\sigma^2}{a \sigma_*^2(z)} \right]^c \right)$$



Halo Mass Function

[Sheth-Tormen, 1999]

Distribution Function

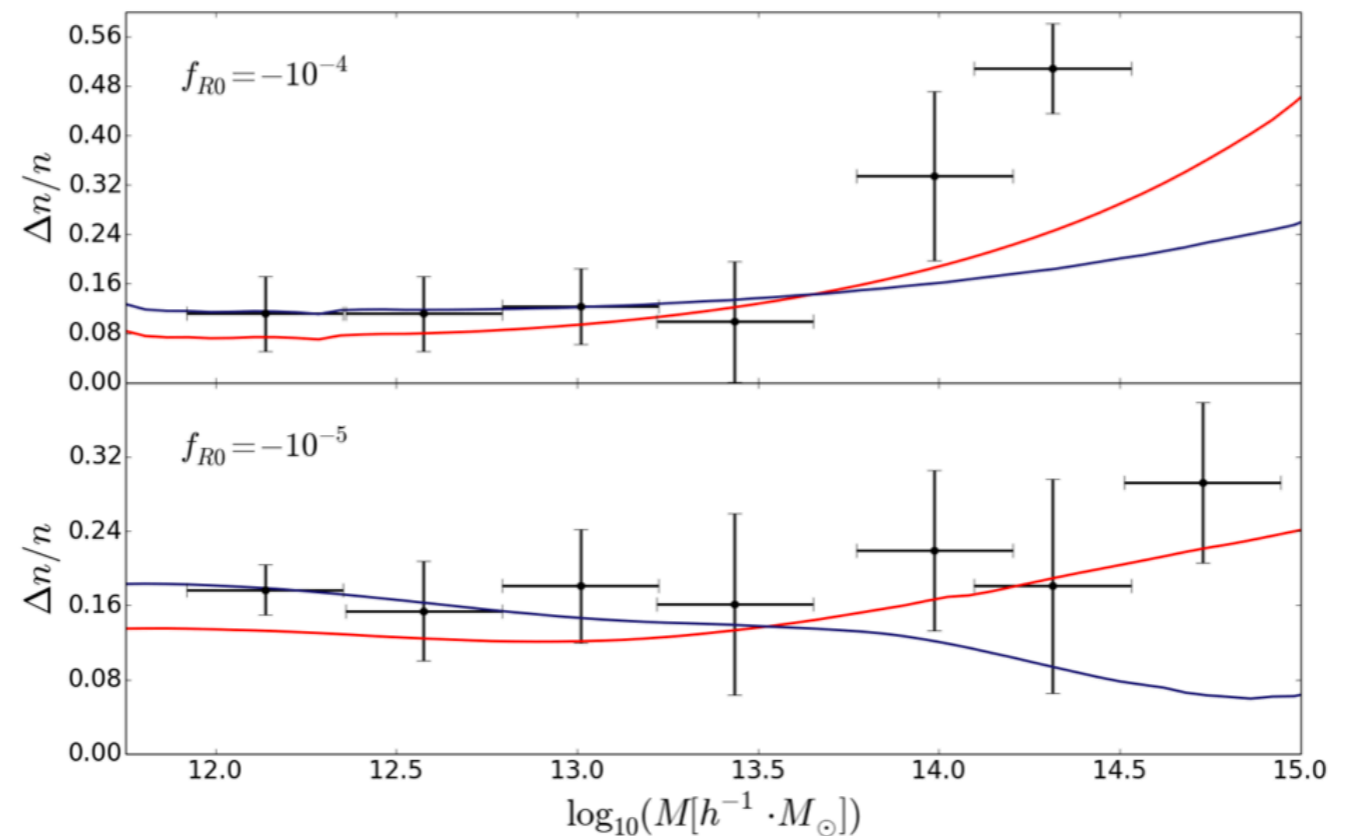


$$\nu f(\nu) = 2A \left(1 + \frac{1}{\nu'^{2q}}\right) \left(\frac{\nu'^2}{2\pi}\right)^{1/2} \exp\left(-\frac{\nu'^2}{2}\right),$$

