

Effective Field Theory approach for Dark Energy/ Modified Gravity

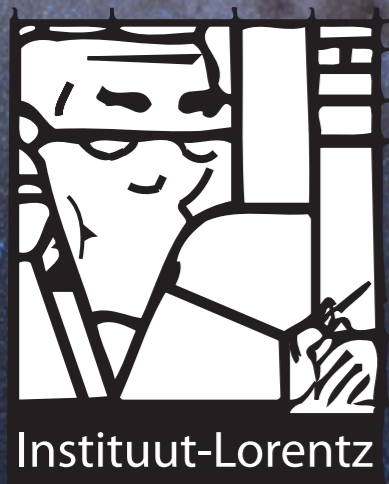


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Beijing/China, Sept. 2015



Outline

1. Evidence of late-time cosmic acceleration
2. Effective Field Theory approach for DE/MG
3. The structure of EFTCAMB
4. Planck-2015 results based on EFTCAMB
5. Conclusion

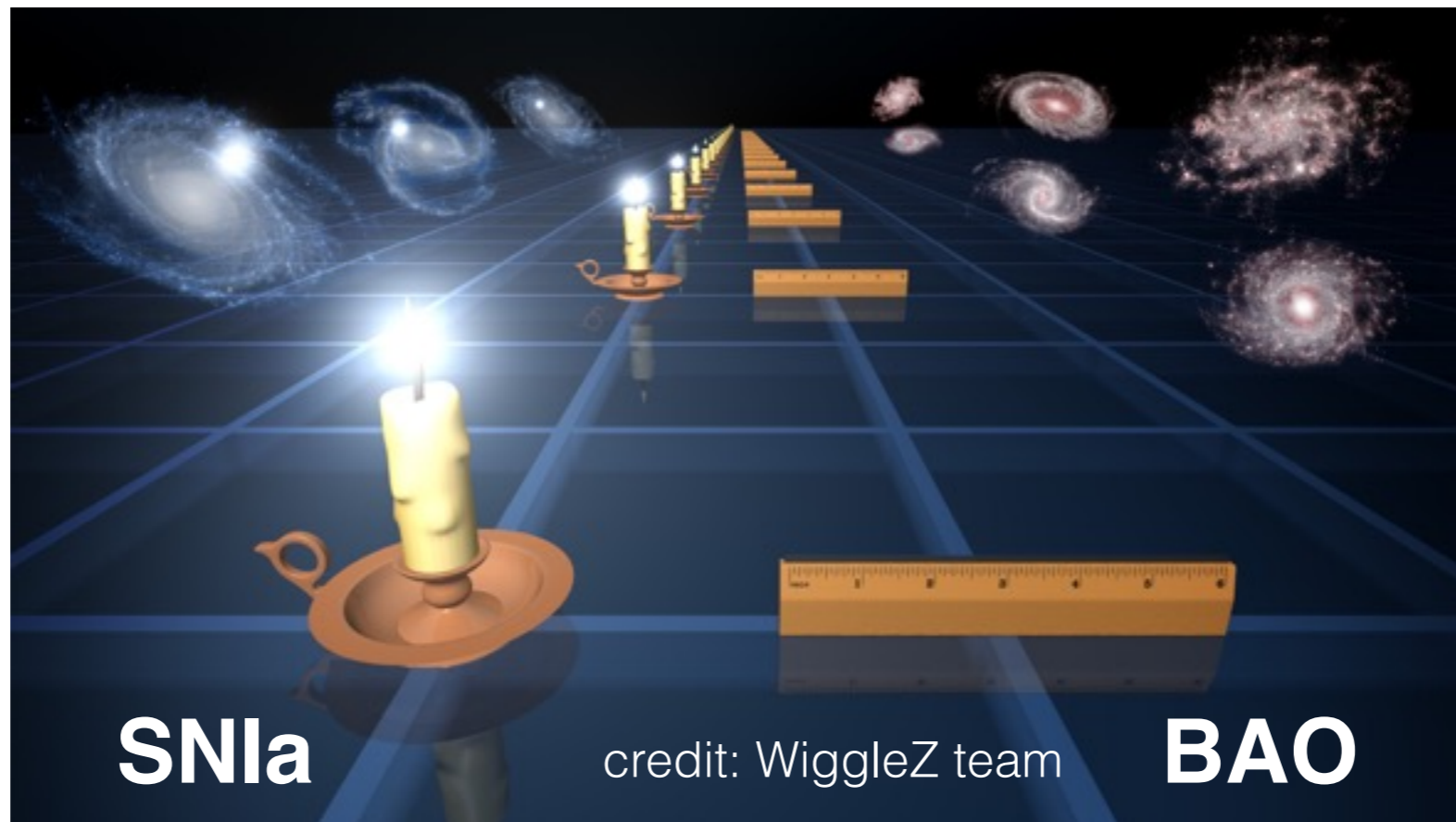
How do we know the Universe is accelerating?

It is via ...

Measurement of the distance of far away object

What we observed is line of sight integration effect

Need to know the intrinsic physics!



Standard
candle

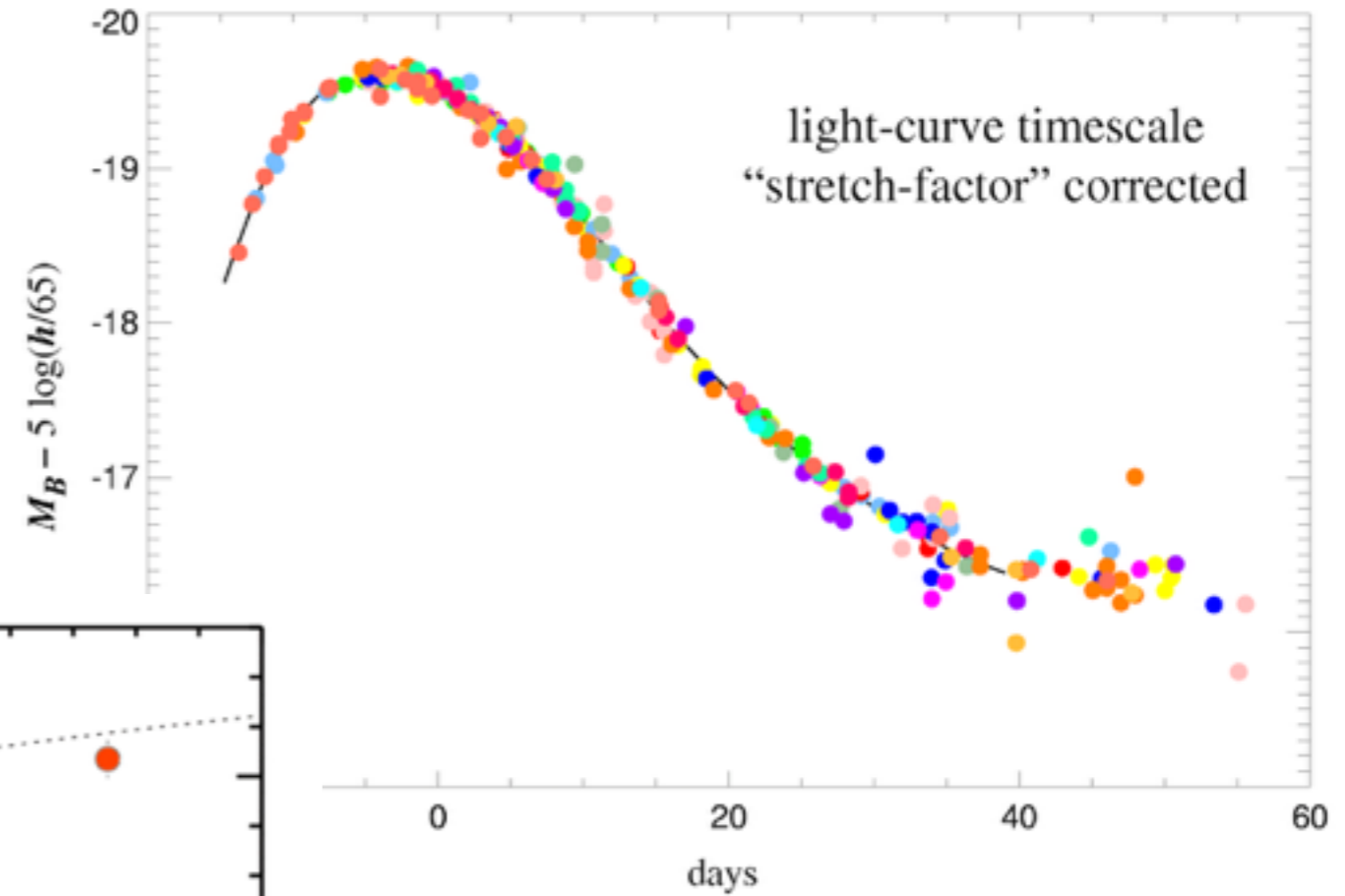
fixed
luminosity

Standard
ruler

fixed
transverse
scale

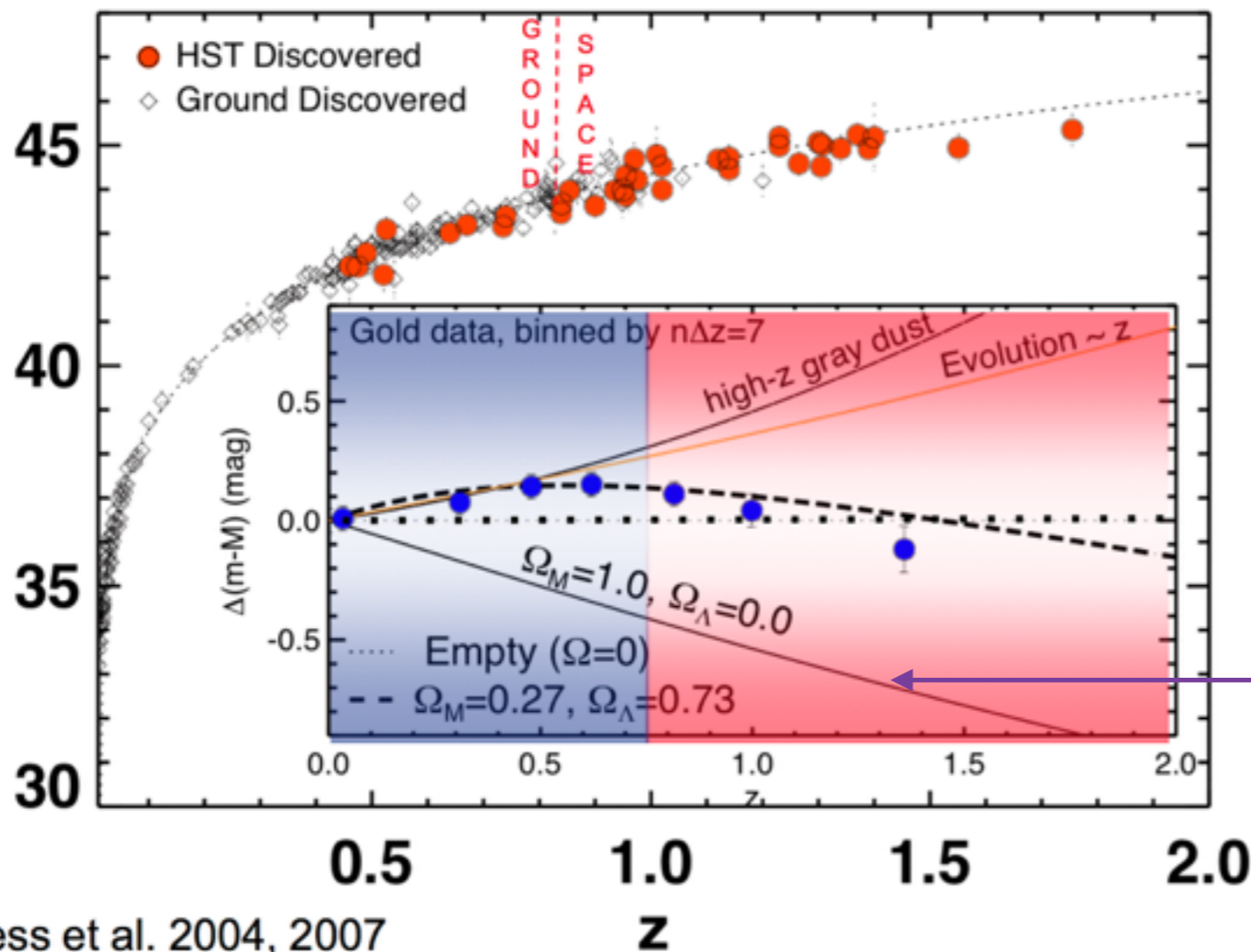
SNIa (White dwarf)

luminosity
module



Kim, *et al.* (1997)

μ

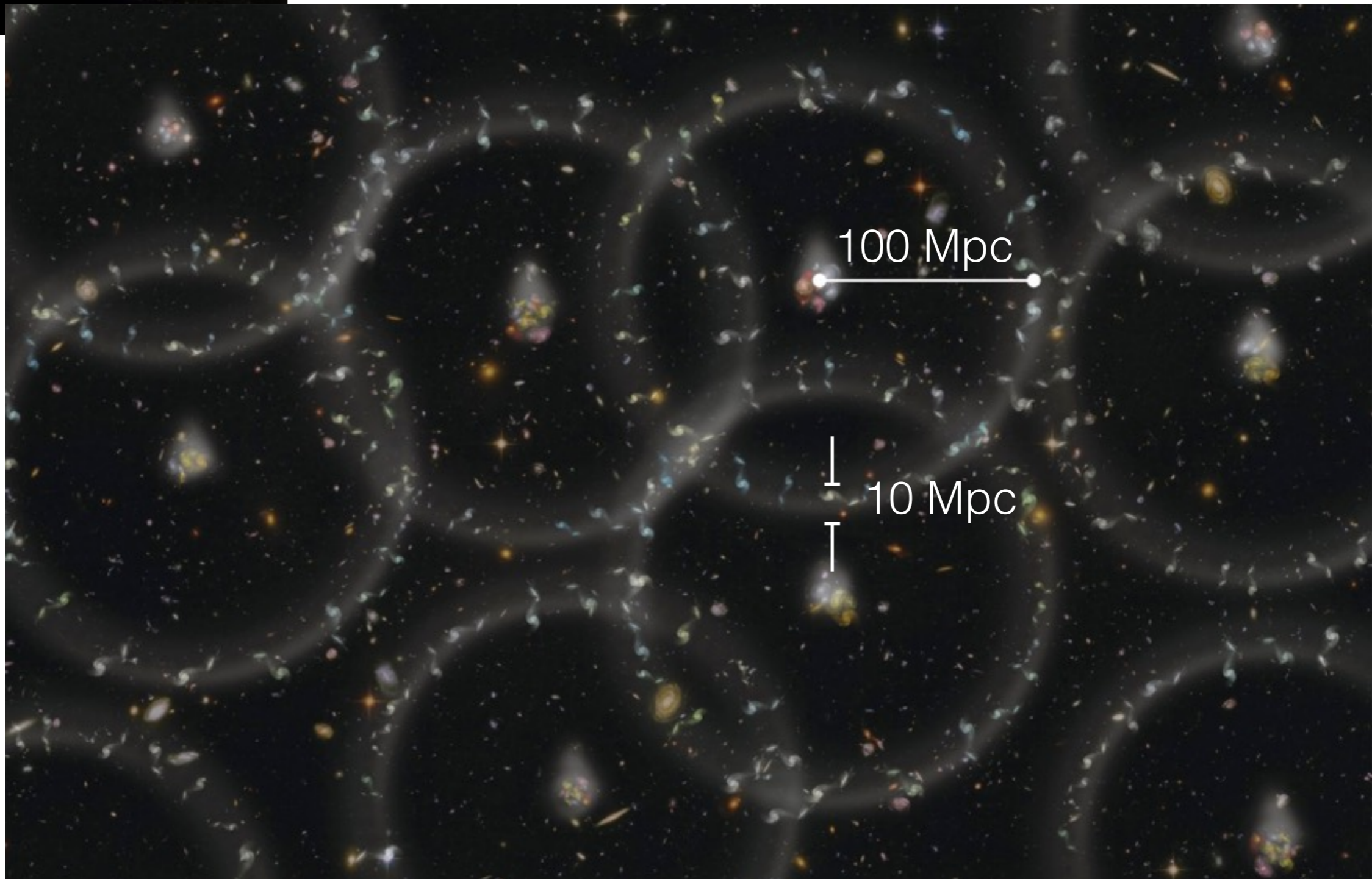
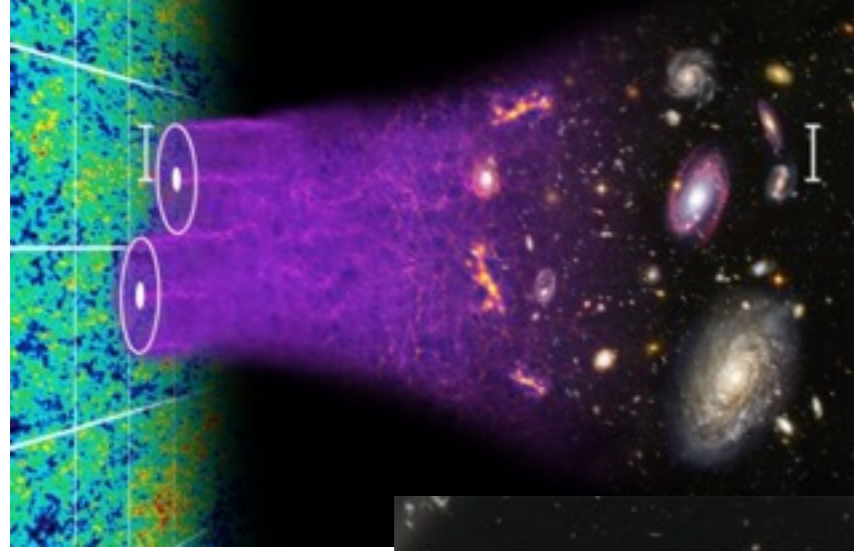


Riess et al. 2004, 2007

Einstein-DeSitter
only CDM+baryon

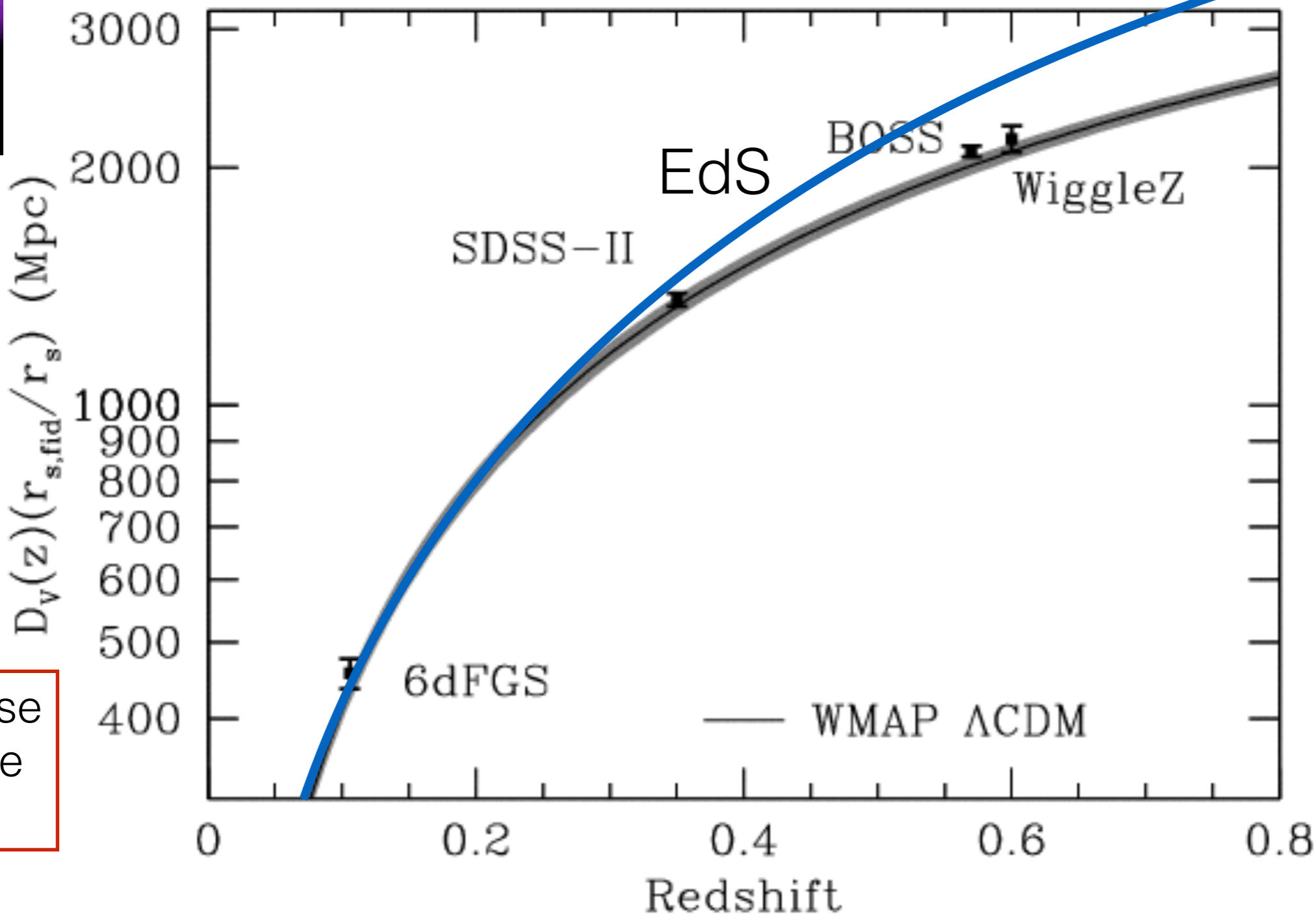
BAO — baryonic acoustic oscillation

The imprint of sound horizon of Recomb epoch on the LSS

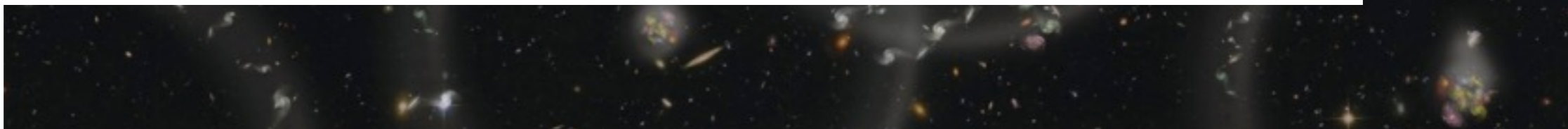
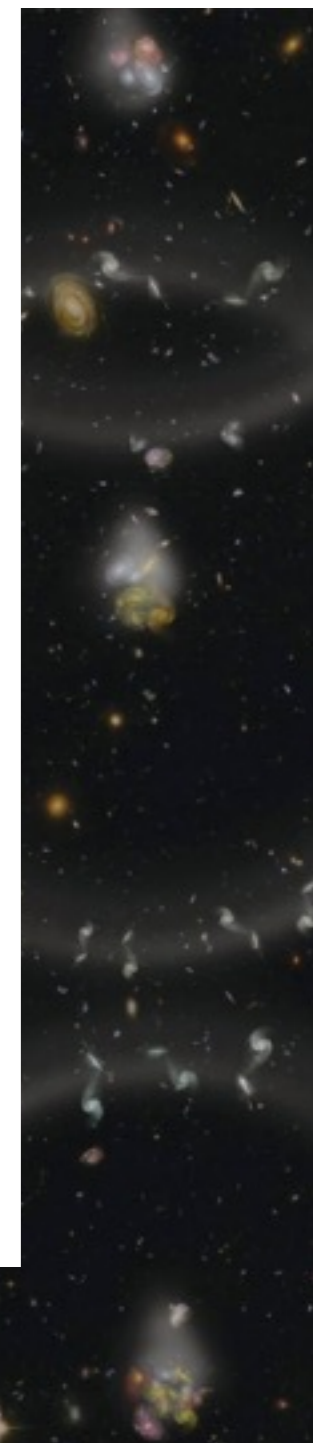
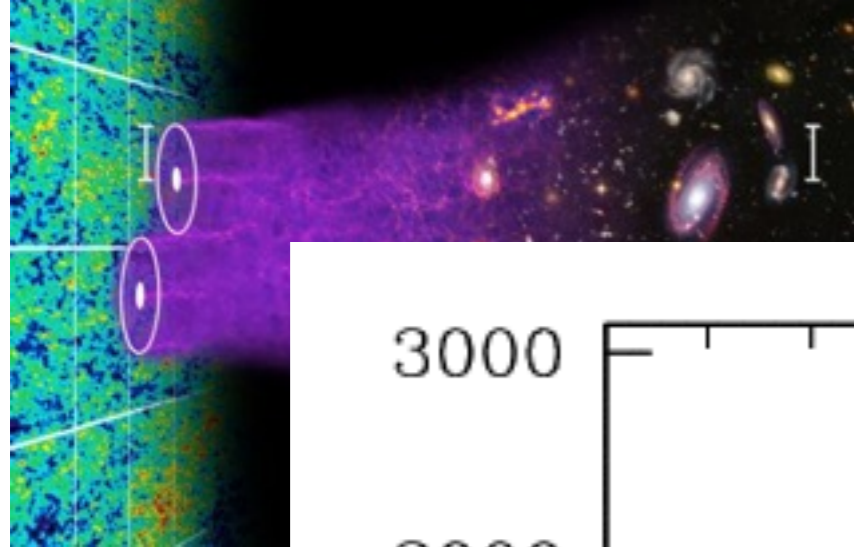


BAO

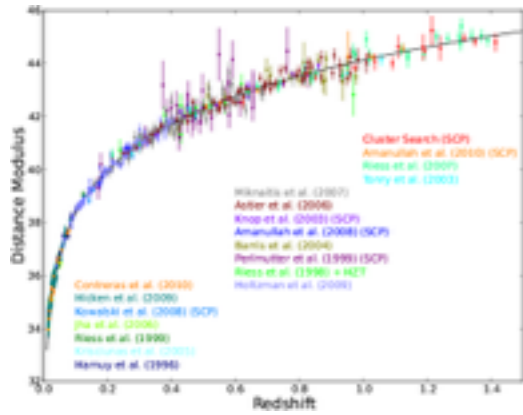
The imprint of sound horizon



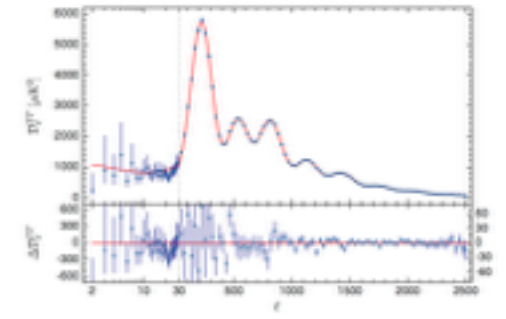
transverse
distance
scale



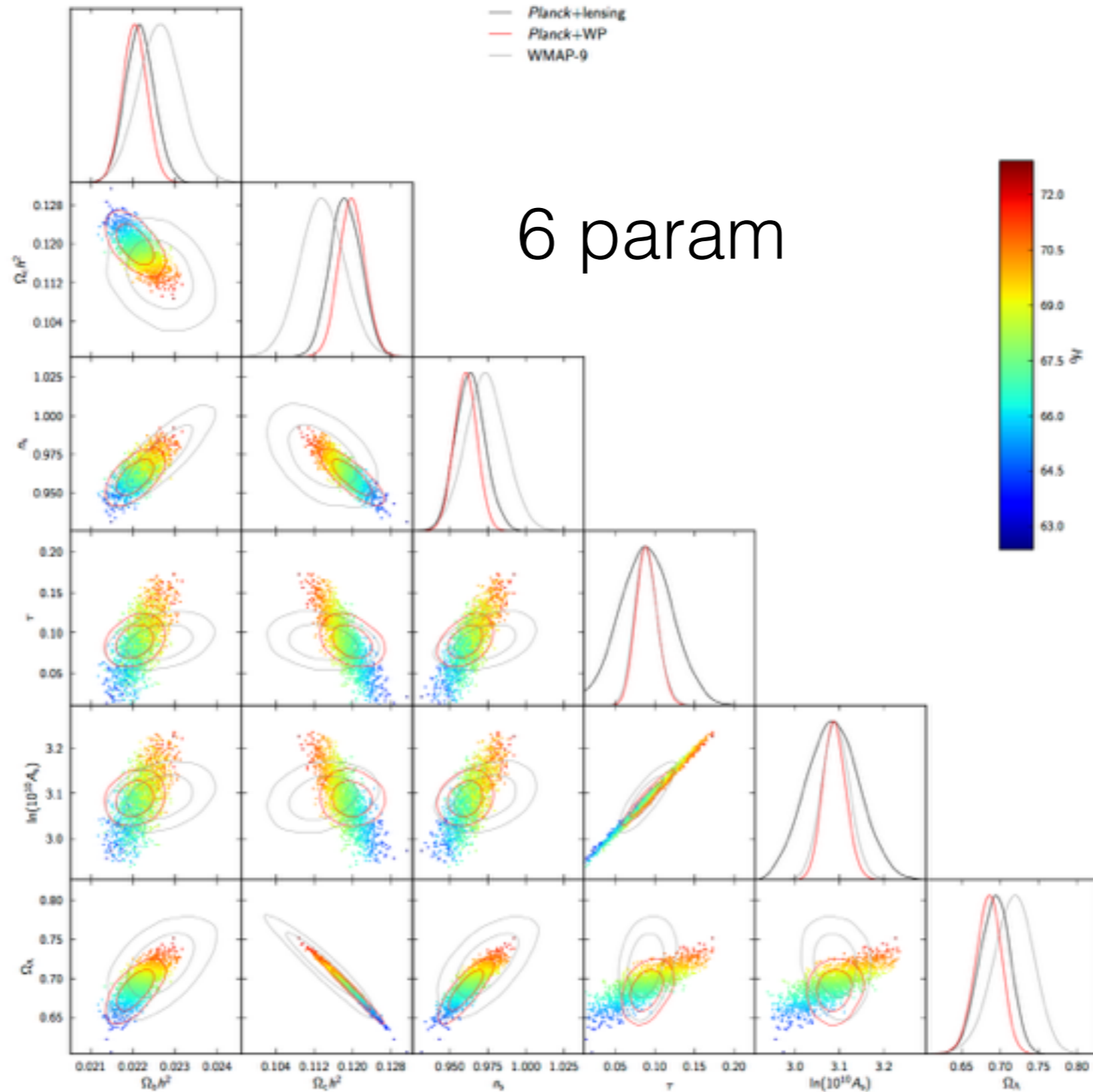
Most simplest explanation — LCDM



SN



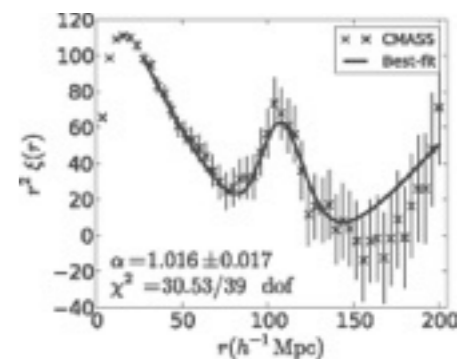
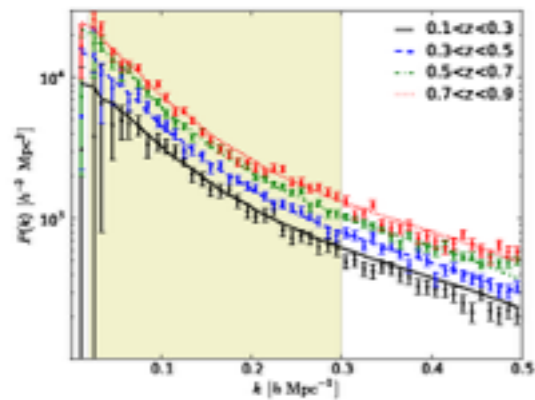
CMB



6 param

LSS

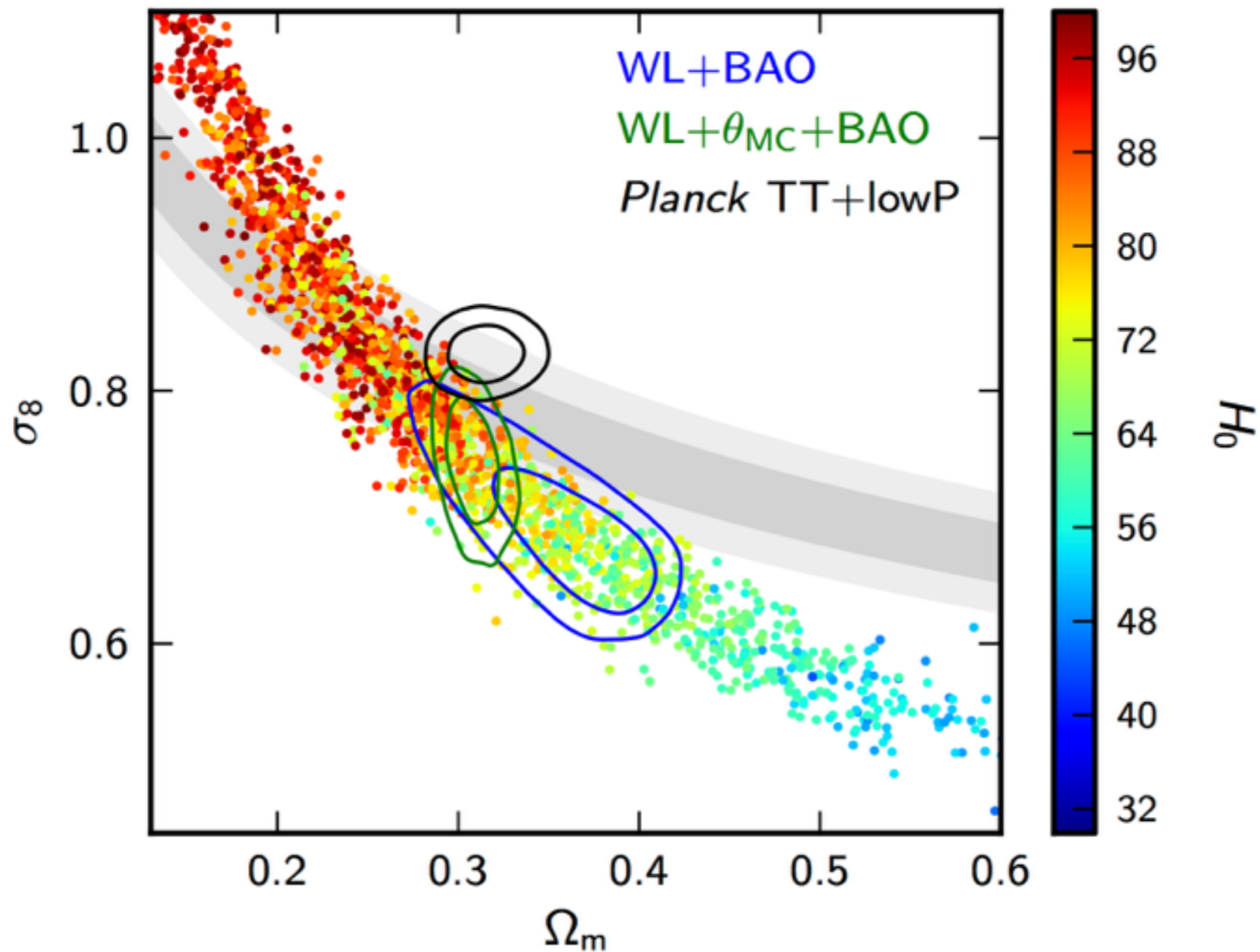
BAO



[Planck13 CP paper]