where $\nu' = \sqrt{a\nu}$, $a = 0.707$, $q = 0.3$ and $A \approx 0.322$

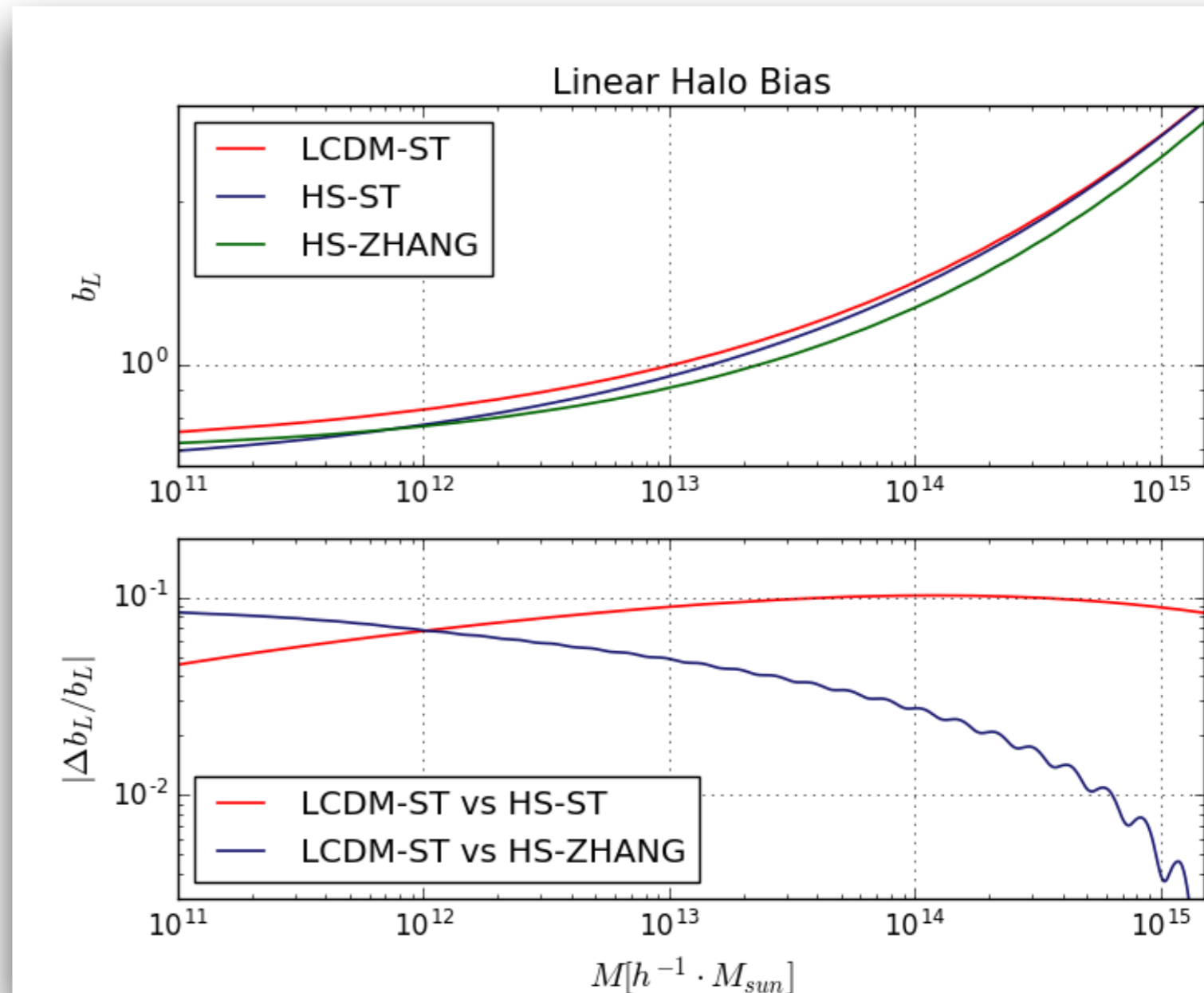
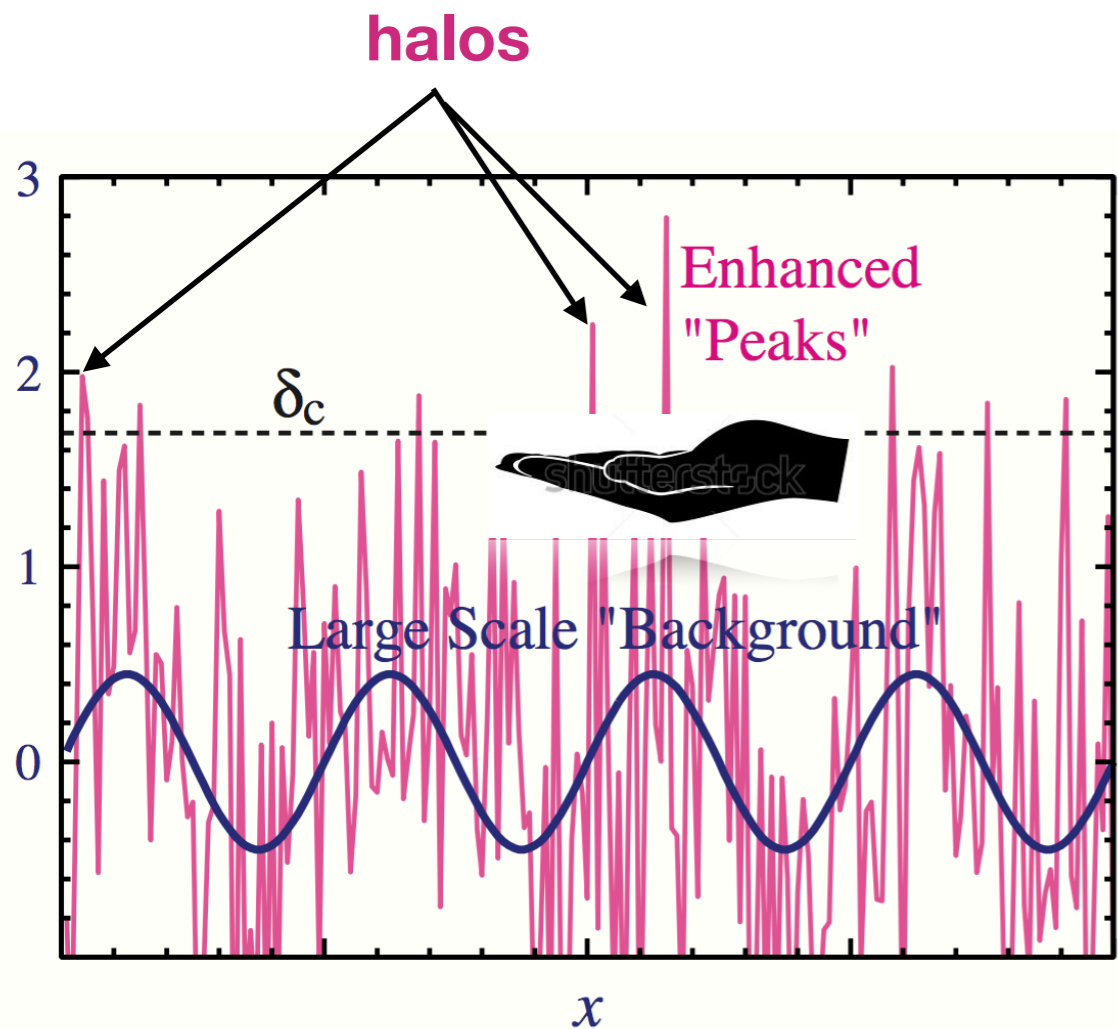
$$n_{\ln M_{\text{vir}}} \equiv \frac{dn}{d \ln M_{\text{vir}}} = \frac{\bar{\rho}_m}{M_{\text{vir}}} f(\nu) \frac{d\nu}{d \ln M_{\text{vir}}}$$