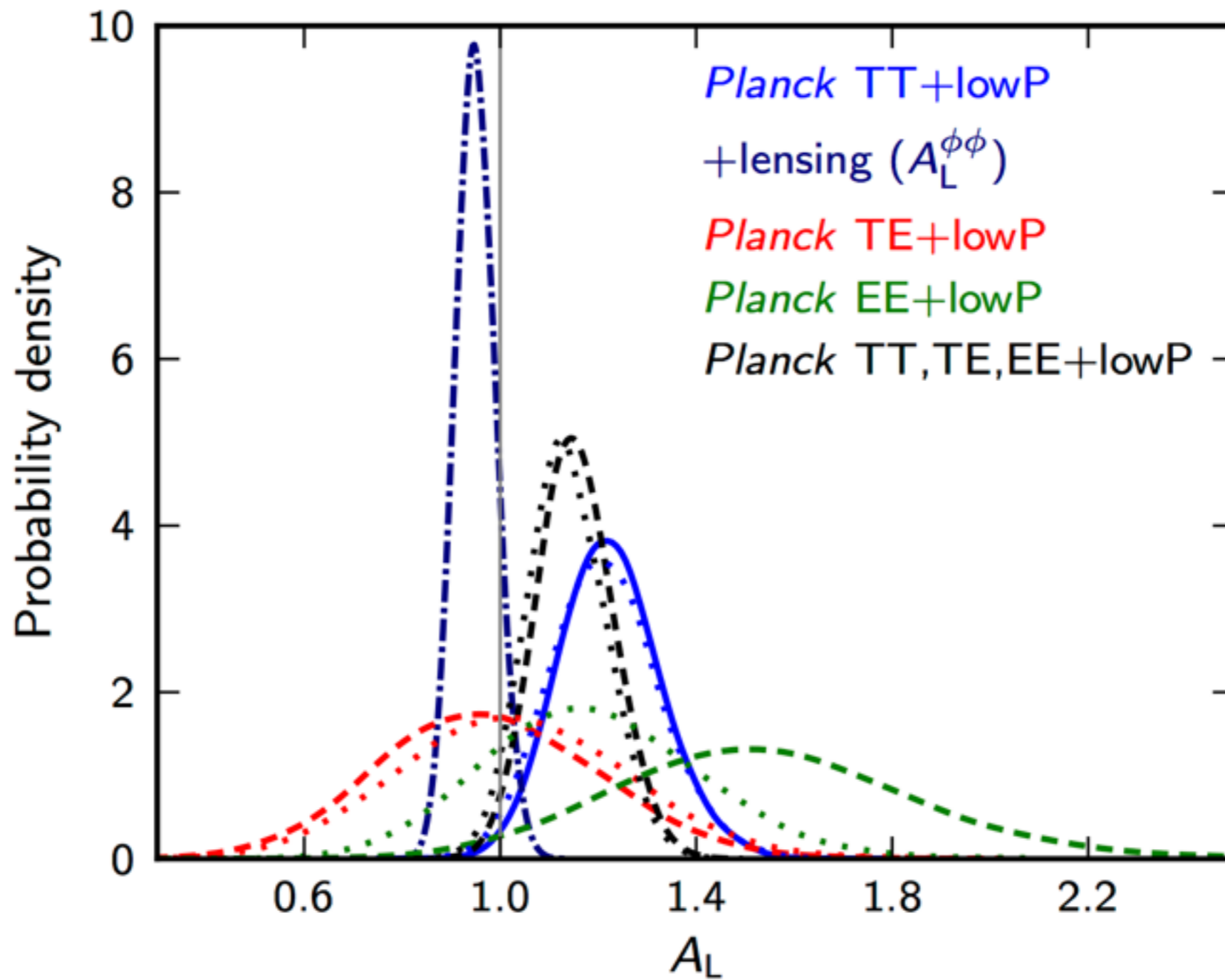
Is this the end of story?

Tension between high-z and low-z



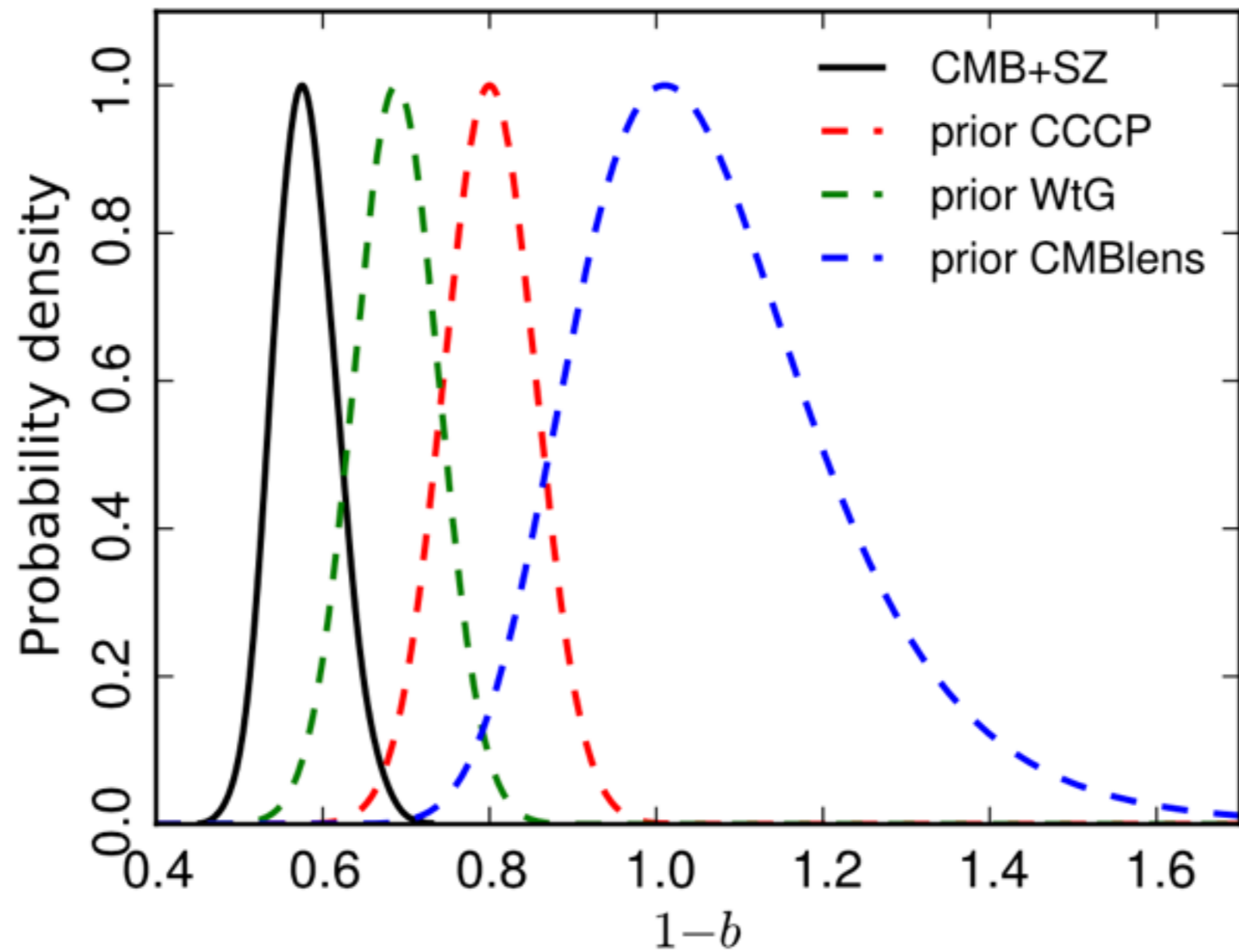
matter fluct.— Planck (CMB) \gg LSS (CFHTLenS)

Tension between high-z and low-z

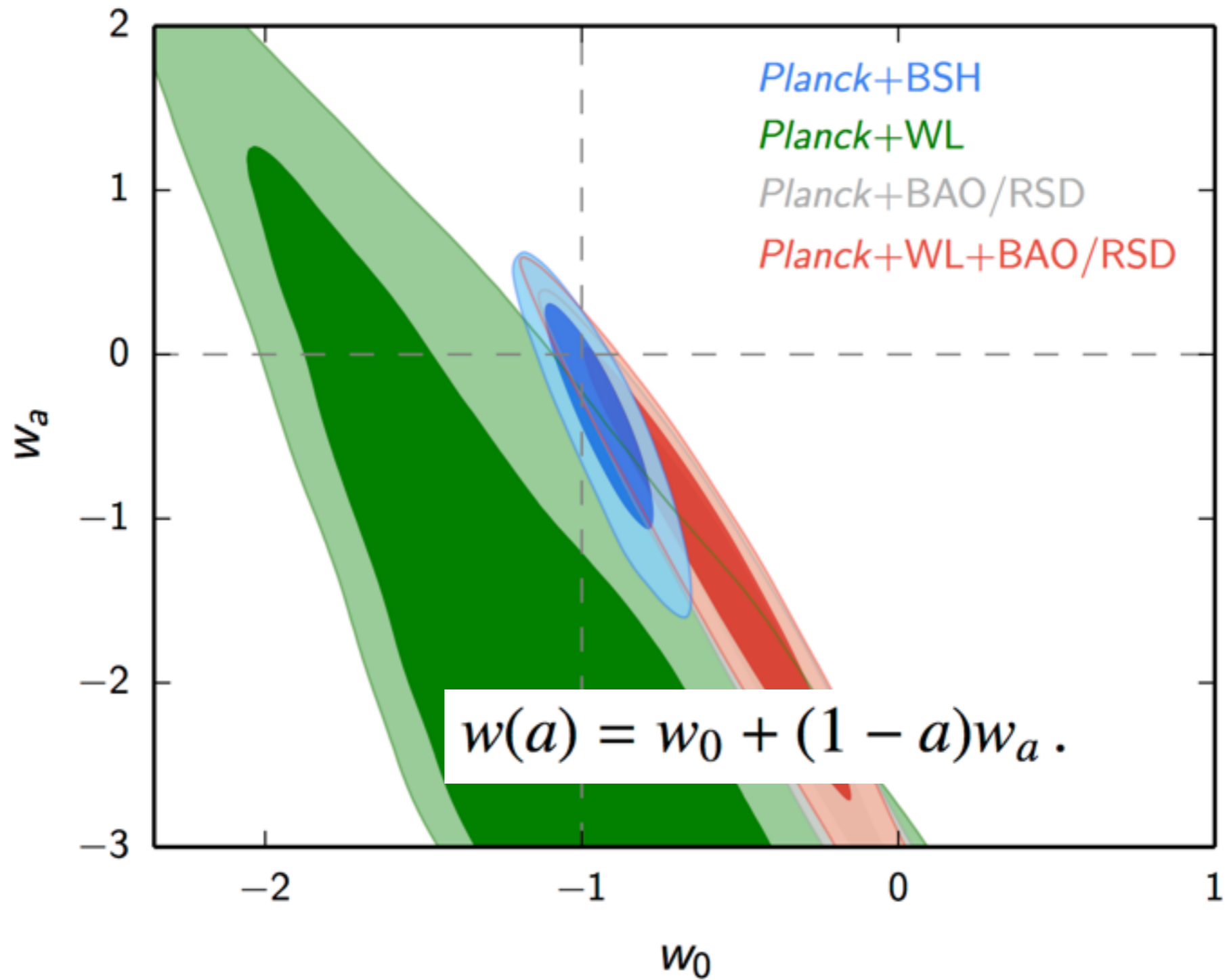


Lensing amplitude — Primary CMB \gg Secondary CMB

Tension between high-z and low-z

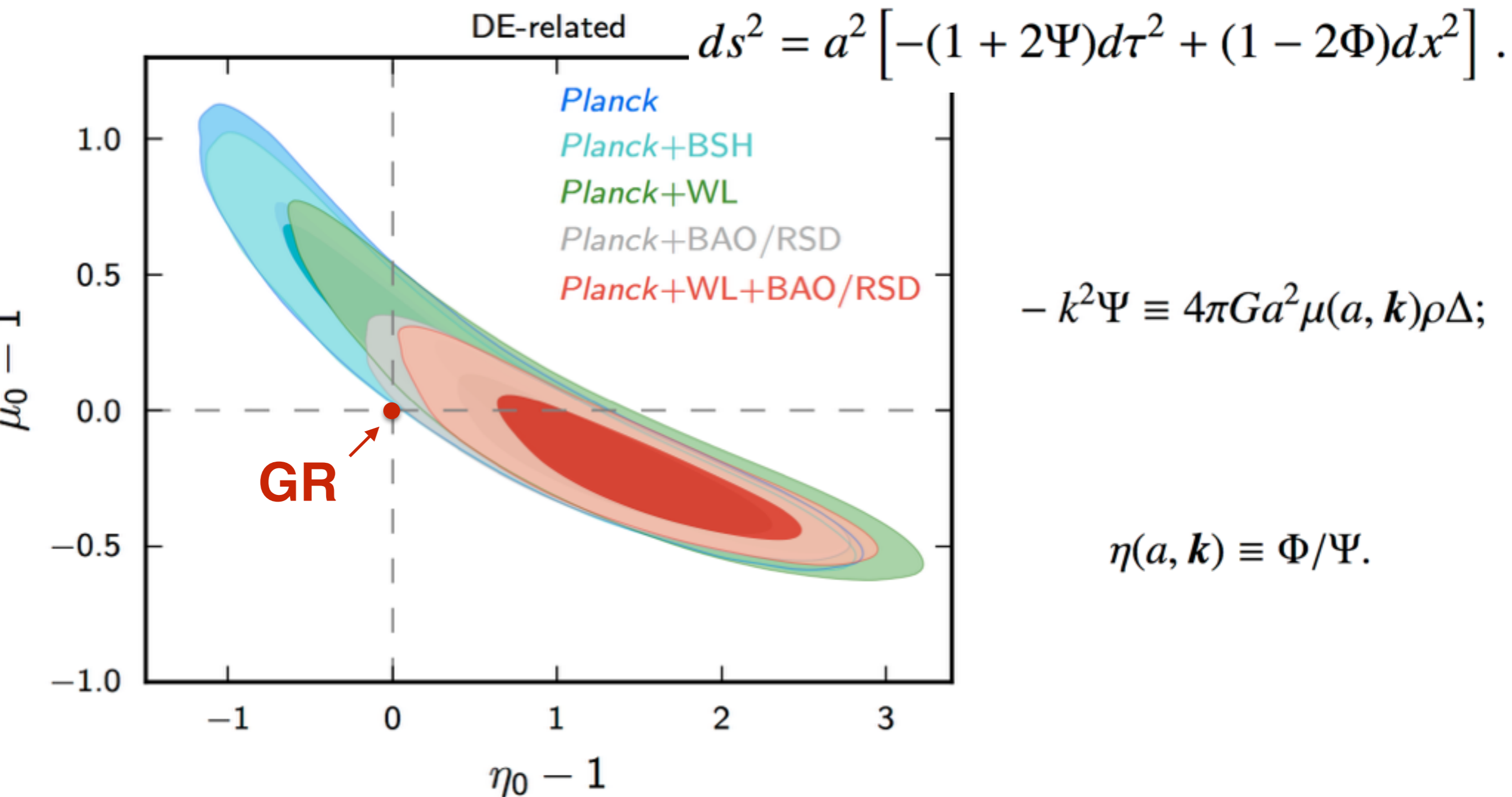


Mass bias of tSZ cluster — CMB \ll LSS



Most part is below phantom divide!

[Planck15-MG paper]



GR predict, on the large scale, the two gravitational potentials are equal, due to the lack of sources of anisotropic stress!

All these motivate us
to



GR!

How to?

DE $G_{\mu\nu} = 8\pi G \left[T_{\mu\nu}^{cdm} + T_{\mu\nu}^b + T_{\mu\nu}^\gamma + T_{\mu\nu}^\nu + T_{\mu\nu}^{DE} \right]$

MG $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G \left[T_{\mu\nu}^{cdm} + T_{\mu\nu}^b + T_{\mu\nu}^\gamma + T_{\mu\nu}^\nu \right]$

Not the math trick of RHS or LHS

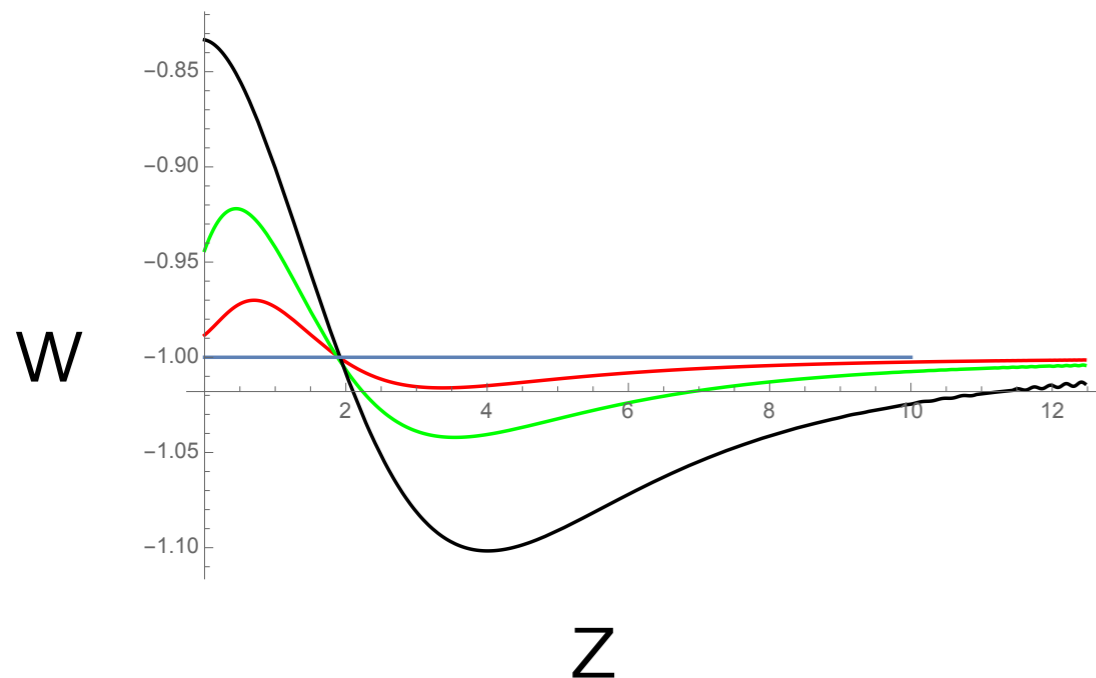
What do I mean by DE and MG?

DE

EoS of exotic fluid

$$w = \frac{P}{\rho}$$

LCDM \longrightarrow $w = -1$



MG

Growth rate of matter fluid

$$g(a) \equiv D(a)/a = \exp \left[\int_0^a (da'/a') [\Omega_M(a')^\gamma - 1] \right]$$

GR \longrightarrow $\gamma = 0.55$

Zeldovich Approximation-II

In the linear sub-Horizon regime, GR gives

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0$$

The growth rate of CDM only depends on time!

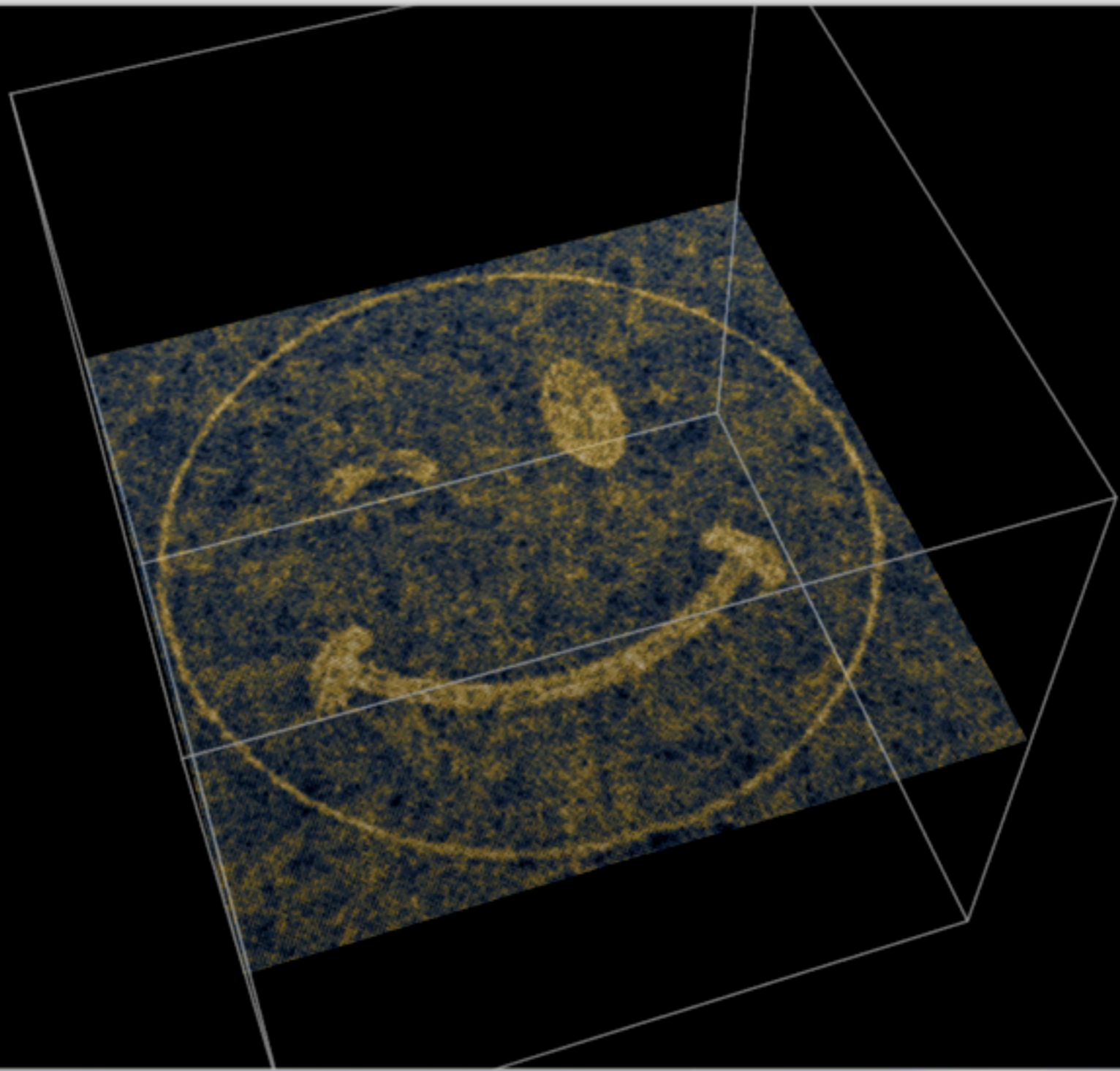
The displacement field $\vec{x} = \vec{y} - \mathcal{D}(\tau) \frac{1}{\nabla_y^2} \vec{\nabla}_y \Delta_c(\tau_i, \vec{y})$

In GR: CDM particles trajectory is **straight** line!

A video of ZA

CPU: 3.3% (8 cores)
RAM: 1.17GB / 16GB
24 bits × 1378px × 770px @ 6.31 FPS = 160.72 Mbps
Compression: 69% (50.17 Mbps)
Displacement: 100%
Slice 128

nG
col
bo
rar
zSt
rei
lin
mu
filt
filt
filt
filt
filt
filt
filt
filt



Quasic-Static Approx: DE/MG:

$$k^2\psi = -4\pi G \mu(a, k) a^2 \rho \Delta ,$$
$$\frac{\phi}{\psi} = \gamma(a, k) .$$

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}}(t, k)\rho_m\delta_m = 0$$

DE/MG: at linear regime
growth rate of CDM
depends on the scales!

GR:

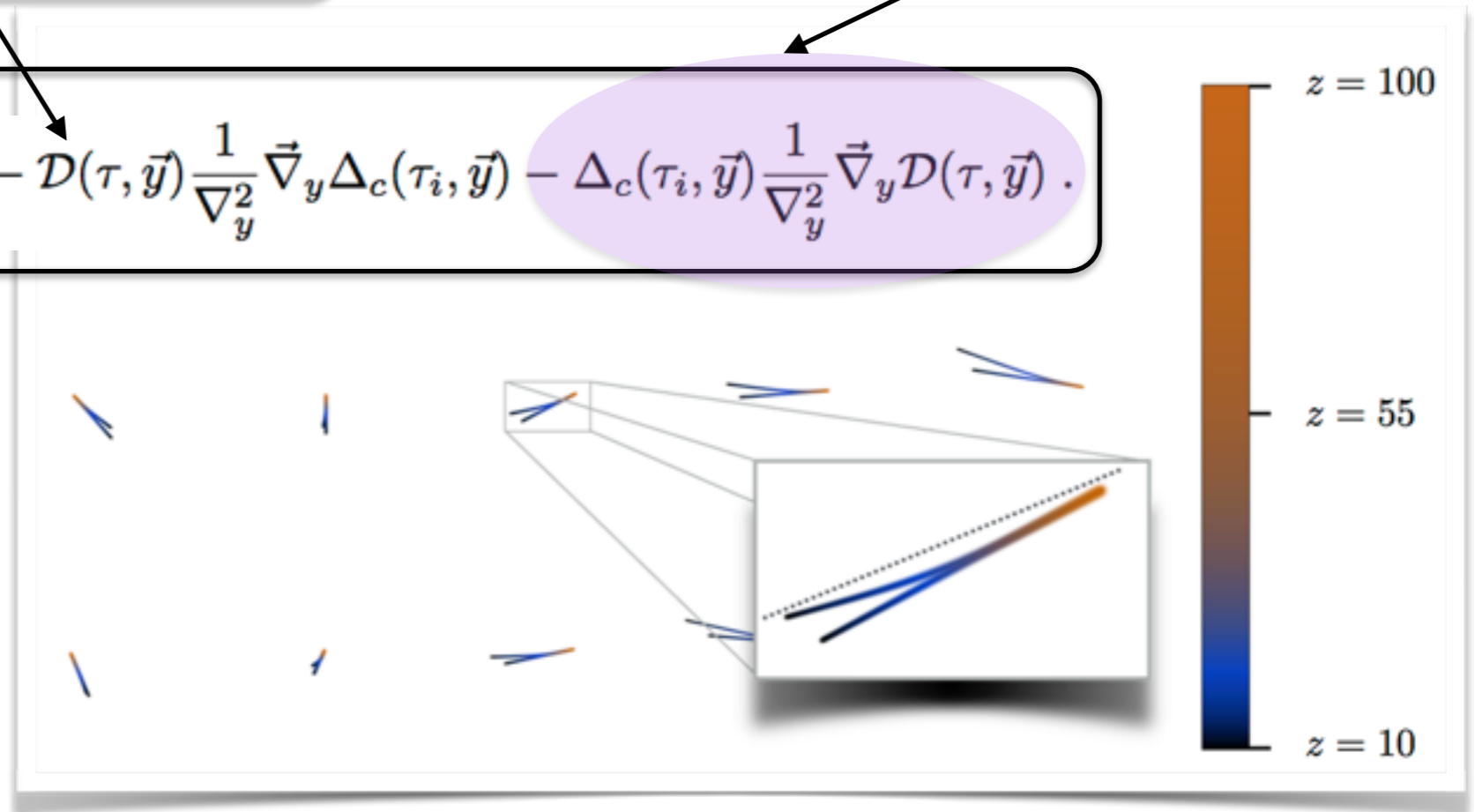
The displacement field $\vec{x} = \vec{y} - \mathcal{D}(\tau) \frac{1}{\nabla_y^2} \vec{\nabla}_y \Delta_c(\tau_i, \vec{y})$

Beyond Zeldovich Approximation

DE/MG: at linear regime
growth rate of CDM
depends on the scales!

Deflection by the
gravitational potential

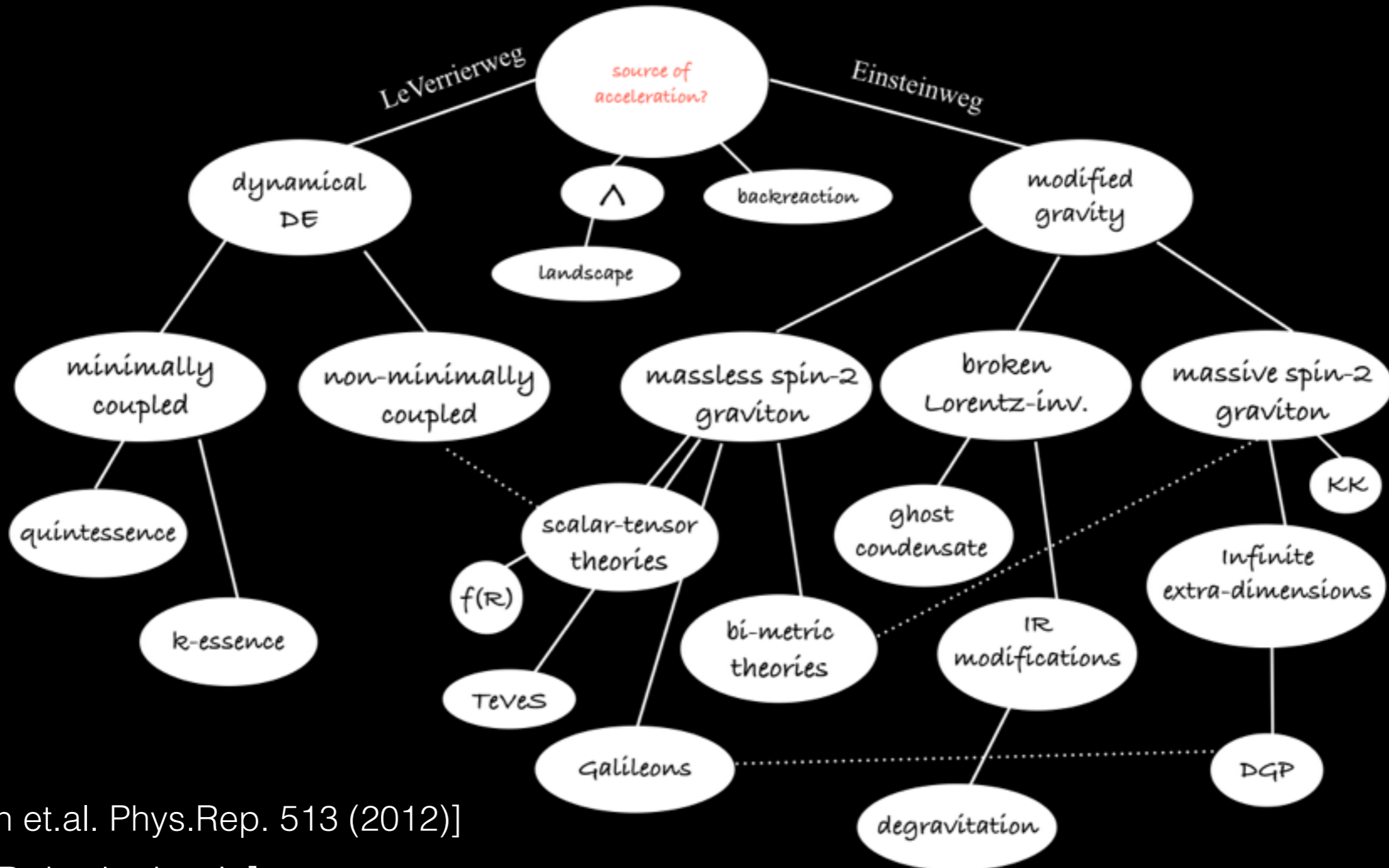
$$\vec{x} = \vec{y} - \mathcal{D}(\tau, \vec{y}) \frac{1}{\nabla_y^2} \vec{\nabla}_y \Delta_c(\tau_i, \vec{y}) - \Delta_c(\tau_i, \vec{y}) \frac{1}{\nabla_y^2} \vec{\nabla}_y \mathcal{D}(\tau, \vec{y}) .$$



Even at linear regime,
trajectory of CDM particles are **curved**!

State-of-the-art of DE/MG models

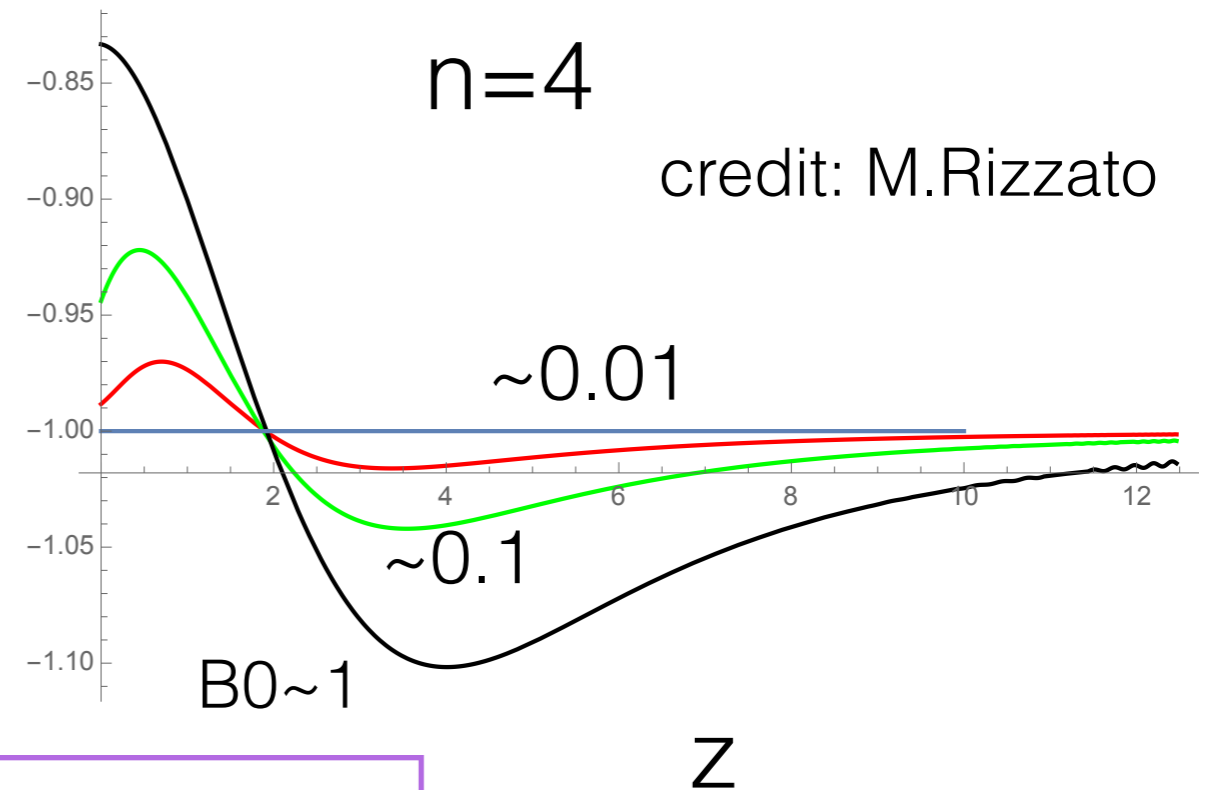
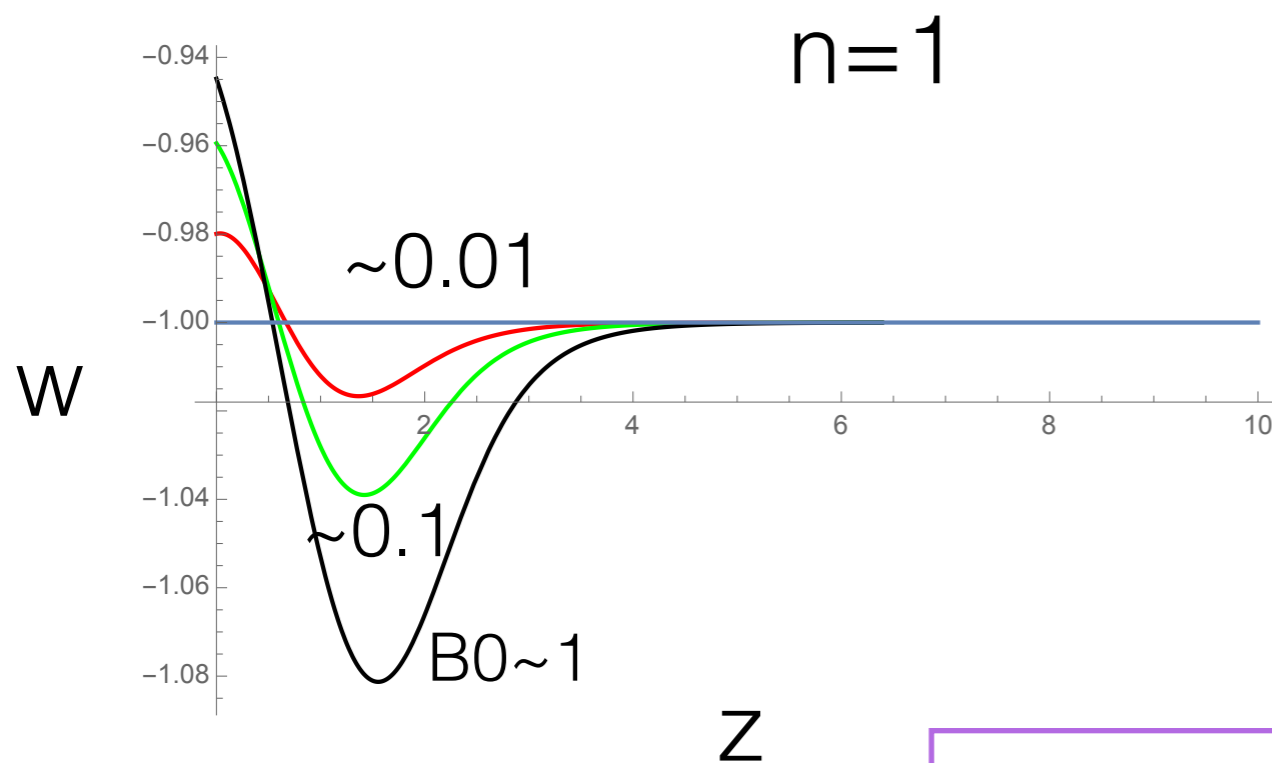
pic credit: A. Silvestri