~20%!



Step-4: Halo mass function gives the mean number density in the range of $(M \sim M + dM)$

Linear Halo bias $\delta_m = (1 + b) \cdot \delta_h$



In denser regime, δ_c is easier to reach

$$\delta_c^{eff} \sim \delta_c - \delta_0$$

1~10%!

$$n(m, \delta_c^{eff}) \sim n(m, \delta_c) - \frac{\partial n}{\partial \delta_c} \cdot \delta_0 + \dots$$

$$b_L \delta_0 = \frac{n(m, \delta_c^{eff})}{n(m, \delta_c)} - 1 = \frac{\partial \log n}{\partial \delta_c} \cdot \delta_0$$

Step-5:

Halo concentration

$$c_v = \frac{r}{r_s}$$

$$\rho_s = \frac{1}{3} \bar{\rho}_m \Delta_{\text{vir}} c_{\text{vir}}^3 \left[\ln(1 + c_{\text{vir}}) - \frac{c_{\text{vir}}}{c_{\text{vir}} + 1} \right]^{-1},$$
$$r_s = \frac{1}{c_{\text{vir}}} \left(\frac{3M_{\text{vir}}}{4\pi \bar{\rho}_m \Delta_{\text{vir}}} \right)^{1/3},$$

Halo density profile (NFW profile)

$$\delta\rho_m(r) = \frac{\rho_s}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

Fourier transform \rightarrow $y(k, M)$

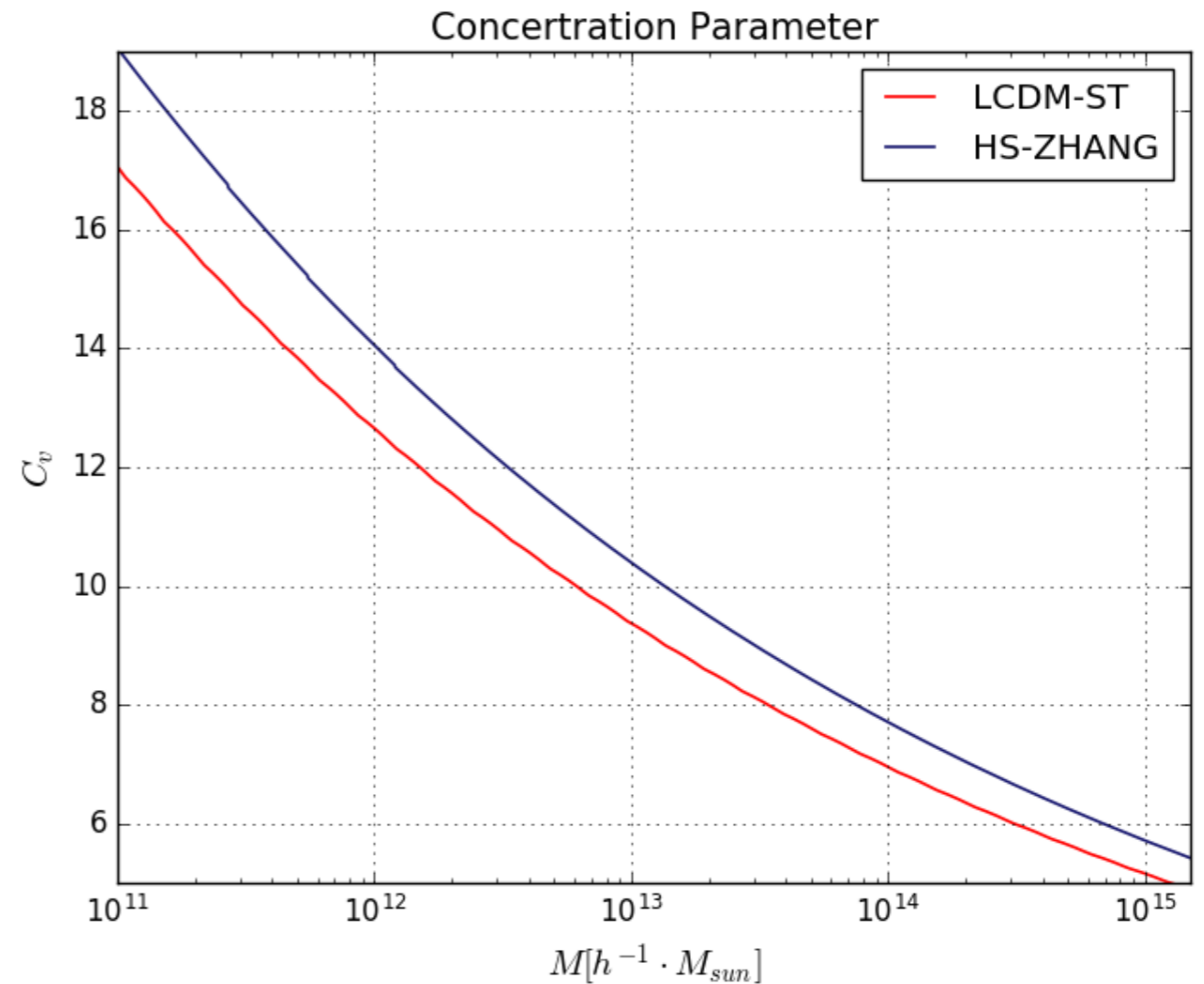
$$c_v(M_v) = 9 \left| \frac{M_*}{M_v} \right|^{0.13}$$