[Clifton et.al. Phys.Rep. 513 (2012)]

[T. Baker's thesis]

Examples— $f(R)$ gravity



at most 10% effect!

Most of viable model gives very similar EoS!

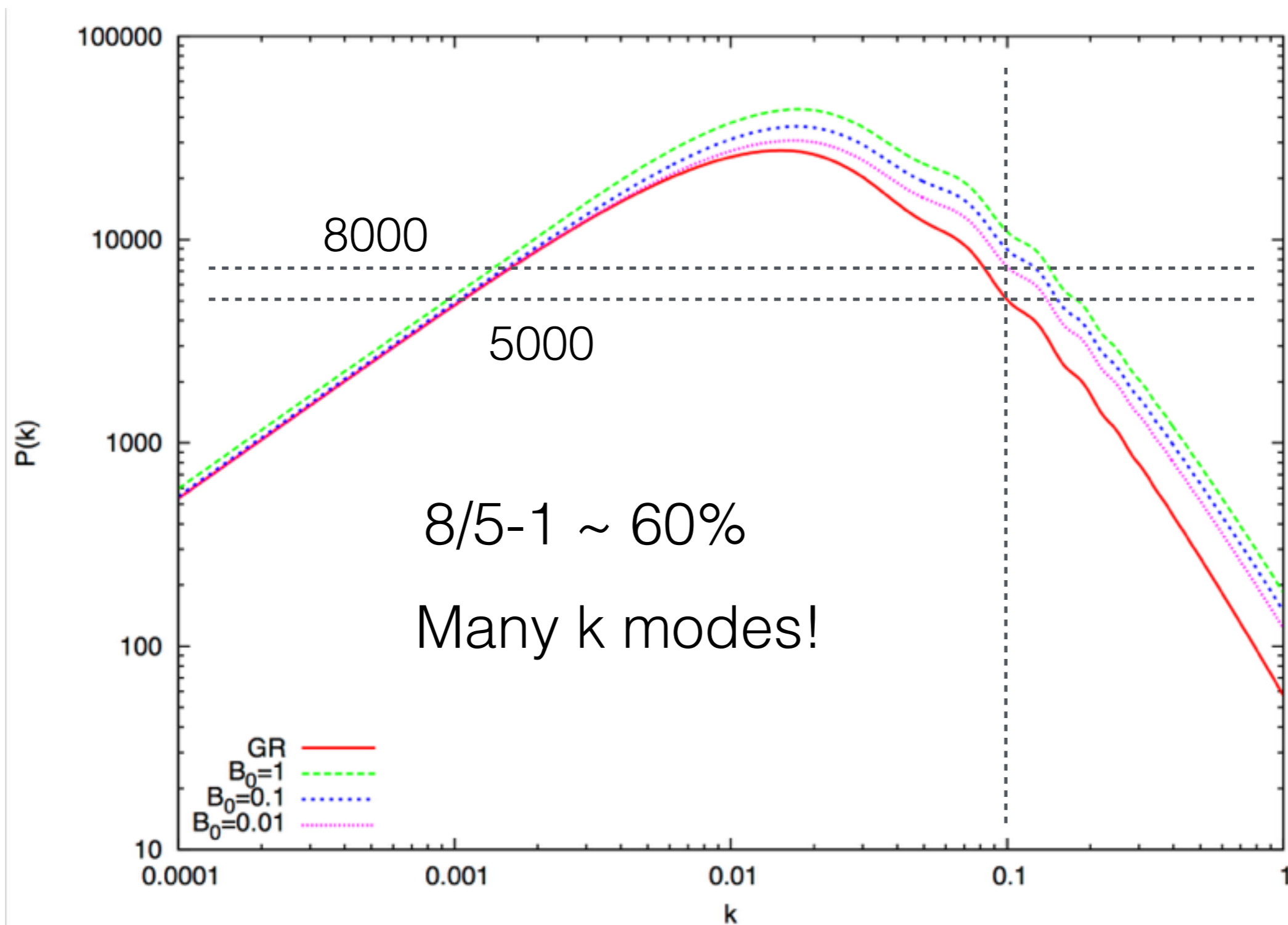
It is hard to distinguish them via only EoS for the on-going and up-coming surveys!

Need other observables to break the theoretical degeneracy!

[Hu, Sawicki, PRD **76**,
064004 (2007)]

$$f(R) = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1},$$

Matter power spectrum— A robust probe!



Take home message: Compared with background probe, we should consider perturbation dynamics!

3. Effective Field Theory of DE/MG

- EFT provides a **unified parametrisation** of the scalar field perturbations in **single** scalar field DE/MG **given background evolution**.

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} [1 + \Omega(\tau)] R + \Lambda(\tau) - a^2 c(\tau) \delta g^{00} + \frac{M_2^4(\tau)}{2} (a^2 \delta g^{00})^2 - \frac{\bar{M}_1^3(\tau)}{2} a^2 \delta g^{00} \delta K^\mu{}_\mu - \frac{\bar{M}_2^2(\tau)}{2} (\delta K^\mu{}_\mu)^2 - \frac{\bar{M}_3^2(\tau)}{2} \delta K^\mu{}_\nu \delta K^\nu{}_\mu + \frac{a^2 \hat{M}^2(\tau)}{2} \delta g^{00} \delta R^{(3)} + m_2^2(\tau) (g^{\mu\nu} + n^\mu n^\nu) \partial_\mu (a^2 g^{00}) \partial_\nu (a^2 g^{00}) + \dots \right\} + S_m[\chi_i, g_{\mu\nu}],$$

[Bloomfield et. al. JCAP08(2013)010]

[Gubitosi et. al. JCAP 1302 (2013) 032]

* There are **7 independent** functions at linear level, EFT functions

* Ω , Λ and c relate with background operators, only one are independent

* EFT functions depend on **time** only

$$\mathcal{H}^2 = \frac{a^2}{3m_0^2(1+\Omega)}(\rho_m + 2c - \Lambda) - \mathcal{H}\frac{\dot{\Omega}}{1+\Omega},$$

$$\dot{\mathcal{H}} = -\frac{a^2}{6m_0^2(1+\Omega)}(\rho_m + 3P_m) - \frac{a^2(c + \Lambda)}{3m_0^2(1+\Omega)} - \frac{\ddot{\Omega}}{2(1+\Omega)},$$

$$c = -\frac{m_0^2\ddot{\Omega}}{2a^2} + \frac{m_0^2\mathcal{H}\dot{\Omega}}{a^2} + \frac{m_0^2(1+\Omega)}{a^2}(\mathcal{H}^2 - \dot{\mathcal{H}}) - \frac{1}{2}(\rho_m + P_m),$$

$$\Lambda = -\frac{m_0^2\ddot{\Omega}}{a^2} - \frac{m_0^2\mathcal{H}\dot{\Omega}}{a^2} - \frac{m_0^2(1+\Omega)}{a^2}(\mathcal{H}^2 + 2\dot{\mathcal{H}}) - P_m.$$

3.1 The logic of construction of the action

1. Choose the time coordinate (clock), by asking

$$\delta\varphi(t, \vec{x}) \equiv \varphi(t, \vec{x}) - \bar{\varphi}(t) = 0$$

(breaking time translation
diffeomorphism)

2. Build the block of the action by the operators which keep the unbroken 3D spatial Diffs

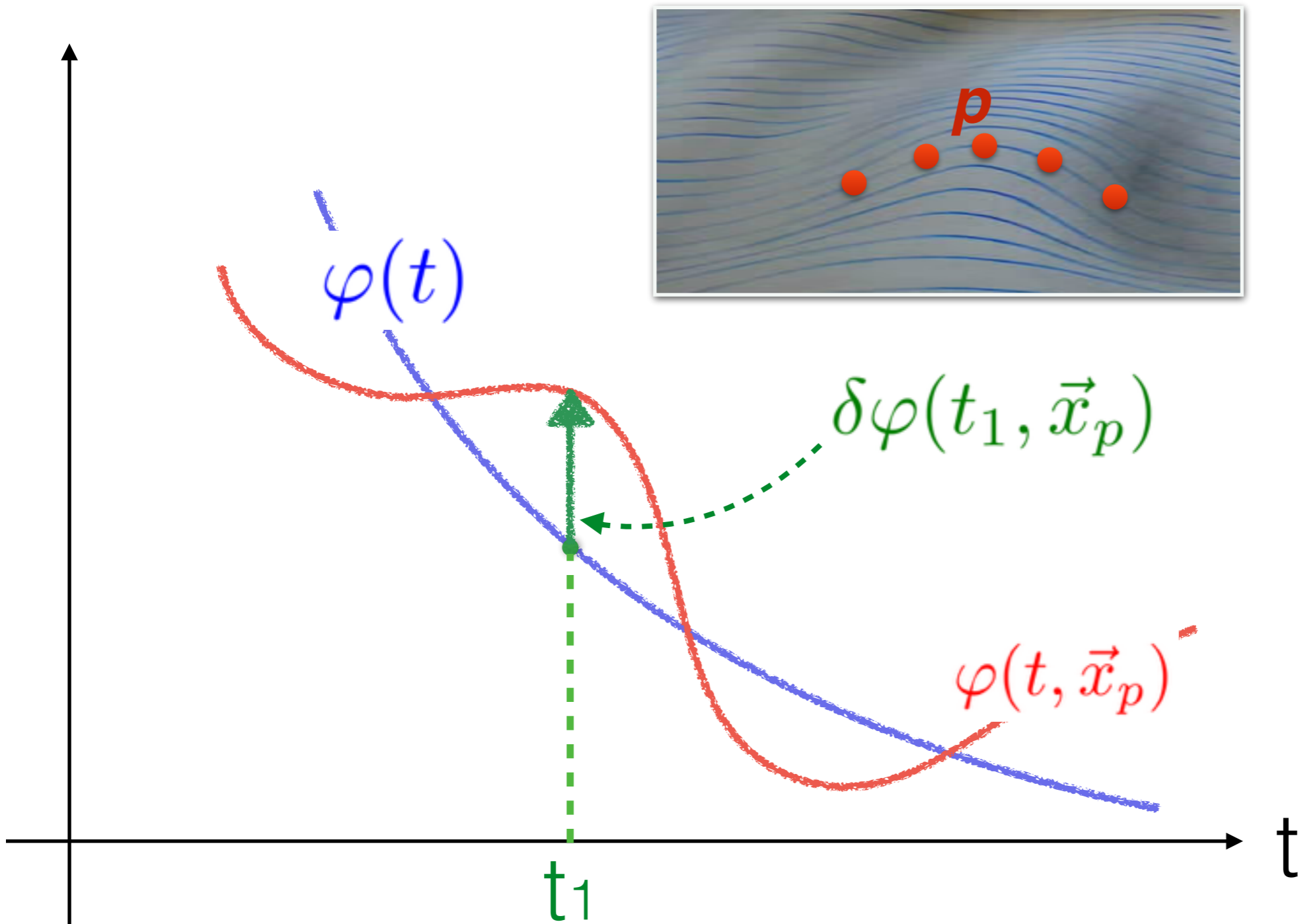
$$\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\rho\sigma} \text{ (or } C_{\mu\nu\rho\sigma}), \delta R_{\mu\nu}, \text{ and } \delta R,$$

3. Multiply these operators by a only time dependent function

$$\begin{aligned}
S = & \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} [1 + \Omega(\tau)] R + \Lambda(\tau) - a^2 c(\tau) \delta g^{00} \right. \\
& + \frac{M_2^4(\tau)}{2} (a^2 \delta g^{00})^2 - \frac{\bar{M}_1^3(\tau)}{2} a^2 \delta g^{00} \delta K^\mu{}_\mu - \frac{\bar{M}_2^2(\tau)}{2} (\delta K^\mu{}_\mu)^2 \\
& - \frac{\bar{M}_3^2(\tau)}{2} \delta K^\mu{}_\nu \delta K^\nu{}_\mu + \frac{a^2 \hat{M}^2(\tau)}{2} \delta g^{00} \delta R^{(3)} \\
& \left. + m_2^2(\tau) (g^{\mu\nu} + n^\mu n^\nu) \partial_\mu (a^2 g^{00}) \partial_\nu (a^2 g^{00}) + \dots \right\} \\
& + S_m[\chi_i, g_{\mu\nu}], \tag{1}
\end{aligned}$$

How we know EFT
approach is equivalent
to the Covariant approach?

Covariant approach



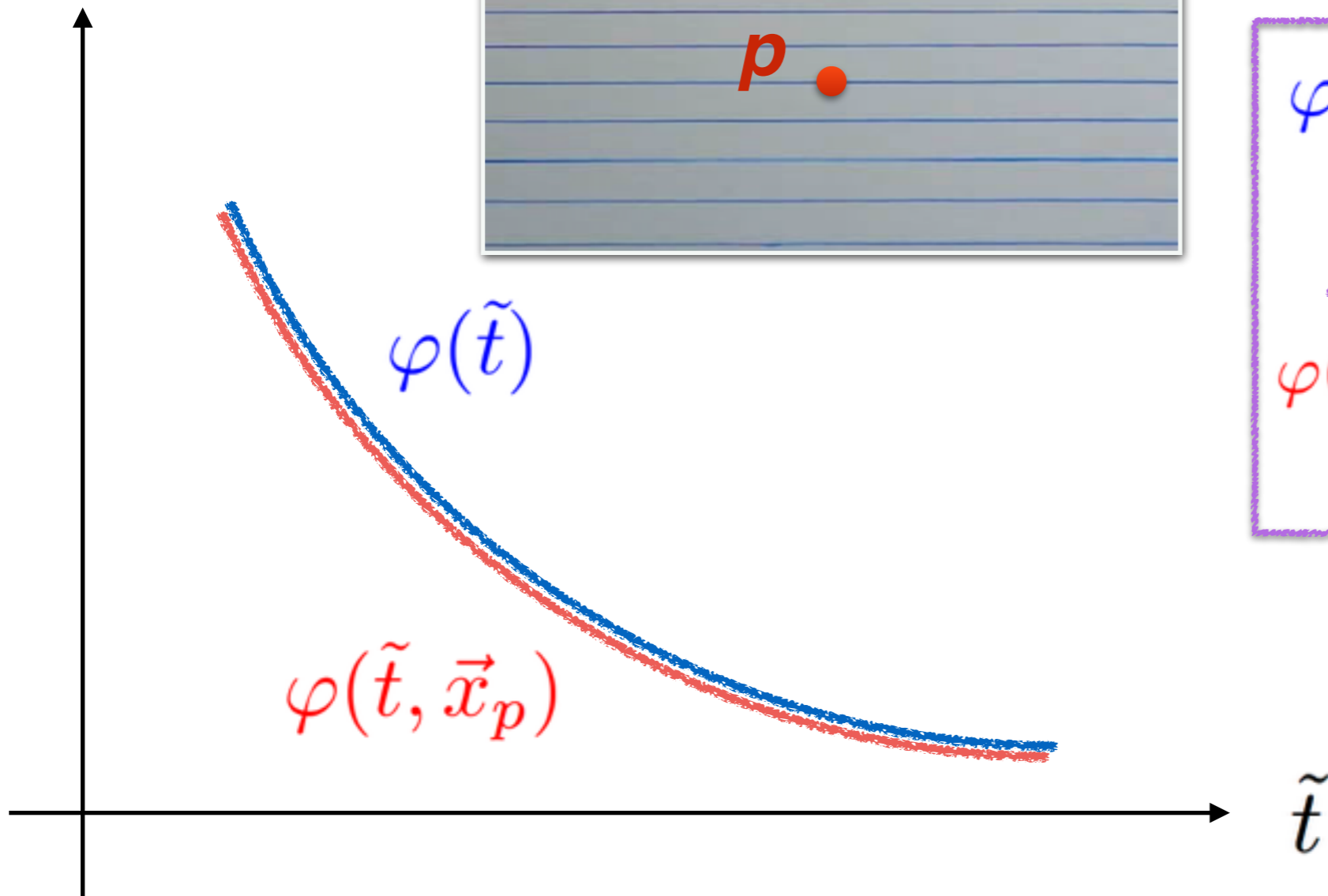
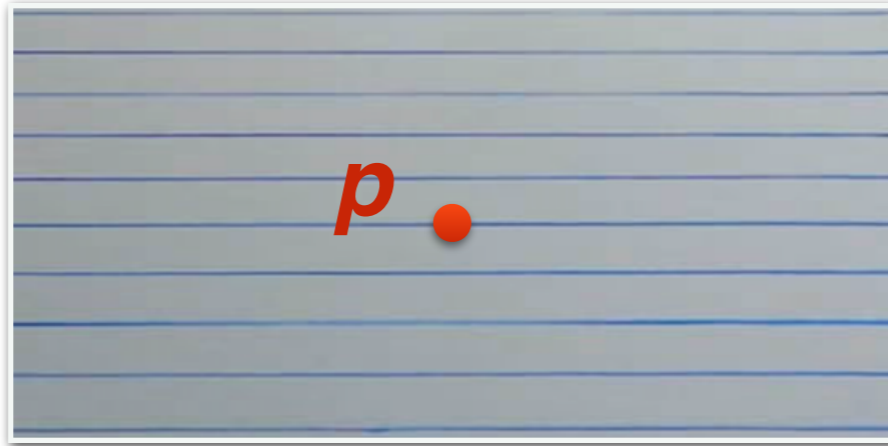
$\varphi(t)$: background
field config

$\varphi(t, \vec{x}_p)$: field config
at point ' p '

$\delta\varphi(t_1, \vec{x}_p)$: field fluct.
at point ' p '

Valid in **ALL** the gauge

EFT approach



$\varphi(\tilde{t})$: background field config

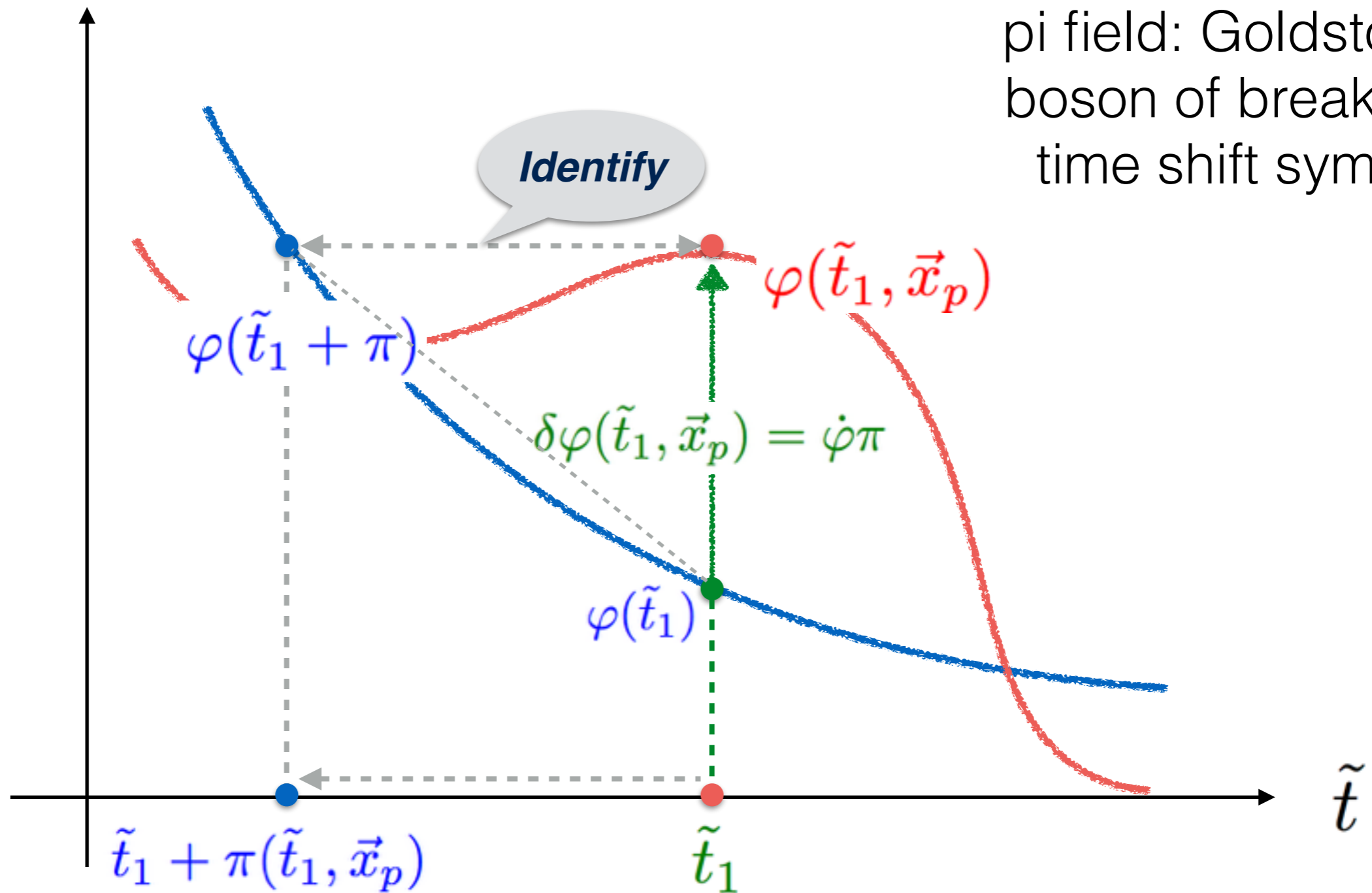
$\varphi(\tilde{t}, \vec{x}_p)$: field config at point 'p'

Only Valid in the unitary gauge

$$\delta\varphi(t, \vec{x}) \equiv \varphi(t, \vec{x}) - \bar{\varphi}(t) = 0$$

EFT approach \Rightarrow Covariant approach

π field: Goldstone boson of breaking time shift symm



Stuckburg trick: restore full covariance

2.3 Parametrizations

1. Full mapping

(From the covariant form)

e.g.

$$f(R) = -m^2 \frac{c_1(R/m^2)^n}{c_2(R/m^2)^n + 1},$$

[Hu,Sawicki PRD76, 064004 (2007)]

$$\Lambda = \frac{m_0^2}{2} [f - Rf_R] \quad ; \quad c = 0 \quad ; \quad \Omega = f_R$$

(Work in progress with Rizzato et. al.)

2. Pure EFT parametrization

(Phenomenological param)

Constant models: $\Omega(a) = \Omega_0$;

Linear models: $\Omega(a) = \Omega_0 a$;

Power law models: $\Omega(a) = \Omega_0 a^s$;

Exponential models: $\Omega(a) = \exp(\Omega_0 a^s) - 1$.

**Have to make sure
that your parametrisation
to be viable, e.g. ghost-free!**

3. The structure of EFTCAMB

We implement the pi field into the Einstein-Boltzmann solver
CAMB → **EFTCAMB**

Evolving the full **Einstein** equation, **Klein-Golden** equation (pi field), **fluid equation** (CDM, baryon, massive neutrino), **Boltzmann hierarchy** equation sets (CMB, massless neutrino)

Effective **F**ield **T**heory with **CAMB**



[Hu et.al. [PRD89,103530\(2014\)](#); [PRD90,043513\(2014\)](#); [PRD91,063524\(2015\)](#)]

<http://wwhome.lorentz.leidenuniv.nl/~hu/codes/>

<http://wwwhome.lorentz.leidenuniv.nl/~hu/codes/>

The screenshot shows a Safari browser window with the URL wwwhome.lorentz.leidenuniv.nl. The browser's address bar and menu bar are visible. The website content includes the title "Effective Field Theory with CAMB" and the authors "By B. Hu, M. Raveri, N. Frusciante and A. Silvestri". Below the text are three scatter plots:

- Left plot: $\text{Log}_{10}(R_0)$ vs Ω_m . The y-axis ranges from -6.0 to -1.5, and the x-axis ranges from 0.270 to 0.330. A color bar on the right indicates values from -1.000 to -0.991.
- Middle plot: Ω_m vs H_0 . The y-axis ranges from 0.27 to 0.36, and the x-axis ranges from 60.0 to 70.0. A color bar on the right indicates values from 0.06 to 0.13.
- Right plot: w_{EFT} vs Ω_m . The y-axis ranges from -1.04 to -0.80, and the x-axis ranges from 0.275 to 0.350. A color bar on the right indicates values from 0.000 to 0.105.

Below the plots, a paragraph of text reads: "EFTCAMB is a patch of the public Einstein-Boltzmann solver CAMB, which implements the Effective Field Theory approach to cosmic acceleration. The code can be used to investigate the effect of different EFT operators on linear perturbations as well as to study perturbations in any specific DE/MG model that can be cast into EFT framework. To interface EFTCAMB with cosmological data sets, we equipped it with a modified version of CosmoMC, namely EFTCosmoMC, creating a bridge between the EFT parametrization of the dynamics of perturbations and observations."

The bottom of the image shows a Mac OS X dock with various application icons.

[Archive](#) | [Notes](#) | [Full Map Key](#)



clicked 1500+ times

EFTCAMB STRUCTURE
(Main EFT flag: **EFTflag**)

0: GR code
Standard CAMB

1: pure EFT
Use some parametrized forms for the EFT functions

2: designer matching EFT
Use a theory whose background mimics exactly the one specified

Background DE equation of state:
(Flag: **EFTwDE**)

Pure EFT Ω model selection:
(Flag: **PureEFTmodelOmega**)

Pure EFT α_1 model selection:
(Flag: **PureEFTmodelAlpha1**)

Pure EFT α_2 model selection:
(Flag: **PureEFTmodelAlpha2**)

Pure EFT α_3 model selection:
(Flag: **PureEFTmodelAlpha3**)

Pure EFT α_4 model selection:
(Flag: **PureEFTmodelAlpha4**)

Pure EFT α_5 model selection:
(Flag: **PureEFTmodelAlpha5**)

Pure EFT α_6 model selection:
(Flag: **PureEFTmodelAlpha6**)

Designer EFT model selection:
(Flag: **DesignerEFTmodel**)

Background DE equation of state:
(Flag: **EFTwDE**)

- 0: LCDM
- 1: wCDM
- 2: CPL
- 3: JBP
- 4: Turning point
- 5: Taylor expansion
- 6: User defined

- 0: Zero
- 1: Constant
- 2: Linear model
- 3: Power law model
- 4: Exponential model
- 5: User defined

- 1: $f(R)$
- 2: minimally coupled quintessence
- 3: non-minimally coupled quintessence
- 4: k-essence
- 5: Horndeski
- 6: Brans-Dicke
- 7: ...

- 0: LCDM
- 1: wCDM
- 2: CPL
- 3: JBP
- 4: Turning point
- 5: Taylor expansion
- 6: User defined

EFTCAMB_V1.1

Structure of EFTCAMB

2.1 Background parametrization—EoS

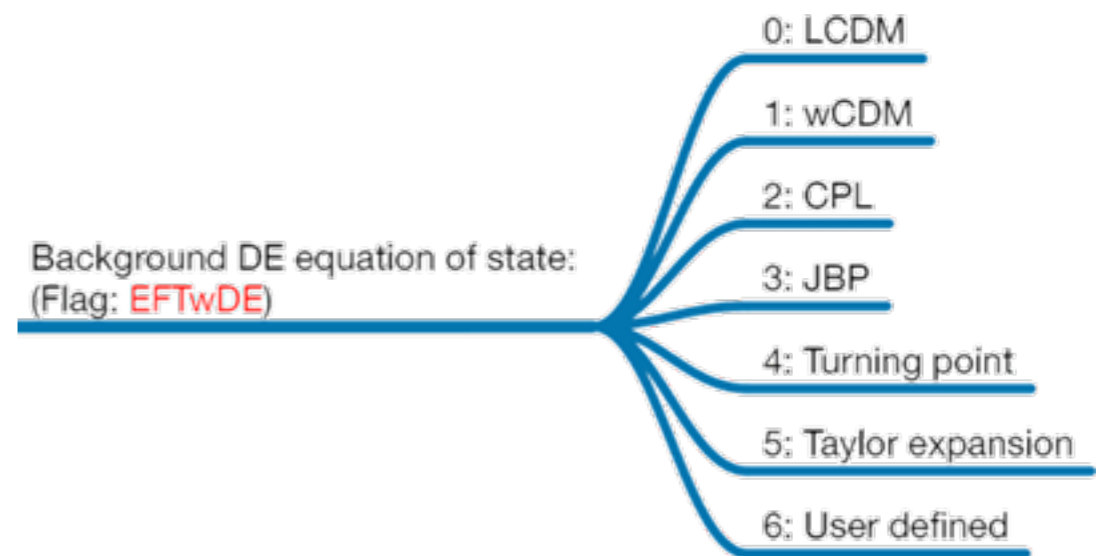
EFTCAMB provides 6 different kinds of parametrization of EoS (Flag: **EFTwDE**), including:

ΛCDM ($w=-1$),

wCDM ($w=w_0$),

CPL ($w=w_0+w_a \cdot a$),

.....



2.2.1 EFT parametrization: Pure EFT

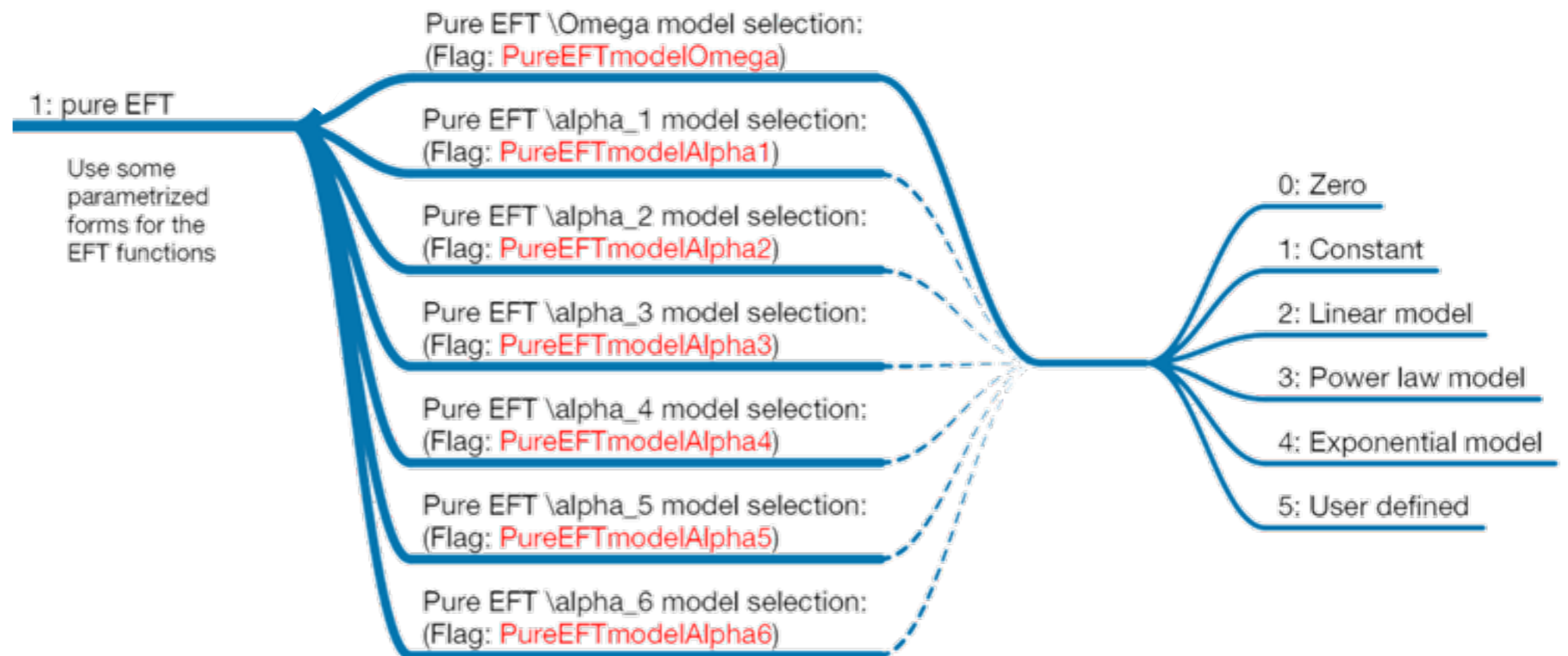
Phenomenological parametrization, e.g.