$$\sigma(M_*) = \delta_c$$

Virial Mass

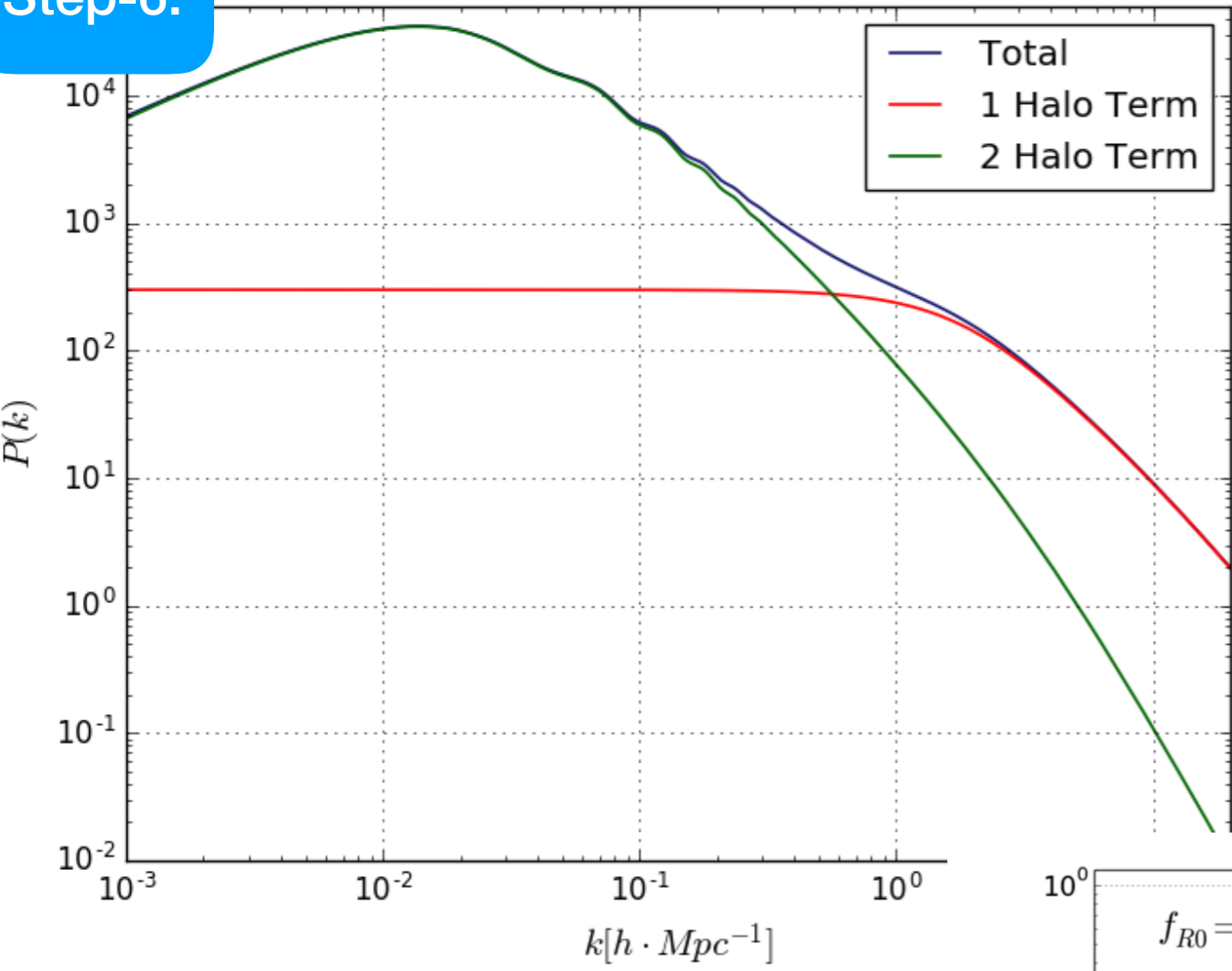
$$\sigma^2(R) = \langle \delta^2(\vec{x}; R) \rangle = \int d \ln k \Delta^2(k) |W(k; R)|^2.$$

~10%!



Step-6:

Hu-Sawicki ($n = 1, f_{R0} = -10^{-5}$)



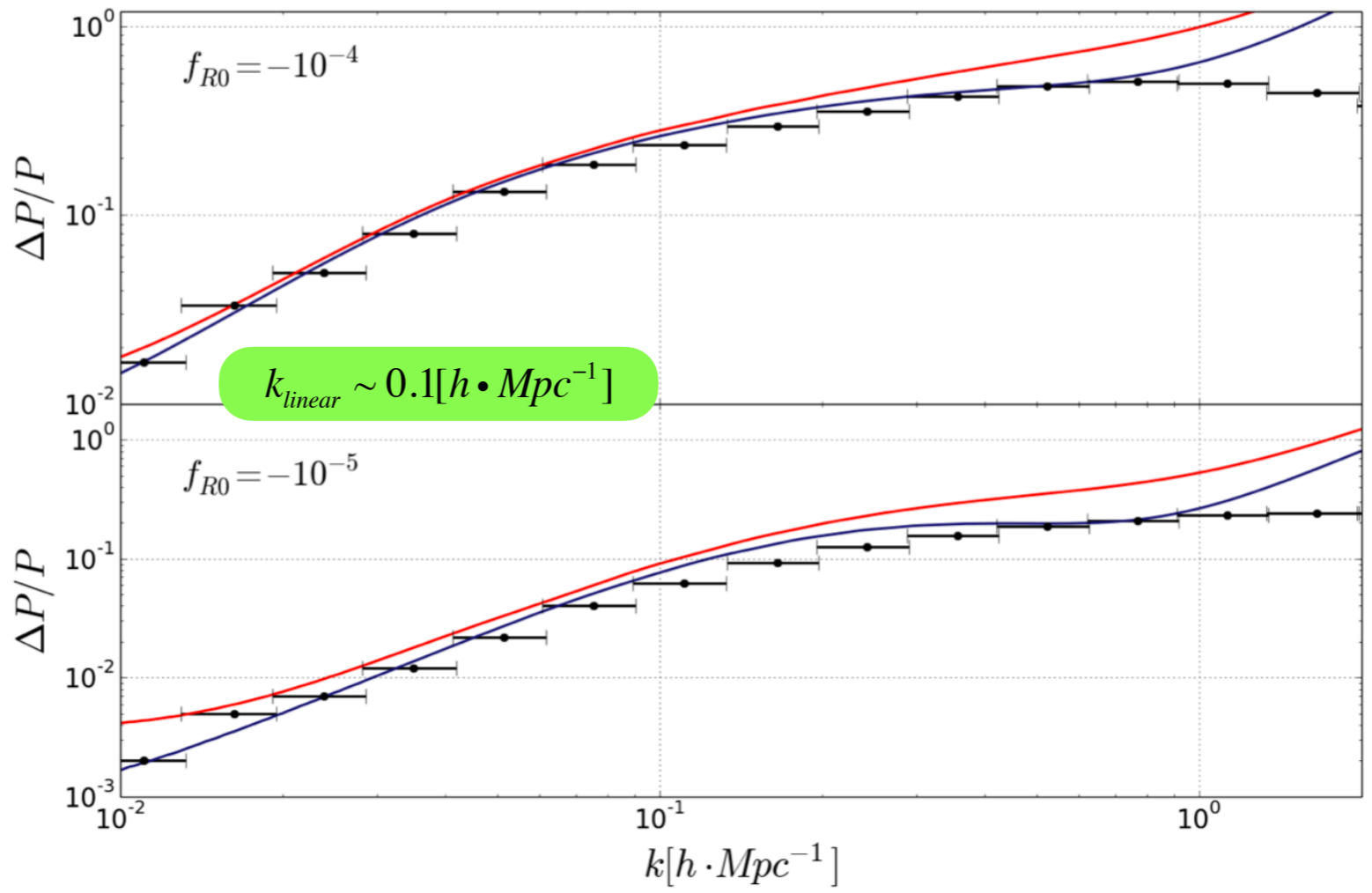
Matter Power spectrum

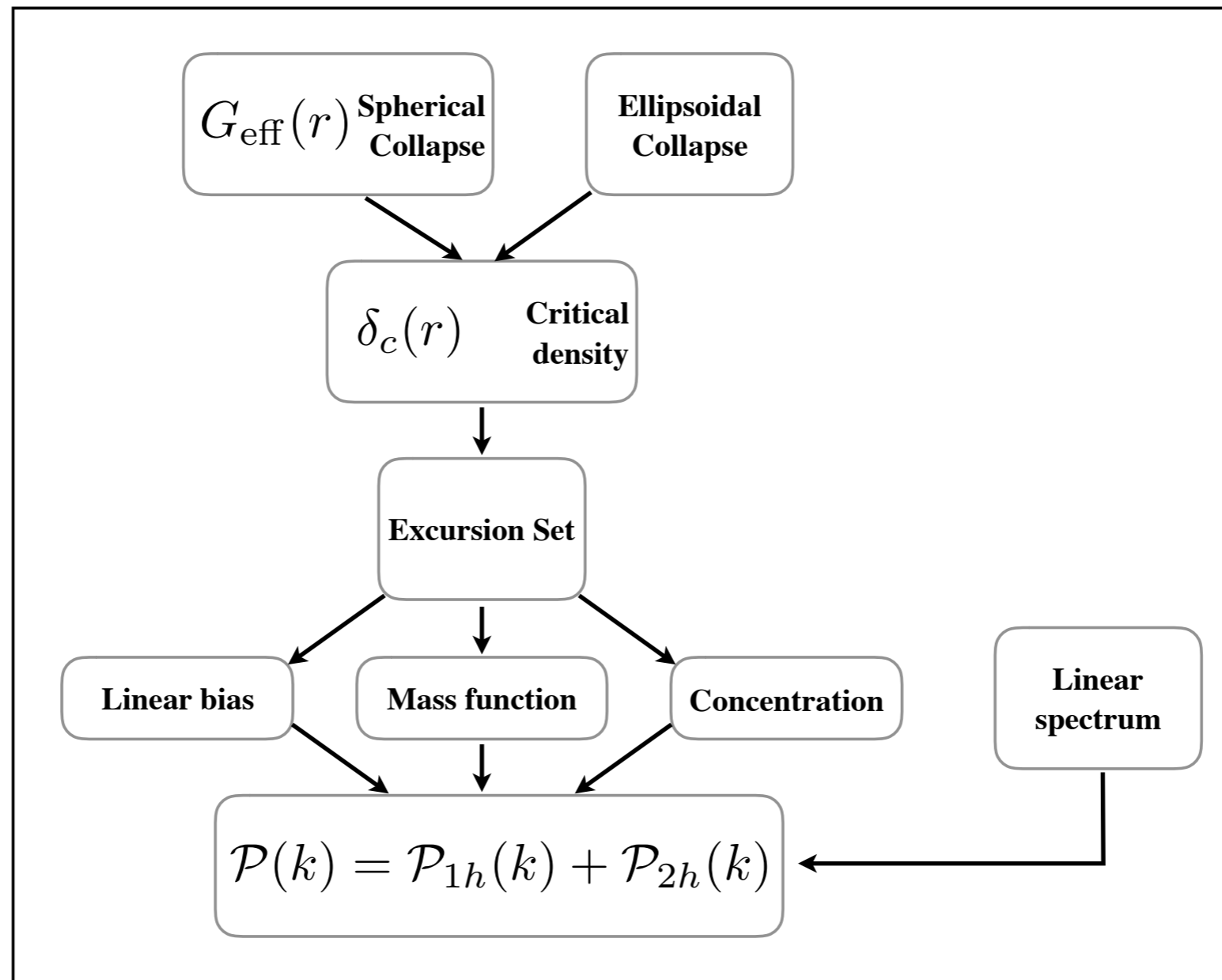
$$P_{mm}(k) \simeq I^2(k)P_L(k) + P^{1h}(k),$$

$$P^{1h}(k) = \int d \ln M_{vir} n_{\ln M_{vir}} \frac{M_{vir}^2}{\bar{\rho}_m^2} |y(k, M_{vir})|^2$$

$$I(k) \simeq \int d \ln M_{vir} n_{\ln M_{vir}} \frac{M_{vir}}{\bar{\rho}_m} y(k, M_{vir}) b_L(M_{vir}),$$

10 times data can be used!





- For the non-linear perturbation, halo model could give a reasonable estimation of the non-linear $\mathcal{P}(k)$ up to $k \sim 1$ [h/Mpc] for Hu-Sawicki $f(R)$ F4/F5 model.

Thanks!