Constant models: $\Omega(a) = \Omega_0$;

Linear models: $\Omega(a) = \Omega_0 a$;

Power law models: $\Omega(a) = \Omega_0 a^s$;

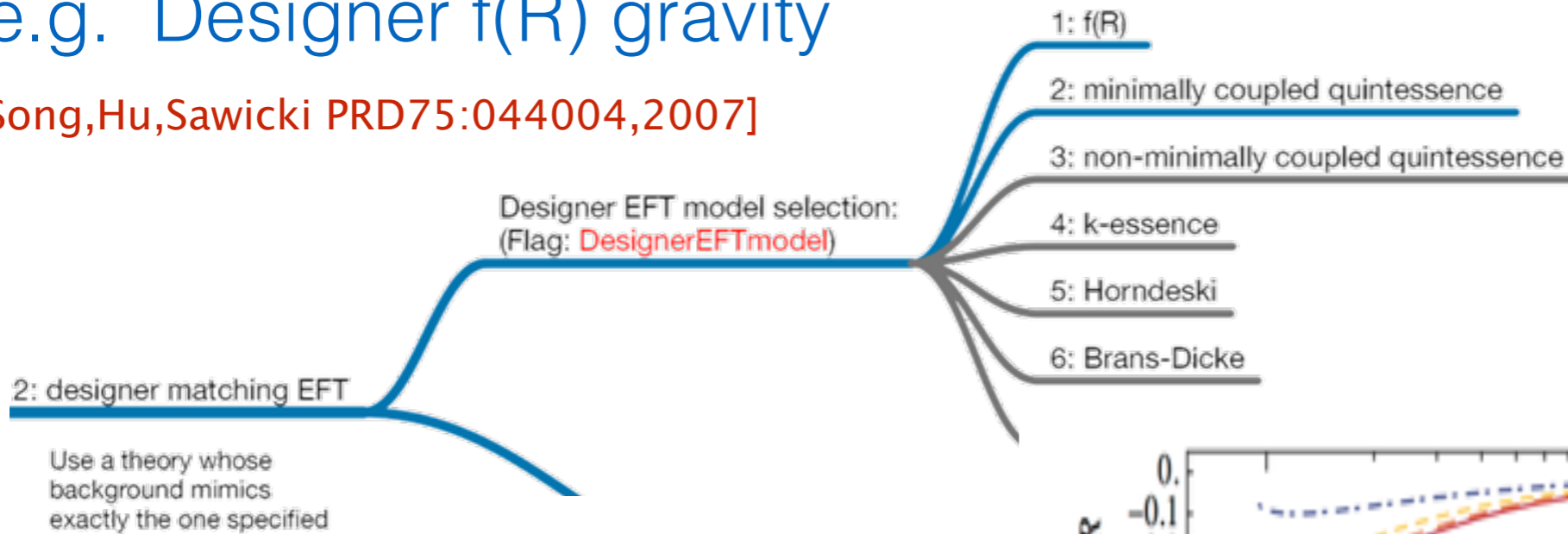
Exponential models: $\Omega(a) = \exp(\Omega_0 a^s) - 1$.



2.2.2 EFT parametrization: Full mapping—designer mapping

e.g. Designer $f(R)$ gravity

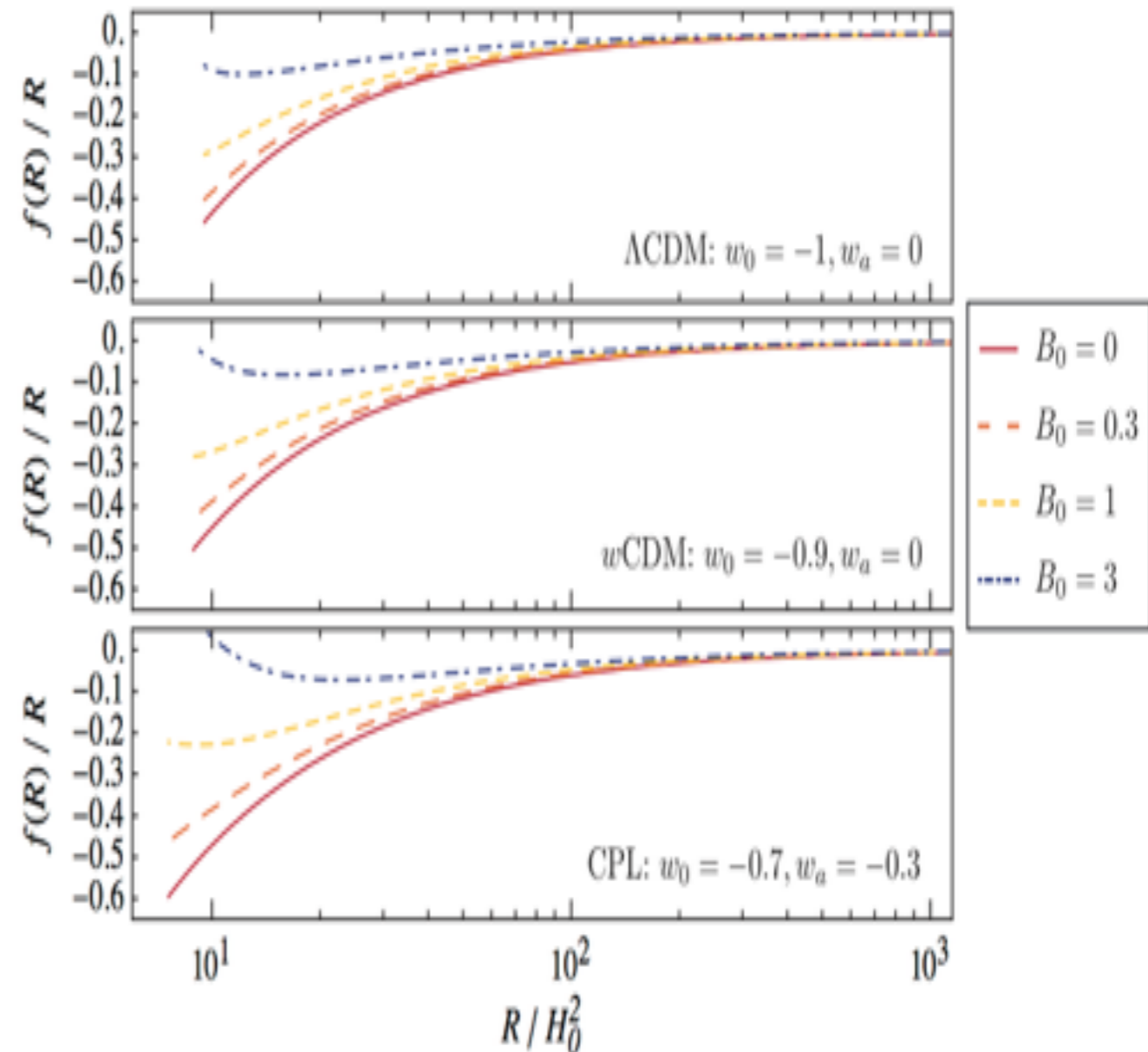
[Song, Hu, Sawicki PRD75:044004, 2007]



$$f'' - \left(1 + \frac{H'}{H} + \frac{R''}{R'}\right) f' + \frac{R'}{6H^2} f = -\frac{R'}{3M_P^2 H^2} \rho_{\text{DE}},$$

$$B_0 \sim \frac{6f_{RR}}{(1+f_R)} H^2 \Big|_{a=1}$$

GR limit: $B_0 \rightarrow 0$,
effective mass $\rightarrow \infty$



- **EFT: Do NOT rely on QS approx!**

time-time Einstein equation:

$$k^2 \eta = -\frac{a^2}{2m_0^2(1+\Omega)} [\delta\rho_m + \dot{\rho}_Q \pi + 2c(\dot{\pi} + \mathcal{H}\pi)] + \left(\mathcal{H} + \frac{\dot{\Omega}}{2(1+\Omega)} \right) k\mathcal{Z} + \frac{\dot{\Omega}}{2(1+\Omega)} [3(3\mathcal{H}^2 - \dot{\mathcal{H}})\pi + 3\mathcal{H}\dot{\pi} + k^2\pi]$$

momentum Einstein equation:

$$\frac{2}{3}k^2(\sigma_* - \mathcal{Z}) = \frac{a^2}{m_0^2(1+\Omega)} [(\rho_m + P_m)v_m + (\rho_Q + P_Q)k\pi] + k\frac{\dot{\Omega}}{(1+\Omega)}(\dot{\pi} + \mathcal{H}\pi),$$

space-space off-diagonal Einstein equation:

$$k\dot{\sigma}_* + 2k\mathcal{H}\sigma_* - k^2\eta = -\frac{a^2 P\Pi_m}{m_0^2(1+\Omega)} - \frac{\dot{\Omega}}{(1+\Omega)}(k\sigma_* + k^2\pi),$$

space-space trace Einstein equation:

$$\begin{aligned} \ddot{h} = & -\frac{3a^2}{m_0^2(1+\Omega)} [\delta P_m + \dot{P}_Q \pi + (\rho_Q + P_Q)(\dot{\pi} + \mathcal{H}\pi)] - 2\left(\frac{\dot{\Omega}}{1+\Omega} + 2\mathcal{H}\right) k\mathcal{Z} + 2k^2\eta \\ & - 3\frac{\dot{\Omega}}{(1+\Omega)} \left[\ddot{\pi} + \left(\frac{\ddot{\Omega}}{\dot{\Omega}} + 3\mathcal{H}\right) \dot{\pi} + \left(\mathcal{H}\frac{\ddot{\Omega}}{\dot{\Omega}} + 5\mathcal{H}^2 + \dot{\mathcal{H}} + \frac{2}{3}k^2\right) \pi \right], \end{aligned}$$

- For Klein-Golden Eq. Of π field

$$\begin{aligned} & \left(c + \frac{3m_0^2}{4a^2} \frac{\dot{\Omega}^2}{(1+\Omega)} \right) \ddot{\pi} + \left[\frac{3m_0^2}{4a^2} \frac{\dot{\Omega}}{(1+\Omega)} \left(\ddot{\Omega} + 4\mathcal{H}\dot{\Omega} + \frac{(\rho_Q + P_Q)a^2}{m_0^2} \right) + \dot{c} + 4\mathcal{H}c - \frac{\dot{\Omega}}{2(1+\Omega)}c \right] \dot{\pi} \\ & + \left[\frac{3m_0^2}{4a^2} \frac{\dot{\Omega}}{(1+\Omega)} \left(\frac{(3\dot{P}_Q - \dot{\rho}_Q + 3\mathcal{H}(\rho_Q + P_Q))a^2}{3m_0^2} + \mathcal{H}\ddot{\Omega} + 8\mathcal{H}^2\dot{\Omega} + 2(1+\Omega)(\ddot{\mathcal{H}} - 2\mathcal{H}^3) \right) \right. \\ & \left. - 2\dot{\mathcal{H}}c + \left(\dot{c} - \frac{\dot{\Omega}}{2(1+\Omega)}c \right) \mathcal{H} + 6\mathcal{H}^2c + \left(c + \frac{3m_0^2}{4a^2} \frac{\dot{\Omega}^2}{(1+\Omega)} \right) k^2 \right] \pi \\ & + \left[c + \frac{3m_0^2}{4a^2} \frac{\dot{\Omega}^2}{(1+\Omega)} \right] k\mathcal{Z} + \frac{1}{4} \frac{\dot{\Omega}}{(1+\Omega)} (3\delta P_m - \delta\rho_m) = 0, \end{aligned}$$

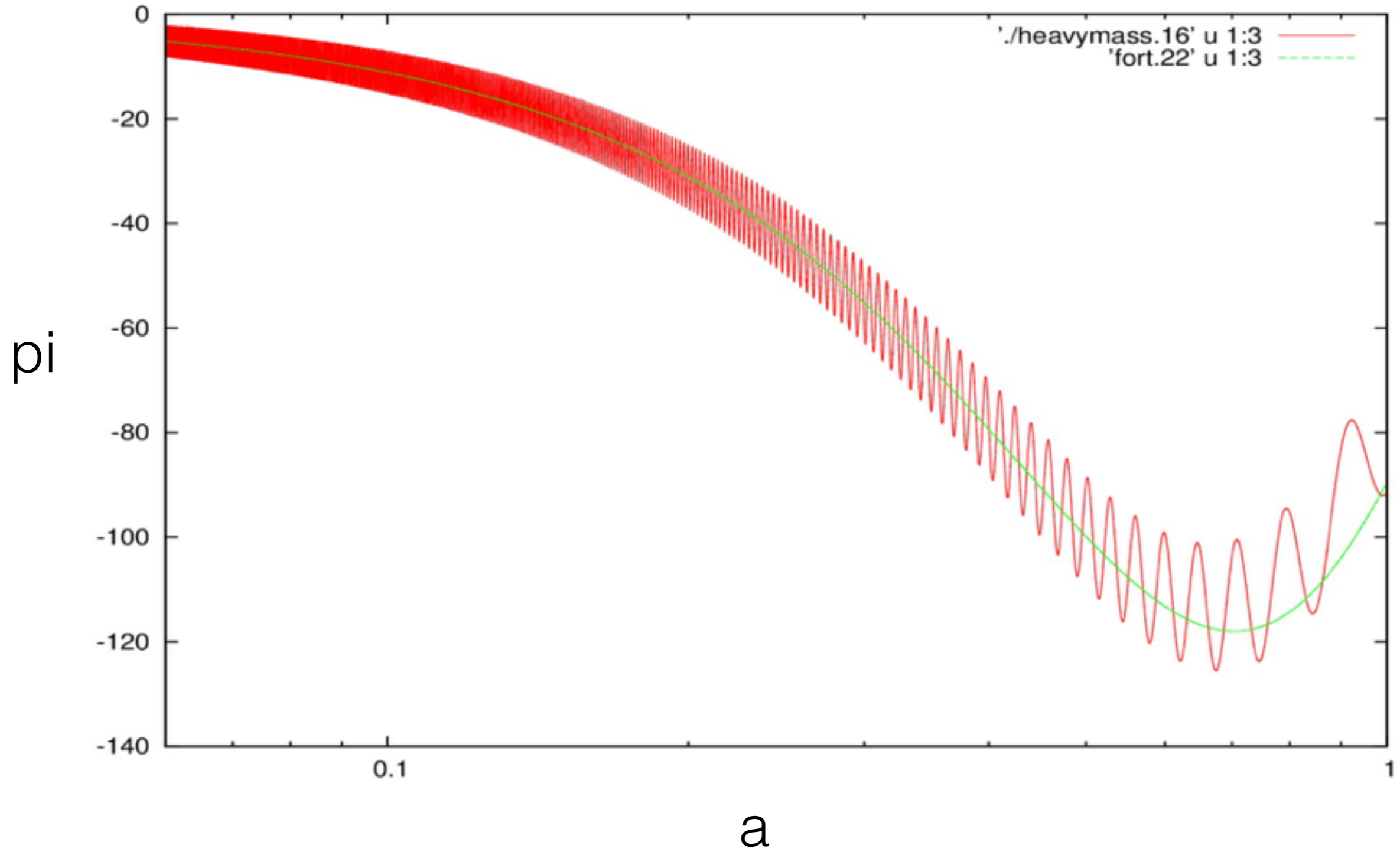
kinetic friction mass sound speed source

$$A(\tau) \ddot{\pi} + B(\tau) \dot{\pi} + C(\tau) \pi + k^2 D(\tau) \pi + E(\tau) = 0$$

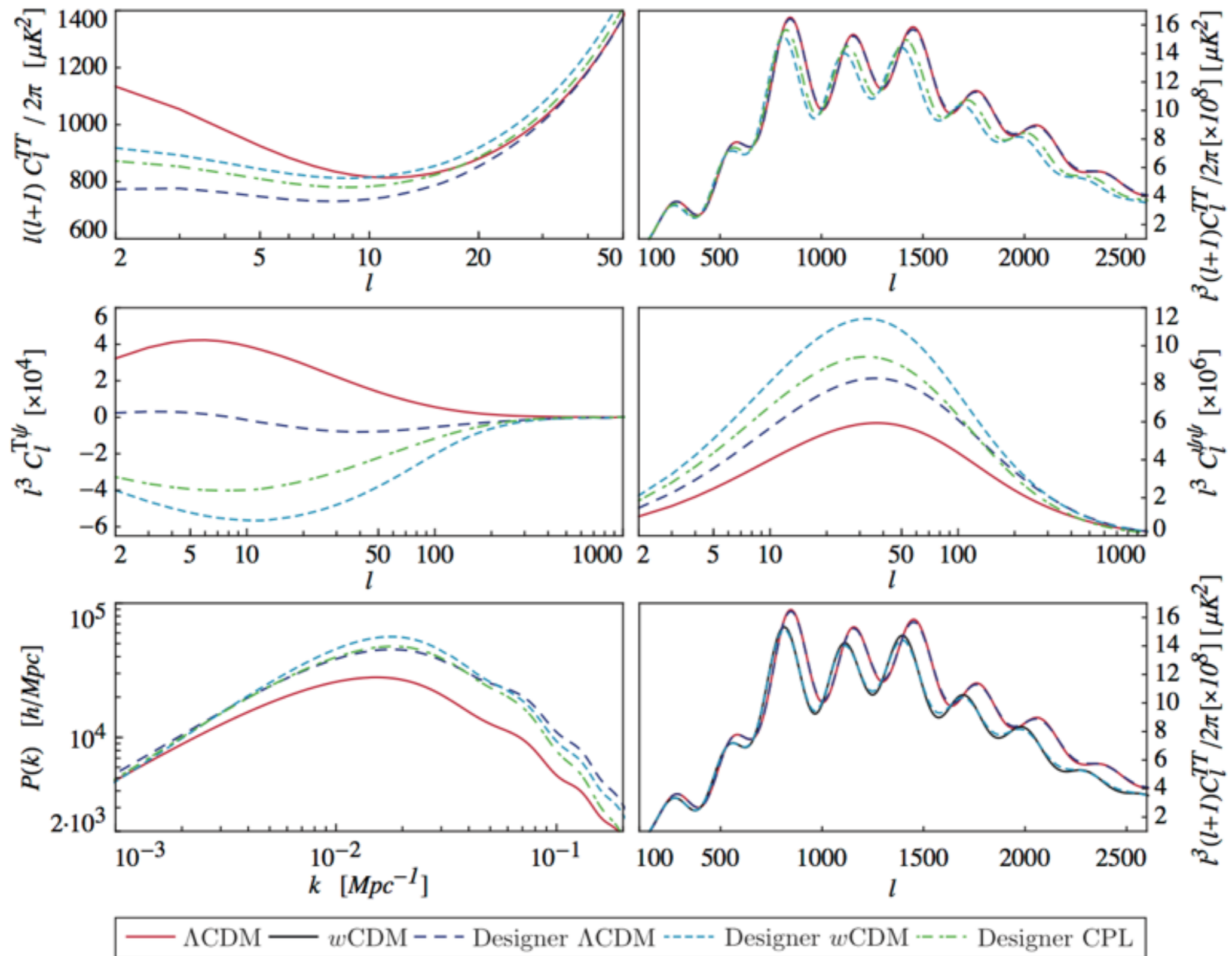
Have pass the viability condition:

1. Effective Newton constant does not change sign: $1+\Omega > 0$
2. ghost instability: $A > 0$
3. sound speed ≤ 1 : $D/A \leq 1$
4. mass square ≥ 0 : $C/A \geq 0$

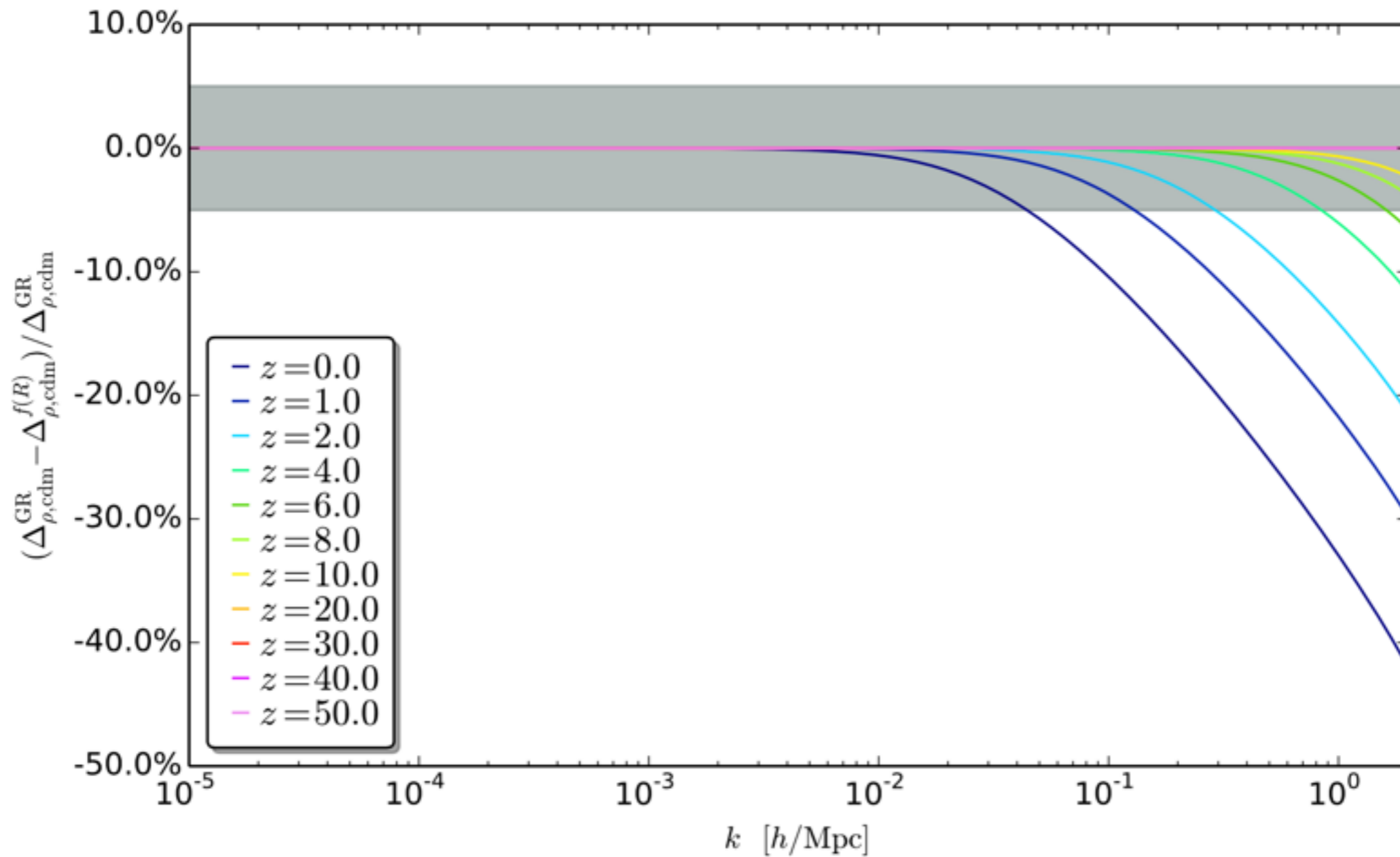
pi field solution: f(R) example



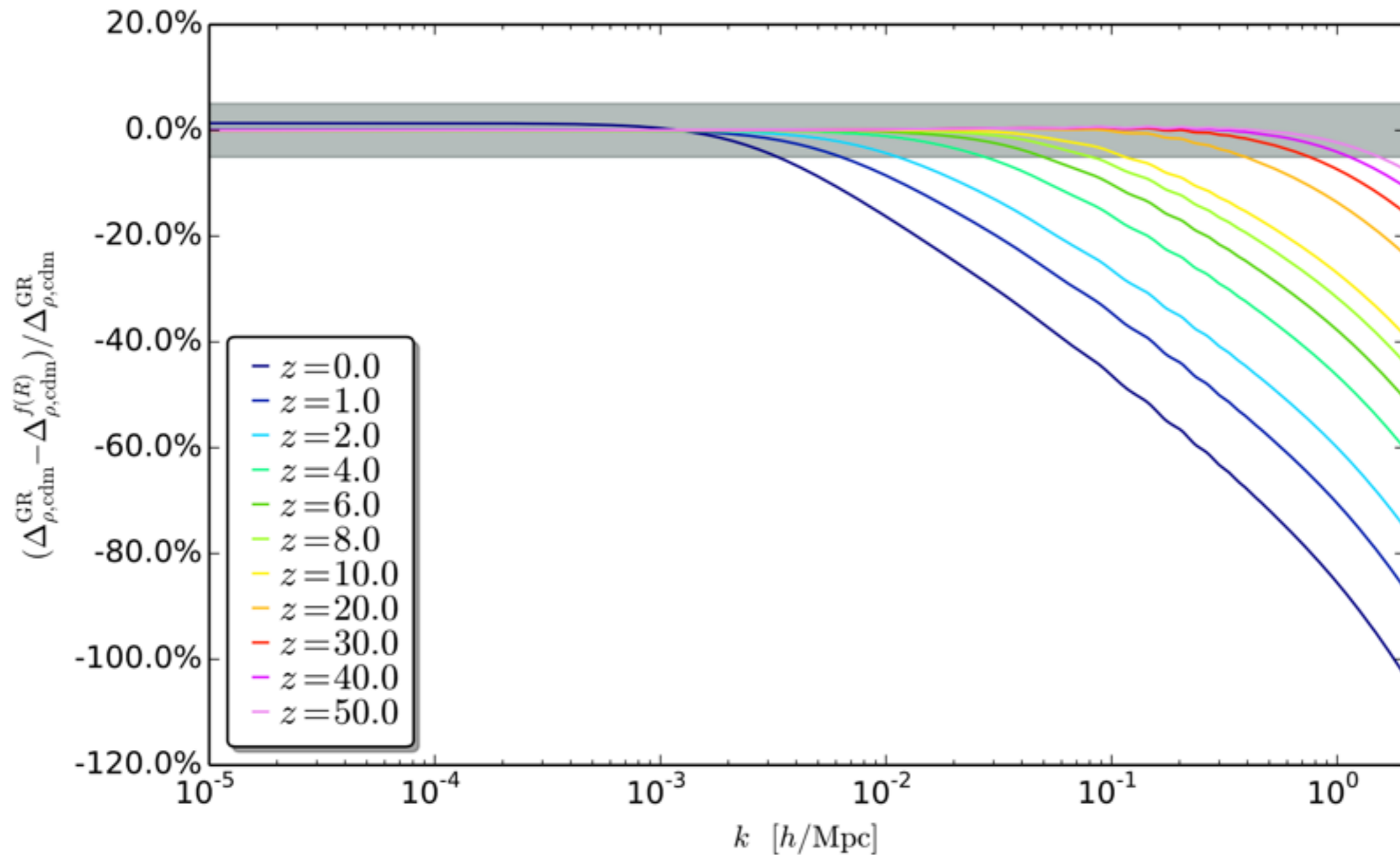
2.4 CMB spectra—example: f(R)



2.5 Transfer function of CDM



Designer $f(R)$ with LCDM background
 $B_0 = 0.001$

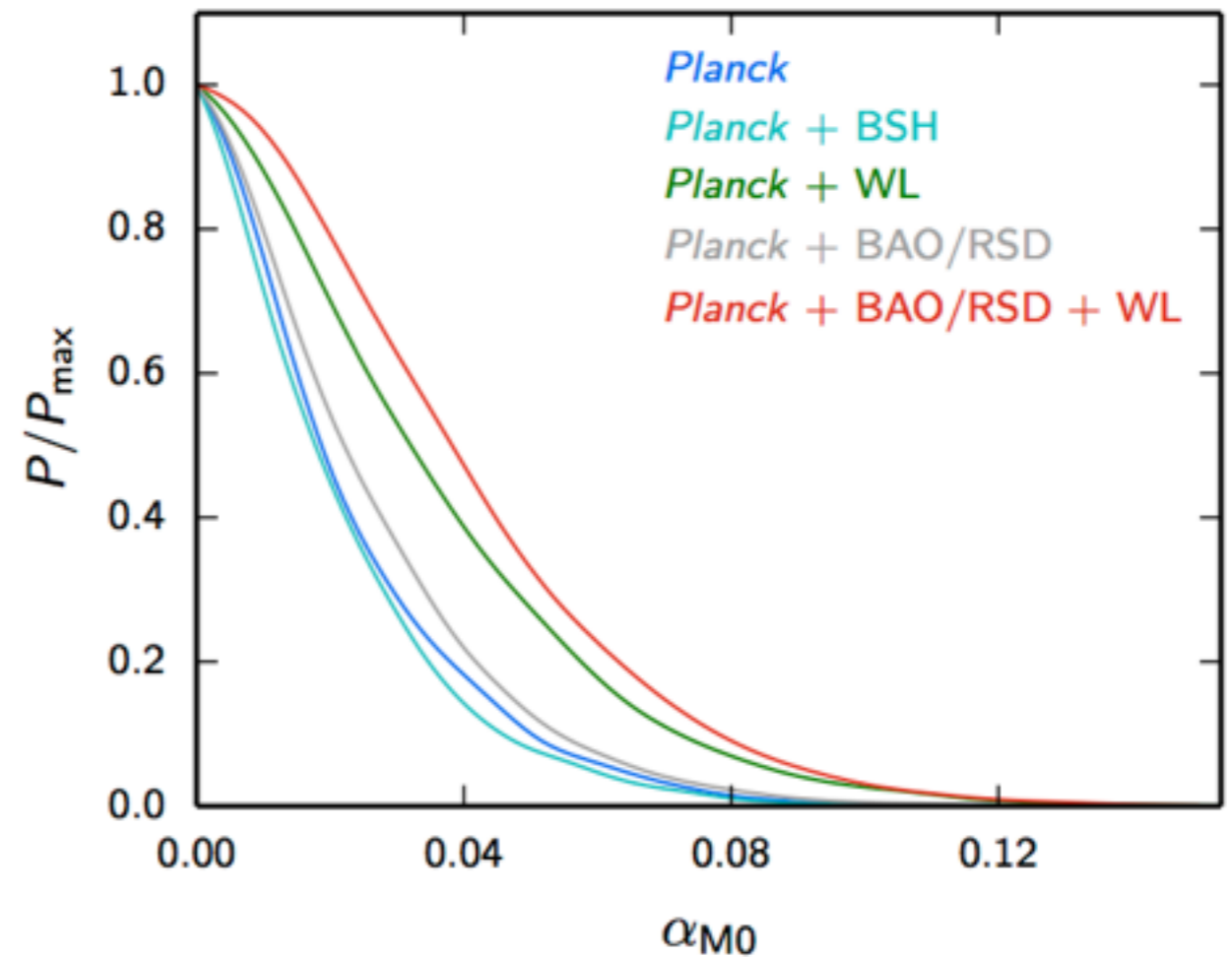
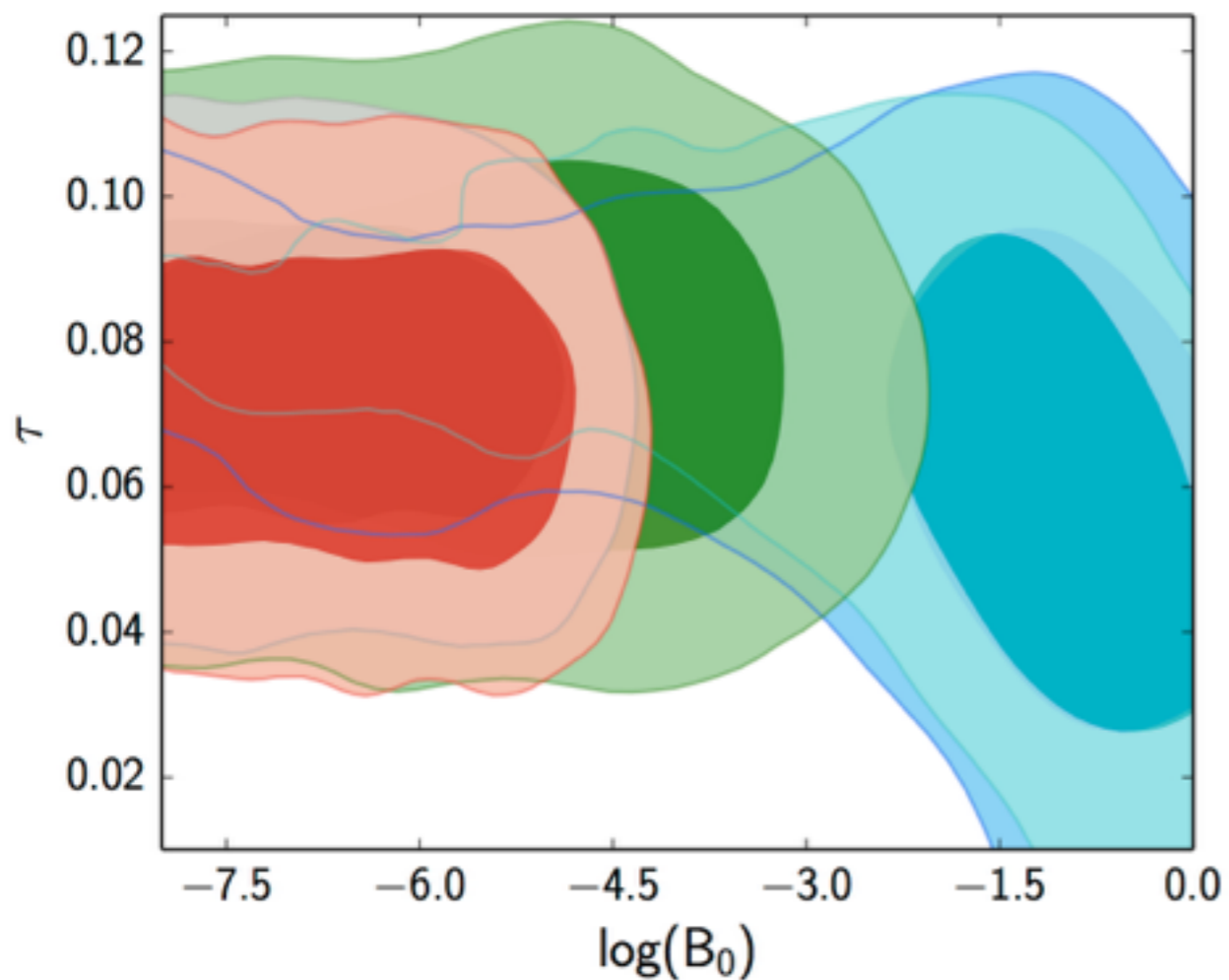


Designer $f(R)$ with $w\text{CDM}$ background
 $B_0=0.01$ and $w=-0.95$

4. Parameter estimation results from EFTCosmoMC and Planck-2015

CosmoMC → **EFTCosmoMC**

Designer $f(R)$



Linear EFT

$$\bar{\Omega}(a) = \alpha_{M0} a.$$

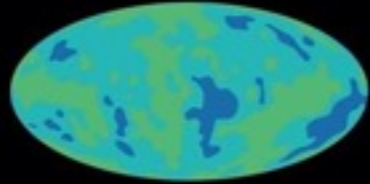
[Planck-2015, MG paper]

5. Conclusion

- **EFTCAMB include most of viable **single** field DE/MG model**
- **For scalar field: full perturbative treatment, does not rely on quasistatic approx**
- **Support various background, LCDM/wCDM/CPL ...**
- **Check the stability for given parameterization**
- **Selected by Planck 2015 data release**
- **Selected by Theory Working Group of Euclid**
- **New release will come soon updated with PLC2.0**

A graphic consisting of several thick, curved lines in shades of blue and green that fan out from the top left towards the center. Below these are many thinner, curved lines in a grid-like pattern, transitioning from blue to yellow to orange.

EFTCAMB



KEEP
CALM
AND
TEST
GRAVITY

the EFTCAMB team

Thank you!